PH 253 Exam I

Instructions

1. Solve 4 of the 6 problems below. All problems have equal weight.
2. Do your work on separate sheets of paper.
3. You are allowed 1 sheet of standard 8.5x11 in paper and a calculator.

1. A train 0.5 km long (as measured by an observer on the train) is traveling at a speed of 44 m/s. Two lightning bolts strike the ends of the train simultaneously as determined by an observer on the ground. What is the time separation as measured by an observer on the train?

2. Pions have a half life of about $\Delta t = 1.8 \times 10^{-8}$ s. A pion beam leaves an accelerator at a speed of $v = 0.8c$. Compute the expected distance in the lab frame over which half the pions should decay by both **both** relativistic **and** classical means.

3. A stick of length L is at rest in a system O and is oriented at an angle θ with respect to the x axis. An observer in system O' travels at velocity v with respect to the system O along the x axis. What is the apparent angle θ' that the stick makes with the x' axis according to the observer in O'? The x and x' axes are parallel.

4. Rocket A travels to the right and rocket B travels to the left, with velocities of $0.8c$ and $0.6c$, respectively, relative to earth. What is the velocity of rocket A measured from rocket B?

5. An electron is released from rest and falls under the influence of gravity. In the first centimeter, what fraction of the potential energy lost is radiated away?

6. Assume that an electron is one Bohr radius ($a_0 = 0.053$ nm) from a proton in a hydrogen atom. **(a)** Find the acceleration of the electron (hint: circular path). **(b)** Calculate the kinetic energy of the electron and determine within an order of magnitude how long it will take the electron to loose all of its energy, assuming a constant acceleration as found in part (a).

Constants:

$$g \approx 9.81\,\mathrm{m/s^2}$$
$$N_A = 6.022 \times 10^{23}\,\text{things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^9\,\mathrm{N \cdot m^2 \cdot C^{-2}}$$
$$\epsilon_o = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$$
$$\mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$$
$$e = 1.60218 \times 10^{-19}\,\mathrm{C}$$
$$h = 6.6261 \times 10^{-34}\,\mathrm{J \cdot s} = 4.1357 \times 10^{-15}\,\mathrm{eV \cdot s}$$
$$\hbar = \frac{h}{2\pi}$$
$$k_B = 1.38065 \times 10^{-23}\,\mathrm{J \cdot K^{-1}} = 8.6173 \times 10^{-5}\,\mathrm{eV \cdot K^{-1}}$$
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792 \times 10^8\,\mathrm{m/s}$$
$$hc = 1240\,\mathrm{eV \cdot nm}$$
$$m_e = 9.10938 \times 10^{-31}\,\mathrm{kg} \qquad m_e c^2 = 510.998\,\mathrm{keV}$$
$$m_p = 1.67262 \times 10^{-27}\,\mathrm{kg} \qquad m_p c^2 = 938.272\,\mathrm{MeV}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Basic Equations:

$$\vec{\mathbf{E}} = \sigma/\epsilon_0 \text{ capacitor}$$
$$C = \epsilon_0 A/d$$
$$\vec{\mathbf{F}}_{\text{net}} = m\vec{a} \text{ Newton's Second Law}$$
$$\vec{\mathbf{F}}_{\text{centr}} = -\frac{mv^2}{r}\hat{\mathbf{r}} \text{ Centripetal}$$
$$K_{\text{classical}} = \frac{1}{2}mv^2$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

E & M

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = q_2 \vec{\mathbf{E}}_1 \qquad \vec{\mathbf{r}}_{12} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$$
$$\vec{\mathbf{E}}_1 = \vec{\mathbf{F}}_{12}/q_2 = k_e \frac{q_1}{r_{12}^2}\hat{\mathbf{r}}_{12}$$
$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$
$$U_{12} = k_e \frac{q_1 q_2}{r_{12}} \quad \text{potential E, 2 charges}$$

Oscillators & waves

$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 m A^2 = 2\pi^2 m f^2 A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$
$$c = \lambda f$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2 \qquad e^x \approx 1 + x + \frac{1}{2}x$$
$$\sin x \approx x - \frac{1}{6}x^3 \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

Radiation

$$P_{\text{rad}} = \frac{q^2 a^2}{6\pi\epsilon_o c^3} \qquad \text{total emitted power, E and B fields}$$

Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t'_{\text{moving}} = \gamma \Delta t_{\text{stationary}} = \gamma \Delta t_p$$
$$L'_{\text{moving}} = \frac{L_{\text{stationary}}}{\gamma} = \frac{L_p}{\gamma}$$
$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$
$$v_{\text{obj}} = \frac{v + v'_{\text{obj}}}{1 + \frac{vv'_{\text{obj}}}{c^2}} \qquad v'_{\text{obj}} = \frac{v_{\text{obj}} - v}{1 - \frac{vv_{\text{obj}}}{c^2}}$$
$$KE = (\gamma - 1)mc^2 = \sqrt{m^2c^4 + c^2p^2} - mc^2$$
$$E_{\text{rest}} = mc^2$$
$$p = \gamma m v$$
$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$

Calculus of possible utility:

$$\int \frac{1}{x}dx = \ln x + c$$
$$\int u\,dv = uv - \int v\,du$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magn} \qquad \theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{dir}$$