University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Fall 2016

Exam 1 practice problems

General comments:

- There will be at least one problem that rewards those with mathematical fluency. I.e., the concept will be straightforward, the mathematics will not be.
- There will be at least one problem that rewards those with good physical intuition. I.e., the mathematics will be straightforward, the concept may not be.
- It is *extremely* unlikely that any exam problems will involve plugging numbers into a single formula. Most problems will not *have* numerical answers.
- A couple of these problems could show up verbatim on the exam.

1. An electron moving initially with constant speed v is brought to rest with uniform deceleration a lasting for a time $t = v/a$. Compare the electromagnetic energy radiated during the deceleration with the electron's initial kinetic energy. Express the ratio in terms of two lengths, the distance light travels in time t and the classical electron radius, $r_e = \frac{e^2}{4\pi\epsilon_o m_e c^2} = \frac{k_e e^2}{m_e c^2}$. Recall the Larmor formula, which gives the total power radiated from an accelerating charge ($v \ll c$):

$$P = \frac{e^2 a^2}{6\pi\epsilon_o c^3} = \frac{2k_e e^2 a^2}{3c^3} \tag{1}$$

2. An atomic clock aboard a spaceship runs slow compared to an Earth-based atomic clock at a rate of 1.0 second per day. What is the speed of the spaceship?

3. The speed of light with respect to a medium is $v'_x = c/n$, where n is the index of refraction. Suppose that the medium, say, flowing water, is moving past a stationary observer in the same direction as the light with speed V. Show that the observer measures the speed of light to be approximately

$$v_x = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) V \tag{2}$$

This effect was first observed by Fizeau in 1851.

4. A mass M at rest decays into two particles of mass m_1 and m_2. Show that the magnitude of the momentum of each of the two particles is

$$\frac{p}{c} = \frac{\sqrt{M^2 - (m_1 + m_2)^2}\sqrt{M^2 - (m_1 - m_2)^2}}{2M} \tag{3}$$

Hint: conserve energy and momentum, and use $E^2 = m^2c^4 + p^2c^2$.

5. An interstellar space probe is moving at a constant speed relative to earth of $0.76c$ toward a distant planet. Its radioisotope generators have enough energy to keep its data transmitter active continuously for 15 years, as measured in their own reference frame. **(a)** How long do the generators last as measured from earth? **(b)** How far is the probe from earth when the generators fail, as measured from earth? **(c)** How far is the probe from earth when the generators fail, *as measured by its built-in trip odometer*?

6. A pion at rest ($m_\pi = 273\, m_{e^-}$) decays to a muon ($m_\mu = 207\, m_{e^-}$) and an antineutrino ($m_{\overline{\nu}} \approx 0$). This reaction is written as $\pi^- \to \mu^- + \overline{\nu}$. Find the kinetic energy of the muon and the energy of the antineutrino in electron volts. *Hint: relativistic momentum is conserved.*

7. An electron and a proton are each accelerated starting from rest through a potential difference of 10.0 million volts (10^7 V). Find the momentum (in MeV/c) and kinetic energy (in MeV) of each, and compare the results with the classical expectation. Recall $PE = q\Delta V$.

8. An electron is released from rest and falls under the influence of gravity. **(a)** How much power does it radiate? **(b)** How much energy is lost after it falls 1 m? (*Hint:* $P = \Delta K/\Delta t$, $y = \frac{1}{2}gt^2$.)

9. How fast must a rocket travel relative to the earth so that time in the rocket "slows down" to half its rate as measured by earth-based observers? Do present-day jet planes approach such speeds?

10. A cube of metal with sides of length a sits at rest in frame S with one edge parallel to the x-axis. Therefore, in S the cube has volume a^3. Frame S' moves along the x-axis with speed u. As measured by an observer in frame S', what is the volume of the metal cube?

11. One of the wavelengths of light emitted by hydrogen atoms under normal laboratory conditions is $\lambda = 656.3\,\text{nm}$, in the red portion of the electromagnetic spectrum. In the light emitted from a distant galaxy this same spectral line is observed to be Doppler-shifted to $\lambda = 953.4\,\text{nm}$, in the infrared portion of the spectrum. How fast are the emitting electrons moving relative to the earth? Are they approaching the earth or receding from it?

12. Two particles in a high-energy accelerator experiment approach each other head-on with a relative speed of $0.890c$. Both particles travel at the same speed as measured in the laboratory. What is the magnitude of the velocity of one particle relative to the other?

13. **(a)** Through what potential difference does an electron have to be accelerated, starting from rest, to achieve a speed of $0.980c$? **(b)** What is the kinetic energy of the electron at this speed? Express your answer in both joules and electron volts.

14. Use the following two equations:

$$\vec{\mathbf{p}} = \frac{m\vec{\mathbf{v}}}{\sqrt{1 - v^2/c^2}} \tag{4}$$

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{5}$$

to derive the relationship:

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

15. At what speed is the momentum of a particle twice as great as the result obtained from the non-relativistic expression mv? Express your answer in terms of the speed of light.

16. Light travels with respect to earth at $3 \times 10^8 \ \frac{m}{s}$. A rocket travels at $2.5 \times 10^8 \ \frac{m}{s}$ with respect to earth in opposite direction of the light. What is the speed of light as viewed from the rocket?