Problem Set 8: Induction

Instructions:
1. Answer all questions below. Show your work for full credit.
2. Due at the end of Friday 10 Nov. 2008
3. You may collaborate, but everyone must turn in their own work

1. 10 points. A thin ring of radius $a$ carries a static charge $q$. This ring is in a magnetic field of strength $B_0$, parallel to the ring’s axis, and is supported so that it is free to rotate about that axis. If the field is switched off, how much angular momentum will be added to the ring? If the ring has mass $m$, show that it will acquire an angular velocity $\omega = qB_0/2mc$.

2. 10 points. There is evidence that a magnetic field exists in most of the interstellar space with a strength between $10^{-9}$ and $10^{-10}$ T. Adopting $3 \times 10^{-10}$ T as a typical value, find the total energy stored in the magnetic field of the galaxy. Assume the galaxy is a disk roughly $10^{21}$ m in diameter and $10^{19}$ m thick. Assuming stars radiate about $10^{37}$ W, how many years of starlight is the magnetic energy worth?

3. 10 points. Find the magnetic field at a point $P$ midway between the plates of capacitor a distance $r$ from the axis of symmetry. A current $I$ is flowing through the capacitor.

4. 10 points. At $t = 0$, the open switch in the figure below is closed. By using Kirchhoff’s rules for the instantaneous currents and voltages in this two-loop circuit, show that the current in the inductor at time $t > 0$ is

$$I(t) = \frac{\Delta V}{R_1} \left[1 - e^{-\left(R'/L\right)t}\right]$$

where $R' = R_1R_2 / (R_1 + R_2)$

Figure 1: Problem 4: an RL circuit.
5. 10 points. A cell membrane typically has a capacitance around $1 \mu F/cm^2$. It is believed the membrane consists of material having a dielectric constant of $\kappa \sim 3$. Find the thickness this implies. Other electrical measurements have indicated that the resistance of 1 cm$^2$ of cell membrane is around 1000 $\Omega$. Show that the time constant of such a leaky capacitor is independent of the area of the capacitor. How large is it in this case? What is the resistivity?

6. 10 points. Two inductors having self-inductance $L_1$ and $L_2$ are connected in parallel to a time-varying source of current $I(t)$. The mutual inductance between the two inductors is $M_{12}$. Determine the equivalent self-inductance $L_{eq}$ for the system.