today: dc circuits
mostly current & resistance
\[ I = \frac{\Delta Q}{\Delta t} \]
not so funny now.

just wait ...

WE WERE GOING TO USE THE TIME MACHINE TO PREVENT THE ROBOT APOCALYPSE, BUT THE GUY WHO BUILT IT WAS AN ELECTRICAL ENGINEER.
$$R = \frac{\rho l}{A} = \frac{\Delta V}{l}$$
\[ I = \text{Cause/Resistance} \]

\( I \) is the current, or flow rate, and describes different scenes:

(a) Heat flow through a wall

(b) Charge flow through a wire

(c) Fluid flow through a pipe

Resistance \( R \) has the same form in most cases,

\[ R = \rho L/A \]
<table>
<thead>
<tr>
<th>Transport what?</th>
<th>Heat</th>
<th>Electric charges</th>
<th>Displacement of a molecule in a fluid</th>
<th>Volume of fluid</th>
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<tbody>
<tr>
<td>Current form</td>
<td>$I = -\Delta T/R$</td>
<td>$I = -\Delta V/R$</td>
<td>$v_{av} \equiv I = -\Delta P/R$</td>
<td>$I = -\Delta P/R$</td>
</tr>
<tr>
<td>(items/second)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current units</td>
<td>J/s or W</td>
<td>C/s or amperes</td>
<td>m/s</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>Resistance form</td>
<td>$R = \rho L/A$</td>
<td>$R = \rho L/A$</td>
<td>$R = \rho L/A$</td>
<td>$R = \rho L/A^2$</td>
</tr>
<tr>
<td>Detail of $\rho$</td>
<td>$\rho = 1$/heat conductivity</td>
<td>$\rho =$ electrical resistivity</td>
<td>$\rho = 6\eta\pi$</td>
<td>$\rho = 8\eta\pi$</td>
</tr>
<tr>
<td>(resistivity)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

battery = pump
voltage = pressure
current = flow
resistor = constriction
capacitor = diaphragm / flexible reservoir
diode = check valve
inductor = paddle wheel
\[ \Delta V = V_b - V_a = -IR \]

\( (a) \)

\( V_a = 0 \quad V_b = -IR \)

\( (b) \)

\( V_a = +IR \quad V_b = 0 \)
real $V$ source = ideal $V$ source + $R$
actual circuit has a parasitic \( r \)

\[ \Delta V \]

\( r \) in series with output ("steals" \( V \))
real current sources

R in parallel with output ("steals" I)
series resistors: conservation of energy

Two Resistors in Series:

\[ R_{eq} = R_1 + R_2 \]

Three or More Resistors in Series:

\[ R_{eq} = R_1 + R_2 + R_3 + \ldots \]

The current through resistors in series is the same.
voltage divider

\[ V_{out} = \frac{R_2}{R_1 + R_2} \cdot V_{in} \]
parallel resistors: conservation of charge

\[ \Delta V_1 = \Delta V_2 = \Delta V \]

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

**Two Resistors in Parallel:**

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

**Three or More Resistors in Parallel:**

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]

**current divider**
rank the currents
more complex arrangements

(a) $I_1 + I_2 = I$

(b) $R_{2-3}$

(c) $R_{1-2-3}$

(d) $R_{eq}$

$\Delta V$
measuring voltage

(a) ! INCORRECT !

(b) CORRECT
real voltmeters

(a) $I \downarrow R_{\text{load}} \uparrow V$

ideal meter

(b) $I \downarrow R_{\text{load}} \uparrow r \uparrow V$

real meter
measuring current

a) ! INCORRECT !

b) CORRECT
a simple ammeter

\[ I \]

\[ R_{\text{precise}} \]
dc Circuits, part II

same thing, just more of it
Thévenin equivalents

This image: Horowitz & Hill, *The art of electronics*

\[ V_{th} = V \text{ (open circuit)} \]

\[ R_{th} = \frac{V \text{ (open circuit)}}{I \text{ (closed circuit)}} \]

any combination of R’s and V’s is equivalent to a SINGLE R and V

disconnect from red dots = open circuit voltage

short red dots, current *there* is closed-circuit current.

(Norton equivalent: a single I source in parallel with R)
series resistors: conservation of energy

Two Resistors in Series:

\[ R_{eq} = R_1 + R_2 \]

Three or More Resistors in Series:

\[ R_{eq} = R_1 + R_2 + R_3 + \ldots \]

The current through resistors in series is the same.

source voltage = sum of voltages on resistors
parallel resistors: conservation of charge

\[ \Delta V_1 = \Delta V_2 = \Delta V \]

\[ \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \]

source current = sum of currents in resistors

Two Resistors in Parallel:

\[ \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \]

Three or More Resistors in Parallel:

\[ \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]
so what?

real sources = ideal sources + R

real meter = ideal meter with R

what about ammeter?
V source loading

\[ \Delta V_{load} = V - Ir \]

for \( r \ll R_{load} \),

\[ \Delta V_{load} \approx V \]

V source wants \( R \) high

extra series resistance

one solution:

large resistor in parallel with load
I source loading

\[ I_{\text{load}} = I \frac{r}{r+R} \]

for \( R_{\text{load}} \ll r \),

\[ I_{\text{load}} \approx I \]

extra parallel resistance

I source wants \( R \) low

sourcing currents at high \( R_{\text{load}} \) is hard
measuring the meter

\[ \Delta V_{load} = IR_{eq} = \frac{R}{1+R/r} I \]

\[ R_{load} \ll r, \quad \Delta V_{load} \approx IR \]
summary

voltmeter wants $R$ **low**!
can use a buffer/follower ... later

$I$ source wants $R$ **low**
transformer pre-amp
consider sourcing $V$

$V$ source wants $R$ **high**
large series + parallel resistors
present large $R$
Sourcing current

This is what a hand meter does.

Why is it no good?

\[ V_{\text{meter}} = I(R_{\text{thing}} + 2R_{\text{wires}}) \]
Sourcing current, properly

No problem. You just need four wires.

\( R_{\text{wires}} \)

\( R_{\text{thing}} \ll R_{\text{internal}} \)

or add buffer
Sourcing voltage

Still have to measure voltage on device
the wires still use up some of $V$
What about current?
Sourcing voltage better

\[ R = \frac{\Delta V}{I} \]

Note we need 4 wires again current meter - not hard still problems?
source/meter resistances

voltmeter wants R low
but V source wants R high

need buffer/amp on V meter
resistor in parallel with source

if V source is problem, R is too low
consider sourcing I
what if I want to measure a *really* high $R$?

source voltage
$R_p$ has same voltage as $R_{\text{thing}}$
$R_s$ has same current
have done $>10^{10}$ Ohm
what if I want to measure a *really* low R?

\[ I \rightarrow R_{\text{wires}} \rightarrow R_{\text{thing}} \rightarrow V \]

this works just fine ...
so long as your V meter is good or you can tolerate large I
v. good amp / part of a bridge
what if I want to measure a small change in $R$?

balance bridge to $V=0$
detect small changes from null

$R_2 = \approx R_3$

make $R_1 - R_3$ about the same
trimming resistor on $R_2 = dR$

$V = \left( \frac{R_x}{R_3 + R_x} - \frac{R_2}{R_1 + R_2} \right) V_s$

$R_x = \frac{R_3 R_2}{R_1}$
Rules for analyzing more complicated circuits
\( \Delta V = V_b - V_a = -IR \) 

\( \Delta V = V_b - V_a = +IR \) 

\( \Delta V = V_b - V_a = +\varepsilon \) 

\( \Delta V = V_b - V_a = -\varepsilon \)
capacitors

Definition of Capacitance: the capacitance $C$ is the ratio of the charge stored on one conductor (or the other) to the potential difference between the conductors:

$$C = \frac{|Q|}{|\Delta V|}$$  \hspace{1cm} (4.12)

frequency-dependent resistor

$I$ and $V$ are $90^\circ$ out of phase

can’t dissipate power, ideally

$$I = \frac{dQ}{dt} = \frac{d(CV)}{dt} \rightarrow C \frac{dV}{dt}$$

Capacitance of a parallel plate capacitor:

$$C = \varepsilon_0 \frac{A}{d}$$

where $d$ is the spacing between the plates, and $A$ is the area of the plates.
combinations of capacitors

Two Capacitors in Parallel:

\[ C_{eq} = C_1 + C_2 \]

Three or More Capacitors in Parallel:

\[ C_{eq} = C_1 + C_2 + C_3 + \ldots \]

Two Capacitors in Series:

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \]

Three or More Capacitors in Series:

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \]
capacitors with stuff inside

Parallel plate capacitor with a dielectric between the plates:

$$C = \kappa \varepsilon_0 \frac{A}{d} = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

the dielectric increases the capacitance by a factor $\kappa$, the dielectric constant. The dielectric constant is also written $\varepsilon_r$, sometimes.

$$\Delta V = \frac{\Delta V_0}{\kappa} = \frac{\Delta V_0}{\varepsilon_r}$$

$$C = \frac{Q_0}{\Delta V} = \frac{\kappa Q_0}{\Delta V_0}$$
rc circuits

\[ V \propto e^{-\tau/RC} \]

Time constant \( \tau \) of an RC circuit:

\[ \tau = RC \]

This gives \( \tau \) in seconds [s] when \( R \) is in Ohms [\( \Omega \)] and \( C \) is in farads [F].

(6.27)
RC differentiator

\[ I = C \frac{d}{dt} (V_{in} - V) = \frac{V}{R} \]

for small RC,

\[ C \frac{dV_{in}}{dt} \approx \frac{V}{R} \]

\[ V(t) \approx RC \frac{d}{dt} V_{in}(t) \]
RC integrator

\[ V_{in}(t) \quad \text{-------------------} \quad R \quad \text{-------------------} \quad C \quad \text{-------------------} \quad V_{out}(t) \]

\[ I = C \frac{dV}{dt} = \frac{V_{in} - V}{R} \]

for large RC \((V \ll V_{in})\)

\[ V(t) = \frac{1}{RC} \int_{0}^{t} V_{in}(t) \, dt + \text{const} \]

\[ C \frac{dV}{dt} \approx \frac{V_{in}}{R} \]
so what?

filtering of signals

unintentional capacitive coupling
see from waveform shape

more later
ac resistive circuits

nothing earth-shattering happens except $P$ is lower than you expect
ac capacitive circuits

\( V \propto V_0 \sin \omega t \)

\[ Z = \frac{1}{i\omega C} = \frac{-1}{2\pi i f C} \]

\( \omega \) is angular frequency

I and V 90° out of phase
average power is ZERO

frequency response?
insulating at dc
conducting at high \( f \)

voltage “lags” current

The University of Alabama

Center for Materials for Information Technology
An NSF Science and Engineering Center
filters

low-pass

\[ V_{in} \xrightarrow{R} V_{out} \]

\[ 3dB \text{ at } \omega = \frac{1}{RC} \]

high-pass

\[ V_{in} \xrightarrow{C} V_{out} \]

\[ \text{low-pass filter} \]

\[ \text{high-pass filter} \]
familiar?

low-pass filter

\[ V_{in} \rightarrow R \rightarrow V_{out} \]

\[ C \]

integrator

\[ V(t) = \frac{1}{RC} \int_{0}^{t} V_{in}(t) \, dt + \text{const} \]

high-pass filter

\[ V_{in} \rightarrow C \rightarrow V_{out} \]

\[ R \]

differentiator

\[ V(t) \approx RC \frac{d}{dt} V_{in}(t) \]
high-pass

\[ V_{in} \xrightarrow{R} V_{out} \]

\[ L \]

\[ V_{out} \]

\[ f \]
low-pass

\[ V_{\text{in}} \xrightarrow{R} V_{\text{out}} \]

\[ V_{\text{out}} \xrightarrow{\text{low-pass}} \]

\[ f \]
audio crossovers

series

parallel

$V_{in}$
audio input

tweeter

tweeter

woofer

woofer

audio crossovers
$$V_{\text{out}} / V_{\text{in}}$$ vs. $$f$$ (Hz)

- **Red** line: high pass
- **Blue** line: low pass
- **Black** line: series combo

$$R = 1.0\, \text{k}\Omega$$, $$C = 0.01\, \mu\text{F}$$
$R = 1.0 \text{k} \Omega, \ C = 0.01 \mu \text{F}$

- **high pass**
- **low pass**
- **series combo**
\[ V_{out} / V_{in} \]

- **R = 1.0kΩ, C = 0.01\mu F**
- **Red line**: high pass
- **Blue line**: low pass
- **Black line**: series combo

\[ dB/\text{decade} \ ... \]
Lab 2: Op Amp Circuits

INTRODUCTION

This lab introduces the operational amplifier or "op amp". The circuit is already constructed for you on a single IC (integrated circuit) and in this lab we will use the IC in several of its most popular configurations. For an introduction to op amps, see section 2.4 in Bobrow.

1. INVERTING AMPLIFIER

The pinout diagram for the LM741 op amp IC is shown in figure 1. Use this to construct the inverting op amp circuit shown in figure 2.

At first, use \( R_1 = 1 \text{k}\Omega \) and \( R_2 = 33 \text{k}\Omega \).

Derive the gain formula \( A = \frac{-R_2}{R_1} \) and experimentally verify the gain for a 100Hz sine wave. Also calculate and measure the gain with \( R_2 = 10 \text{k}\Omega \) and \( R_2 = 100 \text{k}\Omega \). For your lab report, give your derivation and compare the calculated and measured gains. Why is this called an inverting amp?

2. NONINVERTING AMPLIFIER

Construct the noninverting amplifier shown in figure 3 with \( R_1 = 1 \text{k}\Omega \) and \( R_2 = 33 \text{k}\Omega \). For your report, derive and experimentally verify the gain relation \( A = \frac{R_2}{R_1} \).

3. CURRENT-TO-VOLTAGE CONVERTER

An op amp can be used to produce a voltage proportional to a given current. Construct the circuit in figure 4.

Verify that \( V_{out} = \frac{I_{in}}{2} \) for this circuit. (That is, do the following for several voltage settings on the variable power supply: Measure the supply voltage and from this calculate \( I_{in} \). Use the formula to calculate a theoretical \( V_{out} \) and compare this to a measured \( V_{out} \). Include these measured values and calculations in your report along with a brief discussion of the agreement between theory and measurement.)

Today: amplify photodiode signal