electrical energy & capacitance

• today & tomorrow
• first: wrap up Gauss’ law
• rest of the week: circuits/current/resistance
• NEXT MON: exam I

multiple choice, cumulative
more details throughout the week
(a) $E = 0$

(b)
High PE

\[
\begin{align*}
\text{A} & \quad \Delta \vec{x} & \quad \text{B} \\
\text{x}_i & \quad \text{x}_f
\end{align*}
\]

\[\Delta x = x_f - x_i\]

\[
\begin{align*}
q & \quad \vec{E} & \quad q\vec{E}
\end{align*}
\]

Low PE
(a) 

- \( q \vec{E} \) 

\( -q \vec{E} \) 

\( \Delta x \) 

\( \vec{E} \) 

(b) 

\( m \vec{g} \) 

\( \vec{g} \) 

\( \Delta x \)
\[ E = k_e \frac{q}{r^2} \]

\[ V = k_e \frac{q}{r} \]
\[ V_2 = \frac{k_e q_2}{r_{12}} \]
\[ V_2 = \frac{k_e q_2}{r_{12}} \]

\[ V_1 = \frac{k_e q_1}{r_{12}} \]
\( V_2 = \frac{k_e q_2}{r_{12}} \)

\( P \)

\( P' \)

\( r_{12} \)

\( q_2 \)

\( q_1 \)

\( q_2 V_1 = q_1 V_2 \)

\( P E = \frac{k_e q_1 q_2}{r_{12}} \)
PE = (1 due to 2) + (2 due to 1)
(E to bring 1 close to 2)
(E to bring 2 close to 1)
\[ PE = \frac{k_e q_1 q_2}{r_{12}} = q_2 V_1 = q_1 V_2 \]

\[ PE = (1 \text{ due to 2}) + (2 \text{ due to 1}) \]

\[ (E \text{ to bring 1 close to 2}) \]

\[ (E \text{ to bring 2 close to 1}) \]
\[ PE = PE_{1\&2} + PE_{2\&3} + PE_{1\&3} = PE_{2\&1} + PE_{3\&2} + PE_{3\&1} = k_e \left( \frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} \right) \]
\[ PE = \frac{1}{2} \sum_{i=1}^{3} \sum_{\substack{j=1 \atop j \neq i}}^{3} \frac{k_e q_i q_j}{r_{ij}} \]

\[
= \frac{1}{2} \left( \frac{k_e q_2 q_1}{r_{21}} + \frac{k_e q_3 q_1}{r_{31}} + \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_3 q_2}{r_{32}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}} \right) \\
= k_e \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)
\]
what is the potential energy of the “crystal”

\[ \text{potential energy} = \frac{k q^2}{2a^2} \]
we just have to sum the energy of all unique pairs of charges.

so how many are there?
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ways of choosing pairs from five charges = \( \binom{5}{2} = 5C_2 = \frac{5!}{2!(5-2)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 10 \)
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so how many are there?

ways of choosing pairs from five charges = \( \binom{5}{2} = 5C_2 = \frac{5!}{2! (5-2)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 10 \)

(1, 2)  (1, 3)  (1, 4)  (1, 5)
(2, 3)  (2, 3)  (2, 5)
(3, 4)  (3, 5)
(4, 5)
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(1, 2) (1, 3) (1, 4) (1, 5)
(2, 3) (2, 3) (2, 5)
(3, 4) (3, 5)
(4, 5)

<table>
<thead>
<tr>
<th>#, pairing type</th>
<th>separation</th>
<th>pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>4, center-corner</td>
<td>(a)</td>
<td>(1, 5) (2, 5) (3, 5) (4, 5)</td>
</tr>
<tr>
<td>4, adjacent corners</td>
<td>(a\sqrt{2})</td>
<td>(1, 4) (3, 4) (2, 3) (1, 2)</td>
</tr>
<tr>
<td>2, far corner</td>
<td>(2a)</td>
<td>(1, 3) (2, 4)</td>
</tr>
</tbody>
</table>
\[PE_{\text{square}} = 4 \text{ (energy of center-corner pair)} + 2 \text{ (energy of far corner pair)} + 4 \text{ (energy of adjacent corner pair)} \]

\[
= 4 \left[ \frac{k_e q^2}{a} \right] + 2 \left[ \frac{k_e q^2}{2a} \right] + 4 \left[ \frac{k_e q^2}{a \sqrt{2}} \right] \\
= \frac{k_e q^2}{a} \left[ 4 + 1 + \frac{4}{\sqrt{2}} \right] \\
= \frac{k_e q^2}{a} \left[ 5 + 2 \sqrt{2} \right] \approx 7.83 \frac{k q^2}{a}
\]
it works for more complicated stuff
(a) Rocksalt

(b) Rutile

\[ M = -1.75 \]

\[ M = -4.82 \]
travel along surface:

E perpendicular to path everywhere

no work done!

electric force is conservative ...

equipotential lines?
contours of constant $V$
no work to move along them (like gravity)

$x, y =$ spatial coordinates
potential constant on lines

3d

$x, y =$ spatial coordinates
$z =$ electric potential

2d
conductor = mirror for field & potential lines
$q$ conductor $E=0$
Circuit diagram symbol for voltage sources:

Batteries:  

General constant voltage source:  

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\[ W = mg\Delta y \]

\[ W = \frac{1}{2} Q\Delta V = \frac{1}{2} \frac{Q^2}{C} \]
\[ C_{eq} = C_1 + C_2 \]

\[ Q_{tot} = Q_1 + Q_2 \]
\[
\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}
\]

(a) \hspace{4cm} (b)