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Exam 1 practice problems: answers & some solutions

1. An electron moving initially with constant speed v is brought to rest with uniform deceleration a lasting for a time $t = v/a$. Compare the electromagnetic energy radiated during the deceleration with the electron's initial kinetic energy. Express the ratio in terms of two lengths, the distance light travels in time t and the classical electron radius, $r_e = \frac{e^2}{4\pi\epsilon_o m_e c^2} = \frac{k_e e^2}{m_e c^2}$. Recall the Larmor formula, which gives the total power radiated from an accelerating charge ($v \ll c$):

$$P = \frac{e^2 a^2}{6\pi\epsilon_o c^3} = \frac{2k_e e^2 a^2}{3c^3} \tag{1}$$

Solution: The acceleration is $a = v/t$, so we can find the radiated power readily. After the time t required to decelerate completely, the radiated energy is $\Delta E = Pt$ and the initial kinetic energy of the electron is $\frac{1}{2}mv^2$. Thus,

$$\frac{\Delta E}{K} = \frac{2k_e e^2 a^2 t}{3c^3} \frac{1}{\frac{1}{2}mv^2} = \frac{4k_e e^2}{3c^3 mt} \tag{2}$$

The time light travels in time t is $r_l = ct$. Using the definition of the classical electron radius above,

$$\frac{\Delta E}{K} = \frac{k_e e^2}{mc^2}\left(\frac{4}{3ct}\right) = \frac{4r_e}{3r_l} \tag{3}$$

2. An atomic clock aboard a spaceship runs slow compared to an Earth-based atomic clock at a rate of 1.0 second per day. What is the speed of the spaceship?

Solution: The proper time t_p is that measured on earth, while a dilated time $t = \gamma t_p$ is measured on the ship. If the clock aboard the ship is 1 s per day slow, then using $1\,\text{day} = 86400\,\text{s}$

$$\frac{t - t_p}{t_p} = \frac{\gamma t_p - t_p}{t_p} = \gamma - 1 = \frac{1\,\text{s}}{86400\,\text{s}} \tag{4}$$

This gives $\gamma \approx 1 + 1.16 \times 10^{-5}$, or $v/c \approx 4.8 \times 10^{-3}$.

3. The speed of light with respect to a medium is $v'_x = c/n$, where n is the index of refraction. Suppose that the medium, say, flowing water, is moving past a stationary observer in the same direction as the light with speed V. Show that the observer measures the speed of light to be approximately

$$v_x = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) V \tag{5}$$

This effect was first observed by Fizeau in 1851.

Solution: This is just velocity addition in disguise. The velocity in the lab v_x is the relativistic sum of the velocity of the of the flowing water and that of light:

$$v_x = \frac{V + v'_x}{1 + V v'_x/c^2} = \frac{v + c/n}{1 + v/cn} \tag{6}$$

If we restrict ourselves to $v/c \ll 1$, then we may approximate $1/(1+v/cn) \approx 1 - v/nc$:

$$v_x = \frac{v + c/n}{1 + v/cn} \approx \left(v + \frac{c}{n}\right)\left(1 - \frac{v}{cn}\right) = v - \frac{v^2}{cn} + \frac{c}{n} - \frac{v}{n^2} \tag{7}$$

Since we have already required $v/c \ll 1$, we may neglect terms of order v^2, and we have

$$v_x \approx \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right) \tag{8}$$

4. A mass M at rest decays into two particles of mass m_1 and m_2. Show that the magnitude of the momentum of each of the two particles is

$$\frac{p}{c} = \frac{\sqrt{M^2 - (m_1 + m_2)^2}\sqrt{M^2 - (m_1 - m_2)^2}}{2M} \tag{9}$$

Hint: conserve energy and momentum, and use $E^2 = m^2c^4 + p^2c^2$.

Solution: A lot of algebra in this one. First, conservation of momentum implies that the decay products m_1 and m_2 will have equal and opposite momentum, which we will just call p. Conservation of energy relates the rest mass of the initial particle Mc^2 to the total energy of the two decay products:

$$\begin{aligned} Mc^2 &= \sqrt{p^2c^2 + m_1^2c^4} + \sqrt{p^2c^2 + m_2^2c^4} \\ M &= \sqrt{p^2/c^2 + m_1^2} + \sqrt{p^2/c^2 + m_2^2} \end{aligned} \tag{10}$$

Square both sides, rearrange and square again. Much algebra and factoring ensues.

$$\frac{2p^2}{c^2} + m_1^2 + m_2^2 + 2\sqrt{\left(p^2/c^2 + m_1^2\right)\left(p^2/c^2 + m_2^2\right)} = M^2$$

$$4\left(\frac{p^2}{c^2} + m_1^2\right)\left(\frac{p^2}{c^2} + m_2^2\right) = \left[M^2 - \left(m_1^2 + m_2^2\right) - \frac{2p^2}{c^2}\right]^2$$

$$\frac{4p^4}{c^4} + 4\left(m_1^2 + m_2^2\right)\frac{p^2}{c^2} + 4m_1^2m_2^2 = \left[M^2 - \left(m_1^2 + m_2^2\right)\right]^2 - \frac{4p^2}{c^2}M^2 + \frac{4p^2}{c^2}\left(m_1^2 + m_2^2\right) + \frac{4p^4}{c^4}$$

$$\frac{4p^2}{c^2}M^2 = \left[M^2 - \left(m_1^2 + m_2^2\right)\right]^2 - 4m_1^2m_2^2$$

$$\frac{4p^2}{c^2}M^2 = M^4 - 2M^2\left(m_1^2 + m_2^2\right) + \left(m_1^2 + m_2^2\right)^2 - 4m_1^2m_2^2$$

$$\frac{4p^2}{c^2}M^2 = M^4 - 2M^2\left(m_1^2 + m_2^2\right) + \left(m_1^2 - m_2^2\right)^2$$

$$\frac{4p^2}{c^2}M^2 = M^4 - 2M^2\left(m_1^2 + m_2^2\right) + (m_1 - m_2)^2(m_1 + m_2)^2$$

$$\frac{4p^2}{c^2}M^2 = M^4 - M^2\left[(m_1 - m_2)^2 + (m_1 + m_2)^2\right] + (m_1 - m_2)^2(m_1 + m_2)^2$$

$$\frac{p}{c} = \frac{\sqrt{M^2 - (m_1 + m_2)^2}\sqrt{M^2 - (m_1 - m_2)^2}}{2M} \tag{11}$$

5. An interstellar space probe is moving at a constant speed relative to earth of $0.76c$ toward a distant planet. Its radioisotope generators have enough energy to keep its data transmitter active continuously for 15 years, as measured in their own reference frame. **(a)** How long do the generators last as measured from earth? **(b)** How far is the probe from earth when the generators fail, as measured from earth? **(c)** How far is the probe from earth when the generators fail, *as measured by its built-in trip odometer*?

Solution: Just to be clear, we will label quantities measured in the earth's reference frame with primes ($\prime$), and quantities without primes are with respect to the probe's reference frame. The relative velocity between the earth and the probe is the same from both reference frames, $v = v'$. From the probe's (and its generators') reference frame, it is the observers on earth that are moving. The observers on earth should then see a *longer* time interval compared to the proper time measured on the probe:

$$\Delta t' = \gamma \Delta_p = \frac{15\,\text{yrs}}{\sqrt{1 - \frac{(0.76c)^2}{c^2}}} \approx 23\,\text{yrs}$$

According to observers on earth, the generators should fail after a period of $\Delta t'$. Also according to them, the probe should have traveled a distance $d' = v'\Delta t'$ - the earth-bound observers watched the probe travel for an interval $\Delta t'$ at a constant velocity of v' in their reference frame:

$$d' = v'\Delta t' = (23\,\text{yrs})\left(3\times 10^8\,\text{m/s}\right) \approx 2.2\times 10^{17}\,\text{m}$$

Alternatively, we could express the distance in light years - the distance light travels in one year. To do that, we just have to realize that $0.76c$ means the probe travels at 76% of the speed of light:

$$d' = (0.76\,\text{light speed})\,(23\,\text{yrs}) \approx 18\,\text{light-years}$$

Finally, how about the distance traveled according to the probe? That is just the relative velocity multiplied by the elapsed time *from the probe's reference frame, i.e.,* the proper time:

$$d = v\Delta t = (15\,\text{yrs})\left(3\times 10^8\,\text{m/s}\right)(0.76) = 1.1\times 10^{17}\,\text{m} = 11\,\text{light-years}$$

6. A pion at rest ($m_\pi = 273\,m_{e^-}$) decays to a muon ($m_\mu = 207\,m_{e^-}$) and an antineutrino ($m_{\overline{\nu}} \approx 0$). This reaction is written as $\pi^- \rightarrow \mu^- + \overline{\nu}$. Find the kinetic energy of the muon and the energy of the antineutrino in electron volts. *Hint: relativistic momentum is conserved.*

Solution: Before the collision, we have only the pion, and since it is at rest, it has zero momentum and zero kinetic energy. After it decays, we have a muon and an antineutrino created and speed off in opposite directions (to conserve momentum). Both total energy - including rest energy - and momentum must be conserved before and after the collision.

First, conservation of momentum. Before the decay, since the pion is at rest, we have zero momentum. Therefore, afterward, the muon and antineutrino must have equal and opposite momenta. This means we can essentially treat this as a one-dimensional problem, and not bother with vectors. A consolation prize of sorts.

$$\text{initial momentum} = \text{final momentum} \tag{12}$$
$$p_\pi = p_\mu + p_\nu \tag{13}$$
$$0 = p_\mu + p_\nu \tag{14}$$
$$\Longrightarrow p_\nu = -p_\mu = -\gamma_\mu m_\mu v_\mu \tag{15}$$

For the last step, we made use of the fact that relativistic momentum is $p = \gamma m v$. Now we can also write down conservation of energy. Before the decay, we have only the rest energy of the pion. Afterward, we have the energy of both the muon and antineutrino. The muon has both kinetic energy and rest energy, and we can write its total energy in terms of γ and its rest mass, $E = \gamma m c^2$.

The antineutrino has negligible mass, and therefore no kinetic energy, but we can still assign it a total energy based on its momentum, $E=pc$.

$$\text{initial energy} = \text{final energy} \tag{16}$$
$$E_\pi = E_\mu + E_\nu \tag{17}$$
$$m_\pi c^2 = \gamma_\mu m_\mu c^2 + p_\nu c \tag{18}$$
$$m_\pi = \gamma_\mu m_\mu + \frac{p_\nu}{c} \tag{19}$$

Now we can combine these two conservation results and try to solve for the velocity of the muon:

$$m_\pi = \gamma_\mu m_\mu + \frac{p_\nu}{c} = \gamma_\mu m_\mu - \gamma_\mu m_\mu \frac{v_\mu}{c} \tag{20}$$
$$\frac{m_\pi}{m_\mu} = \gamma_\mu - \gamma_\mu \frac{v_\mu}{c} = \gamma \left[1 - \frac{v_\mu}{c}\right] \tag{21}$$

We will need to massage this quite a bit more to solve for v_μ ...

$$\frac{m_\pi}{m_\mu} = \gamma \left[1 - \frac{v_\mu}{c}\right] = \frac{1 - \frac{v_\mu}{c}}{\sqrt{1 - \frac{v_\mu^2}{c^2}}} \tag{22}$$
$$\left(\frac{m_\pi}{m_\mu}\right)^2 = \frac{\left(1 - \frac{v_\mu}{c}\right)^2}{1 - \frac{v_\mu^2}{c^2}} = \frac{\left(1 - \frac{v_\mu}{c}\right)^2}{\left(1 - \frac{v_\mu}{c}\right)\left(1 + \frac{v_\mu}{c}\right)} = \frac{1 - \frac{v_\mu}{c}}{1 + \frac{v_\mu}{c}} \tag{23}$$

Now we're getting somewhere. Take what we have left, and solve it for v_μ ... we will leave that as an exercise to the reader, and quote only the result, using the given masses of the pion and muon:

$$\frac{v_\mu}{c} = \frac{1 - \left(\frac{m_\pi}{m_\mu}\right)^2}{1 + \left(\frac{m_\pi}{m_\mu}\right)^2} \approx -0.270 \tag{24}$$

From here, we are home free. We can calculate γ_μ and the muon's kinetic energy first. It is convenient to remember that the electron mass is $511\,\mathrm{keV/c^2}$.

$$\gamma_\mu = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.27c)^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.27^2}} \approx 1.0386 \tag{25}$$
$$\mathrm{KE}_\mu = (\gamma_\mu - 1)\, m_\mu c^2 = (1.0386 - 1)\,(207 m_{e^-})\, c^2 \tag{26}$$
$$= 0.0386 \left(207 \cdot 511\,\mathrm{keV}/c^2\right) c^2 \approx 4.08 \times 10^6\,\mathrm{eV} = 4.08\,\mathrm{MeV} \tag{27}$$

Finally, we can calculate the energy of the antineutrino as well:

$$E_\nu = p_\nu c = -p_\mu c = -\gamma_\mu m_\mu v_\mu = -1.0386 \cdot \left(207 \cdot 5.11\,\mathrm{keV}/c^2\right) \cdot (-0.270c) \tag{28}$$
$$\approx 2.96 \times 10^7\,\mathrm{eV} = 29.6\,\mathrm{MeV} \tag{29}$$

7. An electron and a proton are each accelerated starting from rest through a potential difference of 10.0 million volts (10^7 V). Find the momentum (in MeV/c) and kinetic energy (in MeV) of each, and compare the results with the classical expectation. Recall $PE = q\Delta V$.

Solution: The key is conservation of energy. Each particle has a charge $|q| = e$, and when accelerated through a potential difference of ΔV changes its potential energy by $e\Delta V$. This must equal the change in kinetic energy of the particle. Thus, $K = e\Delta V = 10^7\,\mathrm{eV} = 1.6 \times 10^{-12}$ J for both. From this, we can use the relativistic kinetic energy expression to find the velocity, and from that the momentum. The algebraic expressions are the same for both - the only difference between the two cases is the particle mass.

$$K = q\Delta V = (\gamma - 1)\, mc^2 \tag{30}$$
$$\gamma = \frac{q\Delta V}{mc^2} + 1 \tag{31}$$
$$p = \gamma m v = \gamma m \sqrt{1 - \frac{1}{\gamma^2}} \tag{32}$$

The last expression is one of convenience – we can just solve for γ this way, rather than bothering to solve for v. Makes no difference in the end, just saves a couple of lines of algebra. Classically, we would expect:

$$K_{cl} = q\Delta V = \frac{1}{2} m v^2 \tag{33}$$
$$v_{cl} = \sqrt{\frac{2q\Delta V}{m}} \tag{34}$$
$$p_{cl} = mv = \sqrt{2mq\Delta V} \tag{35}$$

For the electron, we have:

$$K = 10\,\mathrm{MeV} = 1.6 \times 10^{-12}\,\mathrm{J} \tag{36}$$
$$p = 10.49\,\mathrm{MeV/c} = 5.6 \times 10^{-21}\,\mathrm{kg \cdot m/s} \tag{37}$$
$$K_{cl} = K = 10\,\mathrm{MeV} \tag{38}$$
$$p_{cl} = 3.20\,\mathrm{MeV/c} = 1.71 \times 10^{-21}\,\mathrm{kg \cdot m/s} \tag{39}$$

For the proton, we have

$$K = 10\,\mathrm{MeV} = 1.6\times10^{-12}\,\mathrm{J} \tag{40}$$
$$p = 137\,\mathrm{MeV/c} = 7.34\times10^{-20}\,\mathrm{kg\cdot m/s} \tag{41}$$
$$K_{cl} = K = 10\,\mathrm{MeV} \tag{42}$$
$$p_{cl} = 137\,\mathrm{MeV/c} = 7.31\times10^{-20}\,\mathrm{kg\cdot m/s} \tag{43}$$

For the heavier proton, a potential energy of 10 MeV only accelerates it about $\gamma\approx1.01$, and classical mechanics works just fine. The same energy transferred to the much lighter electron accelerates it to $\gamma\approx20.6$, well into the relativistic regime.

8. An electron is released from rest and falls under the influence of gravity. **(a)** How much power does it radiate? **(b)** How much energy is lost after it falls 1 m? (*Hint:* $P=\Delta K/\Delta t$, $y=\frac{1}{2}gt^2$.)

Solution: The power emitted by a charge e with acceleration a is

$$P = \frac{e^2a^2}{6\pi\epsilon_o c^3} \tag{44}$$

In this case, under free fall the electron's acceleration is $g\approx9.81\,\mathrm{m/s^2}$, which gives

$$P = \frac{e^2g^2}{6\pi\epsilon_o c^3} \approx 5\times10^{-52}\,\mathrm{W} \tag{45}$$

In a time t, starting from rest, an object under the influence of gravity falls a distance $\Delta y=\frac{1}{2}gt^2$. Knowing the electron falls $\Delta y=1\,\mathrm{m}$, the time it takes is

$$t = \sqrt{\frac{2\Delta y}{g}} \approx 0.45\,\mathrm{s} \tag{46}$$

Since the power dissipation is constant, the energy lost is just power times time (since $P=\Delta E/\Delta t$):

$$\Delta E = Pt = \frac{e^2g^2}{6\pi\epsilon_o c^3}\sqrt{\frac{2\Delta y}{g}} \approx 2.5\times10^{-52}\,\mathrm{J} \tag{47}$$

An utterly negligible amount. We don't need to worry about radiation of charges accelerated by gravity.

9. How fast must a rocket travel relative to the earth so that time in the rocket "slows down" to half its rate as measured by earth-based observers? Do present-day jet planes approach such speeds?

Solution: The question wants to know how much time is slowed down compared to the earth-based observers, which means the earth-based observers have the 'proper' time. The rocket must experience dilated time by comparison. Thus, the elapsed time must be related by

$$\Delta t'_{\text{rocket}} = \gamma \Delta t_{\text{earth}} = \frac{\Delta t_{\text{earth}}}{\sqrt{1 - v^2/c^2}} \tag{48}$$

Time slowing down by a factor two implies

$$\frac{\Delta t'_{\text{rocket}}}{\Delta t_{\text{earth}}} = 2 = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{49}$$

$$\implies \frac{1}{4} = 1 - \frac{v^2}{c^2} \tag{50}$$

$$\implies v = \frac{\sqrt{3}}{2} c \approx 0.866c \approx 2.6 \times 10^8 \text{ m/s} \tag{51}$$

10. A cube of metal with sides of length a sits at rest in frame S with one edge parallel to the x-axis. Therefore, in S the cube has volume a^3. Frame S' moves along the x-axis with speed u. As measured by an observer in frame S', what is the volume of the metal cube?

Solution: Since relative motion is involved, there must be length contraction for the moving observer - the person in S' since we are observing the block. Since there is relative motion only along the x axis, there is length contraction only along that axis. The cube therefore appears shortened by a factor γ along the x axis, but its dimensions along y and z are the same. The volume in the two frames is thus:

$$V = a \cdot a \cdot a = a^3 \qquad \text{in } S \tag{52}$$

$$V' = \frac{a}{\gamma} \cdot a \cdot a = \frac{a^3}{\gamma} \qquad \text{in } S' \tag{53}$$

11. One of the wavelengths of light emitted by hydrogen atoms under normal laboratory conditions is $\lambda = 656.3\,\text{nm}$, in the red portion of the electromagnetic spectrum. In the light emitted from a distant galaxy this same spectral line is observed to be Doppler-shifted to $\lambda = 953.4\,\text{nm}$, in the infrared portion of the spectrum. How fast are the emitting electrons moving relative to the earth? Are they approaching the earth or receding from it?

Solution: The relativistic Doppler shift is given by

$$\lambda_o = \lambda_s \sqrt{\frac{c+v}{c-v}} \tag{54}$$

where λ_o is the wavelength observed in relative motion at velocity v with respect to the source, and λ_s is the wavelength observed in the source's frame. Positive velocities correspond to observers approaching the source. We are given both wavelengths: $\lambda_o = 656.3\,\text{nm}$ and $\lambda_s = 953.4\,\text{nm}$. Since $\lambda_o > \lambda_s$, from the equation above it is clear that we must have $v < 0$ for this to be true, which already tells the source is receding. All we need to do is solve the above for v to find the speed, which gives

$$\frac{v}{c} = \frac{\left(\frac{\lambda_o}{\lambda_s}\right)^2 - 1}{\left(\frac{\lambda_o}{\lambda_s}\right)^2 + 1} \approx 0.357 \tag{55}$$

12. Two particles in a high-energy accelerator experiment approach each other head-on with a relative speed of $0.890c$. Both particles travel at the same speed as measured in the laboratory. What is the magnitude of the velocity of one particle relative to the other?

Solution: What we are given is the speed of the two particles relative to each other. That is, if we were in the reference frame of one of the particles, we would say the other approaches with $u = 0.890c$. In the observer's frame (call it S'), we see the particles moving toward each other, each with the same speed v'. In the S' frame of reference, we would have to say that adding the two velocities v' together gives us $u = 0.890c$.

$$u = \frac{v' + v'}{1 + v'v'/c^2} = 0.890c \tag{56}$$

Solving this for v' gives us velocity of one particle relative to the other in the lab frame. The result is

$$v' = \frac{2 \pm \sqrt{4 - 4u^2/c^2}}{2u/c^2} = \frac{c^2}{u}\left(1 \pm \sqrt{1 - u^2/c^2}\right) \approx \{0.611c, 1.64c\} \tag{57}$$

Clearly, the second root, while mathematically allowed, does not make physical sense - we've already established velocities can't be greater than c. Therefore we reject it as unphysical, and the remaining valid solution is $v \approx 0.611c$.

13. **(a)** Through what potential difference does an electron have to be accelerated, starting from rest, to achieve a speed of $0.980c$? **(b)** What is the kinetic energy of the electron at this speed?

Express your answer in both joules and electron volts.

Solution: The key is to remember that a charge q moving through a potential difference ΔV changes its potential energy by $\Delta U = q\Delta V$. If we are not worrying about resistive forces, this change in potential energy is equal to the charge's change in kinetic energy. Starting from rest, we know that must be $\Delta K = (\gamma - 1)\, mc^2$, with $\gamma = 1/\sqrt{1 - v^2/c^2}$.

For an electron $q = e$, and $mc^2 = 511\,\mathrm{keV}$, and with $v = 0.980c$ we have $\gamma \approx 5.025$. Putting it all together,

$$\Delta K = \Delta U = e\Delta V = (\gamma - 1)\, mc^2 = (5.025 - 1)\left(511 \times 10^3\,\mathrm{eV}\right) \approx 2.06 \times 10^6\,\mathrm{eV} \tag{58}$$

Since $e\Delta V$ is the particle's change in both potential and kinetic energy, this is already the answer to the second part of the question: the particle's kinetic energy is about $2.06\,\mathrm{MeV}$, or about $0.33\,\mathrm{pJ}$ ($\mathrm{p} = 10^{-12}$). The corresponding potential difference is by definition $2.06\,\mathrm{MV}$, illustrating how handy a unit the electron volt is.

14. Use the following two equations:

$$\vec{\mathbf{p}} = \frac{m\vec{\mathbf{v}}}{\sqrt{1 - v^2/c^2}} \tag{59}$$

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{60}$$

to derive the following relationship:

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

Solution: No big trick, just grind through it. Since you know the result you want to get to, start by finding p^2c^2.

$$p^2c^2 = \frac{m^2v^2c^2}{1 - v^2/c^2} = m^2c^4\left(\frac{v^2}{c^2 - v^2}\right) \tag{61}$$

Now add m^2c^4 and rearrange, and you've got it.

$$p^2c^2 + m^2c^4 = m^2c^4\left(1 + \frac{v^2}{c^2 - v^2}\right) = m^2c^4\left(\frac{c^2 + v^2 - v^2}{c^2 - v^2}\right) \tag{62}$$

$$= m^2c^4\frac{c^2}{c^2 - v^2} = m^2c^4\left(\frac{1}{1 - v^2/c^2}\right) = \gamma^2m^2c^4 \tag{63}$$

$$\sqrt{p^2c^2 + m^2c^4} = \gamma mc^2 = E \tag{64}$$

15. At what speed is the momentum of a particle twice as great as the result obtained from the non-relativistic expression mv? Express your answer in terms of the speed of light.

Solution: Relativistic momentum is $p = \gamma mv$, classically we would write $p = mv$. The latter is off by a factor of two when

$$\gamma mv = 2mv \tag{65}$$

$$\gamma = 2 \tag{66}$$

$$|v| = \frac{\sqrt{3}}{2}c \tag{67}$$

16. Light travels with respect to earth at $3 \times 10^8\ \frac{m}{s}$. A rocket travels at $2.5 \times 10^8\ \frac{m}{s}$ with respect to earth in opposite direction of the light. What is the speed of light as viewed from the rocket?

Solution: It is light, the speed is always c in vacuum.