PH 253 Exam II

Solve 4 of the 6 problems below. All problems have equal weight.

1. The maximum energy of photoelectrons from aluminum is 2.3 eV for radiation of 200 nm wavelength and 0.90 eV for radiation of 313 nm. Use this data to calculate Planck's constant and the work function of aluminum.

2. Show that it is impossible for a photon striking a free electron to be absorbed and not scattered.

3. A nucleus emits a gamma ray of energy 1.0 MeV from a state that has a lifetime of 1.2 ns. **(a)** What is the uncertainty in the energy of the gamma ray? **(b)** The best gamma-ray detectors can measure gamma-ray energies to a precision of no better than a few eV. Will this uncertainty be directly measurable?

4. Why does the wave nature of particles come as such a surprise to most people? If, as de Broglie says, a wavelength can be associated with *every* moving particle, then why are we not forcibly made aware of this property in our everyday experience? In answering, calculate the de Broglie wavelength of each of the following "particles":

(a) an automobile of mass 2 metric tons (2000 kg) traveling at a speed of 50 mph (22 m/s),
(b) a marble of mass 10 g moving with a speed of 10 cm/sec,
(c) a smoke particle of diameter 10^{-7} m (and a density of, say, 2 g/cm^3) being jostled about by air molecules at room temperature (300 K). Assume the particle has the same translational kinetic energy as the thermal average of the air molecules, $p^2/2m = 3k_bT/2$ with $k_b = 1.38 \times 10^{-23}$ = Boltzmann's constant.

5. A particle of mass m is trapped in a one dimensional oscillator potential, $V(x) = \frac{1}{2}m\omega^2x^2$ for $-\infty < x < \infty$. The wave functions for the first two states are shown below.

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar} \qquad \psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{2m\omega}{\hbar}}\, x\, e^{-m\omega x^2/2\hbar} \tag{1}$$

(a) Find the energy of the ψ_0 state.
(b) What are $\langle x\rangle$ and $\langle p\rangle$ for each state? An explicit calculation is unnecessary if you can provide a physical explanation for your result.

To save you some time, we note $\frac{d}{dx}(e^{-ax^2}) = -2axe^{-ax^2}$ and $\frac{d^2}{dx^2}(e^{-ax^2}) = 2a(2ax^2-1)e^{-ax^2}$. Make use of the integral table on the formula sheet.

6. At $t=0$, the (normalized) wave function of a free particle of mass m is given by

$$\psi(x,0) = \begin{cases} \frac{e^{ik_o x}}{\sqrt{x_2 - x_1}} & x_1 \leq x \leq x_2 \\ 0 & x < x_1 \quad \text{and} \quad x > x_2 \end{cases} \tag{2}$$

(a) Find $\langle x\rangle$ at $t=0$. Your answer should agree with your commonsense expectation. *Think carefully about how to calculate* $|\psi|^2$.
(b) What is the probability that the particle is found in the interval $[x_1, \frac{1}{2}(x_1+x_2)]$?

Constants:

$$N_A = 6.022 \times 10^{23}\,\text{things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^9\,\text{N}\cdot\text{m}^2\cdot\text{C}^{-2}$$
$$\epsilon_o = 8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$$
$$\mu_0 \equiv 4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$$
$$e = 1.60218 \times 10^{-19}\,\text{C}$$
$$h = 6.6261 \times 10^{-34}\,\text{J}\cdot\text{s} = 4.1357 \times 10^{-15}\,\text{eV}\cdot\text{s}$$
$$\hbar = \frac{h}{2\pi} \qquad hc = 1239.84\,\text{eV}\cdot\text{nm}$$
$$k_B = 1.38065 \times 10^{-23}\,\text{J}\cdot\text{K}^{-1} = 8.6173 \times 10^{-5}\,\text{eV}\cdot\text{K}^{-1}$$
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792 \times 10^8\,\text{m/s}$$
$$m_e = 9.10938 \times 10^{-31}\,\text{kg} \qquad m_e c^2 = 510.998\,\text{keV}$$
$$m_p = 1.67262 \times 10^{-27}\,\text{kg} \qquad m_p c^2 = 938.272\,\text{MeV}$$
$$m_n = 1.67493 \times 10^{-27}\,\text{kg} \qquad m_n c^2 = 939.565\,\text{MeV}$$
$$u = 1.66054 \times 10^{-27}\,\text{kg} \qquad uc^2 = 931.494\,\text{MeV}$$

Schrödinger

$$i\hbar\frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi \quad \text{1D time-dep}$$
$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi \quad \text{1D time-indep}$$
$$\int_{-\infty}^{\infty} |\psi(x)|^2\,dx = 1 \qquad P(\text{in }[x, x+dx]) = |\psi(x)|^2 \qquad \text{1D}$$
$$\int_0^{\infty} |\psi(r)|^2\,4\pi r^2\,dr = 1 \qquad P(\text{in }[r, r+dr]) = 4\pi r^2|\psi(r)|^2 \qquad \text{3D}$$
$$\langle x^n\rangle = \int_{-\infty}^{\infty} x^n P(x)\,dx \qquad \text{1D} \qquad \langle r^n\rangle = \int_0^{\infty} r^n P(r)\,dr \qquad \text{3D}$$
$$\langle p\rangle = \int_{-\infty}^{\infty} \psi^*\left(-i\hbar\frac{d}{dx}\right)\psi\,dx$$
$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} \qquad \Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2}$$

Oscillators

$$E = \left(n + \frac{1}{2}\right)hf$$
$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA^2 = 2\pi^2 mf^2A^2$$
$$\omega = 2\pi f = \sqrt{k/m} \qquad k = 2\pi/\lambda$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2 \qquad \tan x \approx x + \frac{1}{3}x^3$$
$$e^x \approx 1 + x + \frac{1}{2}x \qquad \sin x \approx x - \frac{1}{6}x^3 \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

Misc Quantum/Relativity

$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$
$$E = hf \qquad p = h/\lambda = E/c \qquad \lambda f = c \qquad \text{photons}$$
$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta) \quad \text{Compton}$$
$$\lambda = \frac{h}{|\vec{\mathbf{p}}|} = \frac{h}{\gamma mv} \approx \frac{h}{mv}$$
$$\Delta x\Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2} \qquad \Delta E\Delta t \geq \frac{h}{4\pi} = \frac{\hbar}{2}$$
$$eV_{\text{stopping}} = KE_{\text{electron}} = hf - \varphi = hf - W$$

Calculus of possible utility:

$$\int \frac{1}{x}dx = \ln x + c$$
$$\int udv = uv - \int vdu$$
$$\int \sin ax dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax dx = \frac{1}{a}\sin ax + C$$
$$\int e^{-ax}\,dx = -\frac{1}{a}e^{-ax} + C$$
$$\int_0^{\infty} x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$
$$\int_0^{\infty} xe^{-ax^2}\,dx = \frac{1}{2a}$$
$$\int_0^{\infty} x^2e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$
$$\int_{-\infty}^{\infty} x^3e^{-ax^2}\,dx = \int_{-\infty}^{\infty} xe^{-ax^2}\,dx = 0$$
$$\int_0^{\infty} x^4e^{-ax^2}\,dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$