Name & CWID ____

PH 253 Exam I

Instructions

1. **Solve four of the six problems below. All problems have equal weight.**
2. Clearly mark your which problems you have chosen using the tick box.
3. Do your work on separate sheets. Staple them to this exam paper when you are finished.
4. You are allowed 1 sheet of standard 8.5x11 in paper and a calculator.

□ **1.** In a test of the relativistic time dilation effect, physicists compared the rates of vibration of nuclei of iron moving at different speeds. One sample of iron nuclei was placed on the rim of a high-speed rotor; another sample of similar nuclei was placed at the center. The radius of the rotor was 0.1 m, and it was rotated at 35000 rev/min. **(a)** Under these conditions, what was the speed of the rim of the rotor relative to the center? **(b)** What was the time dilation factor of the sample at the rim compared with the sample at the center? Note the following useful approximations for $v \ll c$:

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2} \approx 1-\frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{1}$$

$$\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2} \approx 1+\frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{2}$$

□ **2.** A K^0 particle *at rest* spontaneously decays into a π^+ particle and a π^- particle. What will be the speed of each of the latter? The mass of the K^0 is 8.87×10^{-28} kg, and the masses of the π^+ and π^- particles are 2.49×10^{-28} kg each.

□ **3.** Show that at long wavelengths, Planck's radiation law (see formula sheet) reduces to the Rayleigh-Jeans law,

$$I(\lambda, T) = \frac{(\text{const})\, k_b T}{\lambda^4} \tag{3}$$

Note the series expansion $e^x = \sum x^n/n!$.

□ **4.** A 0.700 MeV photon (Compton) scatters off a free electron such that the scattering angle of the photon is twice the scattering angle of the electron. Determine **(a)** the scattering angle for the electron and **(b)** the final speed of the electron.

Name & CWID ______________________________

□ 5. X-ray photons of wavelength $0.154\,\text{nm}$ are produced by a copper source. Suppose that 1.00×10^{18} of these photons are absorbed by the target each second.

(a) What is the total momentum p transferred to the target each second?
(b) What is the total energy E of the photons absorbed by the target each second?
(c) If the beam shines perpendicularly onto a perfectly reflecting surface, what force does it exert on the surface? Recall $F = \Delta p / \Delta t$.
(d) For these values, verify that the force on the target is related to the rate of energy transfer by

$$\frac{dp}{dt} = \frac{1}{c}\frac{dE}{dt} \tag{4}$$

□ 6. A student studying the photoelectric effect from two different metals records the following information: (i) the stopping potential for photoelectrons released from metal 1 is $1.48\,\text{V}$ larger than that for metal 2, and (ii) the threshold frequency for metal 1 is 40.0% smaller than that for metal 2. Determine the work function of each metal.

Constants:

$$N_A = 6.022\times10^{23}\text{ things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755\times10^9\ \text{N}\cdot\text{m}^2\cdot\text{C}^{-2}$$
$$\epsilon_o = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$$
$$\mu_o \equiv 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$$
$$e = 1.60218\times10^{-19}\ \text{C}$$
$$h = 6.6261\times10^{-34}\ \text{J}\cdot\text{s} = 4.1357\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = \frac{h}{2\pi}$$
$$k_B = 1.38065\times10^{-23}\ \text{J}\cdot\text{K}^{-1} = 8.6173\times10^{-5}\ \text{eV}\cdot\text{K}^{-1}$$
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792\times10^8\ \text{m/s}$$
$$hc = 1240\ \text{eV}\cdot\text{nm}$$
$$m_e = 9.10938\times10^{-31}\ \text{kg} \qquad m_e c^2 = 510.998\ \text{keV}$$
$$m_p = 1.67262\times10^{-27}\ \text{kg} \qquad m_p c^2 = 938.272\ \text{MeV}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Basic Equations:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a} \qquad \text{Newton's Second Law}$$
$$\vec{F}_{centr} = -\frac{mv^2}{r}\hat{r} \qquad \text{Centripetal}$$

E & M

$$\vec{F}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

EM Waves:

$$c = \lambda f = \frac{|\vec{E}|}{|\vec{B}|}$$
$$I = \left[\frac{\text{photons}}{\text{time}}\right]\left[\frac{\text{energy}}{\text{photon}}\right]\left[\frac{1}{\text{Area}}\right]$$
$$I = \frac{\text{energy}}{\text{time}\cdot\text{area}} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{\text{power }(\mathscr{P})}{\text{area}} = \frac{E_{max}^2}{2\mu_0 c}$$

Blackbody

$$E_{tot} = \sigma T^4 \qquad \sigma = 5.672\times10^{-8}\ \text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$$
$$T\lambda_{max} = 0.29\times10^{-2}\ \text{m}\cdot\text{K} \qquad \text{Wien}$$
$$E_{quantum} = hf$$
$$E_{oscillator} = hf/\left(e^{hf/k_BT}-1\right)$$
$$I(\lambda,T) = \frac{(\text{const})}{\lambda^5}\left[e^{\frac{hc}{\lambda k_b T}}-1\right]^{-1}$$
$$I(f,t) = (\text{const})\,f^3\left[e^{\frac{hf}{k_b T}}-1\right]^{-1}$$

Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\Delta t'_{moving} = \gamma\Delta t_{stationary} = \gamma\Delta t_p$$
$$L'_{moving} = \frac{L_{stationary}}{\gamma} = \frac{L_p}{\gamma}$$
$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$v_{obj} = \frac{v + v'_{obj}}{1+\frac{vv'_{obj}}{c^2}} \qquad v'_{obj} = \frac{v_{obj}-v}{1-\frac{vv_{obj}}{c^2}}$$
$$KE = (\gamma-1)mc^2 = \sqrt{m^2c^4+c^2p^2} - mc^2$$
$$E_{rest} = mc^2$$
$$p = \gamma mv$$
$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$

Quantum

$$E = hf \qquad p = h/\lambda = E/c \qquad \lambda f = c \qquad \text{photons}$$
$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)$$
$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{\gamma mv} \approx \frac{h}{mv}$$
$$\Delta x\Delta p \geqslant \frac{h}{4\pi}$$
$$\Delta E\Delta t \geqslant \frac{h}{4\pi}$$
$$eV_{stopping} = KE_{electron} = hf - \varphi = hf - W$$

Calculus of possible utility:

$$\int\frac{1}{x}dx = \ln x + c$$
$$\int u\,dv = uv - \int v\,du$$
$$\int\sin ax\,dx = -\frac{1}{a}\cos ax + C$$
$$\int\cos ax\,dx = \frac{1}{a}\sin ax + C$$
$$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$$

Vectors:

$$|\vec{F}| = \sqrt{F_x^2+F_y^2} \quad \text{magnitude} \qquad \theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$
$$\hat{r} = \vec{r}/|\vec{r}| \quad \text{construct any unit vector}$$
$$\text{let}\quad \vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z} \quad\text{and}\quad \vec{b} = b_x\hat{x} + b_y\hat{y} + b_z\hat{z}$$
$$\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z = \sum_{i=1}^{n}a_ib_i = |\vec{a}||\vec{b}|\cos\theta$$
$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$
$$\vec{a}\times\vec{b} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} = (a_yb_z - a_zb_y)\hat{x} + (a_zb_x - a_xb_z)\hat{y} + (a_xb_y - a_yb_x)\hat{z}$$