Back-of-the-envelope Physics

From “Physics for Scientists & Engineers, 6 Ed.,” Serway & Jewett

It is often useful to compute an approximate answer to a given physical problem even when little information is available. This answer can then be used to determine whether or not a more precise calculation is necessary. Such an approximation is usually based on certain assumptions, which must be modified if greater precision is needed. We will sometimes refer to an order of magnitude of a certain quantity as the power of ten of the number that describes that quantity. Usually, when an order-of-magnitude calculation is made, the results are reliable to within about a factor of 10. If a quantity increases in value by three orders of magnitude, this means that its value increases by a factor of about 1e3=1000. We use the symbol **~** *for “is on the order of.” Thus,*

$$0.0086 \sim 10^{-2} \qquad 0.0021 \sim 10^{-3} \qquad 720 \sim 10^{3}$$

The spirit of order-of-magnitude calculations, sometimes referred to as “guesstimates” or “ball-park figures,” is given in the following quotation: “Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle.” Inaccuracies caused by guessing too low for one number are often canceled out by other guesses that are too high. You will find that with practice your guesstimates become better and better. Estimation problems can be fun to work as you freely drop digits, venture reasonable approximations for unknown numbers, make simplifying assumptions, and turn the question around into something you can answer in your head or with minimal mathematical manipulation on paper. Because of the simplicity of these types of calculations, they can be performed on a small piece of paper, so these estimates are often called “back-of-the-envelope calculations.”

Plan:
Below, there are several problems listed, along with a URL to access a further list of problems. You are to come up with the best estimate of the desired quantity that you can, using whatever resources at your disposal. You must be able to explain your reasoning. Solve as many problems as you can. Answers resulting from a simple internet search will be disregarded; calculations must be involved.

Procedure:
When you solve a problem, write the problem number (and section, if applicable) on the board, along with your numerical estimate. Your team has taken this question out of consideration, unless another team chooses to challenge you (below). Once you answer a question, your answer is final, and cannot be changed. Your score is determined by:

+1 point for a plausible numerical answer and reasoning
+0.5 point if the answer is implausible but the reasoning is (or vice versa)

-0.5 points for a reason and no answer, or an answer and no reason
-1 point for a haphazard guess (no reason, very implausible answer)

Basically: you must be “right,” and for the “right” reason!

If you feel one of the other teams’ answers is in error, you can challenge their answer by writing your own next to it. Multiple challenges can be issued for a single question, but only one answer per team. At then end of the period, Dr. LeClair will arbitrate the winner of the challenge, and the winner’s score will be determined as above. The non-winning answers will receive 0 points.

The team with the highest score at the end of all questions will receive prizes at the next session ...

Problems:

“Fermi problems”

http://www.physics.umd.edu/perg/fermi/fermi.htm

AKA, ‘back of the envelope’ calculations, used to estimate the order of magnitude of a quantity. What do you know, what can you guess, and what doesn’t really matter?

Additional problems

What can you look up, what do you really need, and how accurate does the answer really need to be?

1. The distance from the Sun to the nearest star is about 4e16m. The Milky Way galaxy is roughly a disk of diameter ~1e21m and thickness ~1e19m. Find the order of magnitude of the number of stars in the Milky Way. Assume the distance between the Sun and our nearest neighbor is typical.

2. Assume there are 100 million passenger cars in the United States and that the average fuel consumption is 20mi/gal of gasoline. If the average distance traveled by each car is 10000 mi/yr, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?

3. Grass grows densely everywhere on a quarter-acre plot of land. What is the order of magnitude of the number of blades of grass on this plot? Explain your reasoning.

4. Approximately how many raindrops fall on a one-acre lot during a one-inch rainfall? Explain your reasoning.

5. Estimate the number of breaths taken during an average life span.

6. Estimate the number of steps a person would take walking from New York to Los Angeles.

7. Estimate the total ticket sale revenue to fill Bryant-Denny stadium for an average regular-season game.

8. For many electronics applications, such as in computer chips, it is desirable to make components as small as possible to keep the temperature of the components low and to increase the speed of the device. Thin metallic coatings (films) can be used instead of wires to make electrical connections. Gold is especially useful because it does not oxidize readily. Its atomic mass is 197 u. A gold film can be no thinner than the size of a gold atom. Calculate the minimum coating thickness, assuming that a gold atom occupies a cubical volume in the film that is equal to the volume it occupies in a large piece of metal. This geometric model yields a result of the correct order of magnitude.

9. **WORTH DOUBLE POINTS** - The CO molecule can be modeled as a system of two masses connected by a spring. If the spring has force constant k, and the spring is extended from equilibrium by x, the energy required is $0.5kx^2$.

Given the bond strength of CO is ~1000kJ/mol, and the bond length is ~1e-10m, estimate the spring constant and vibrational frequency of the CO molecule. This is the region of characteristic absorption of the molecule; is it in the visible range, infrared, ultraviolet, or what?

Note - the oscillation frequency of a simple mass-spring system with two identical masses is $f^2 \sim k/m$, where m is the mass connected to the ends of the spring.