University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Spring 2010

Exam 1 practice problems

General comments:

- There will be at least one problem that rewards those with mathematical fluency. I.e., the concept will be straightforward, the mathematics will not be.
- There will be at least one problem that rewards those with good physical intuition. I.e., the mathematics will be straightforward, the concept may not be.
- It is *extremely* unlikely that any exam problems will involve plugging numbers into a single formula. Most problems will not *have* numerical answers.
- A couple of these problems could show up verbatim on the exam.

1. Show that the angular frequency of a charge moving in a uniform magnetic field is given by

$$\omega = \frac{qB}{m}\sqrt{1-\frac{u^2}{c^2}} \tag{1}$$

2. An excited atom of mass m, initially at rest in frame S, emits a photon and recoils. The internal energy of the atom decreases by ΔE and the energy of the photon is hf. Show that

$$hf = \Delta E\left(1-\frac{\Delta E}{2mc^2}\right) \tag{2}$$

3. An electron moving initially with constant speed v is brought to rest with uniform deceleration a lasting for a time $t\!=\!v/a$. Compare the electromagnetic energy radiated during the deceleration with the electron's initial kinetic energy. Express the ratio in terms of two lengths, the distance light travels in time t and the classical electron radius, $r_e = \frac{e^2}{4\pi\epsilon_o m_e c^2} = \frac{k_e e^2}{m_e c^2}$. Recall the Larmor formula, which gives the total power radiated from an accelerating charge ($v\!\ll\!c$):

$$P = \frac{e^2a^2}{6\pi\epsilon_o c^3} = \frac{2k_e e^2 a^2}{3c^3} \tag{3}$$

4. An atomic clock aboard a spaceship runs slow compared to an Earth-based atomic clock at a rate of 1.0 second per day. What is the speed of the spaceship?

5. The speed of light with respect to a medium is $v'_x = c/n$, where n is the index of refraction. Suppose that the medium, say, flowing water, is moving past a stationary observer in the same direction as the light with speed V. Show that the observer measures the speed of light to be approximately

$$v_x = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right) V \tag{4}$$

This effect was first observed by Fizeau in 1851.

6. A mass M at rest decays into two particles of mass m_1 and m_2. Show that the magnitude of the momentum of each of the two particles is

$$\frac{p}{c} = \frac{\sqrt{M^2 - (m_1 + m_2)^2}\sqrt{M^2 - (m_1 - m_2)^2}}{2M} \tag{5}$$

Hint: conserve energy and momentum, and use $E^2 = m^2c^4 + p^2c^2$.

7. For an experiment on the Compton effect, you want the X rays emerging at 90° from the incident direction to suffer an increase of wavelength by a factor 2. What wavelength to you need for your incident X rays?

8. X-ray photons of wavelength 0.154 nm are produced by a copper source. Suppose that 1.00×10^{18} of these photons are absorbed by the target each second.

(a) What is the total momentum p transferred to the target each second?
(b) What is the total energy E of the photons absorbed by the target each second?
(c) For these values, verify that the force on the target is related to the rate of energy transfer by

$$\frac{dp}{dt} = \frac{1}{c}\frac{dE}{dt} \tag{6}$$

9. An interstellar space probe is moving at a constant speed relative to earth of $0.76c$ toward a distant planet. Its radioisotope generators have enough energy to keep its data transmitter active continuously for 15 years, as measured in their own reference frame. **(a)** How long do the generators last as measured from earth? **(b)** How far is the probe from earth when the generators fail, as measured from earth? **(c)** How far is the probe from earth when the generators fail, *as measured by its built-in trip odometer*?

10. A pion at rest ($m_\pi = 273\, m_{e^-}$) decays to a muon ($m_\mu = 207\, m_{e^-}$) and an antineutrino ($m_{\overline{\nu}} \approx 0$). This reaction is written as $\pi^- \to \mu^- + \overline{\nu}$. Find the kinetic energy of the muon and the energy of the antineutrino in electron volts. *Hint: relativistic momentum is conserved.*

11. The electromagnetic wave equation in one dimension is given by

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = 0 \tag{7}$$

Prove that wave equation retains its form under Lorentz transformations. That is, show that

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = \frac{\partial^2 \varphi}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t'^2} = 0 \tag{8}$$

where x, x', t, and t' are related by the Lorentz transformations. You may want to recall the chain rule: if $f = f(x', t')$, then

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial x} \tag{9}$$