1

a) $v_{rim} = r\omega$ $\qquad \omega = \frac{35000 \text{ rev}}{\text{min}} \cdot \frac{1 \text{ min}}{60 s} \cdot \frac{2\pi \text{ rad}}{\text{rev}} = 3665 \text{ rad/s}$

$$v_{rim} = (0.1 m)(3665 \text{ rad/sec}) \approx \underline{370 \text{ m/s}}$$

or, $v_{rim} = \left(\frac{35000 \text{ rev}}{\text{min}}\right)\cdot\left(\frac{1 \text{ min}}{60 \sec}\right)\cdot\underbrace{(2\pi r)}_{\text{circ.}} \approx 370 \text{ m/s}$

b) center is at rest, measures proper time

$$\Delta t_{rim} = \gamma \Delta t_p$$

so dilation factor $= \frac{\Delta t_{rim}}{\Delta t_p} = \gamma = \frac{1}{\sqrt{1-v^2/c^2}} \approx 1 + \frac{1}{2}\left(\frac{v}{c}\right)^2$

$$\approx \underline{1 + 7\times 10^{-13}}$$

or, $\Delta t_{rim} - \Delta t_p = \Delta t_p(\gamma - 1) = \Delta t_p\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) \approx \Delta t_p\left(\frac{1}{2}\frac{v^2}{c^2}\right)$

$$\approx 7\times 10^{-13} \Delta t_p$$

$$\frac{\Delta t_{rim} - \Delta t_p}{\Delta t_p} \approx 7\cdot 10^{-13}$$

(2)

$\overset{\circ}{K^0}$ $\quad\quad \pi^- \longleftarrow \bullet \longrightarrow \pi^+$

p conv: $\quad P_- = P_+ \quad$ or $\quad \gamma_+ m_+ v_+ = \gamma_- m_- v_-$

Since $m_- = m_+ \quad \gamma_+ v_+ = \gamma_- v_-$

Since γ is a function of v,

$\Rightarrow \quad v_+ = -v_- \;, \quad \gamma_+ = \gamma_-$

π^+ and π^- have equal speed, opp. dir

call it v, $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$; let $m_+ = m_- \equiv m$

Conv. E: $\quad \underset{\text{has rest mass}}{m_K c^2} = \underset{\text{total } E}{\gamma_+ m_+ c^2 + \gamma_- m_- c^2} = 2\gamma m c^2$

$$\Rightarrow \quad m_K = 2m\gamma = 2m \cdot \left(\frac{1}{\sqrt{1-v^2/c^2}}\right)$$

$$1 - \frac{v^2}{c^2} = \frac{4m^2}{m_K^2}$$

$$\boxed{\frac{v}{c} = \sqrt{1 - \frac{4m^2}{m_K^2}} \simeq 0.83}$$

or $v \simeq 2.5 \times 10^8$ m/s

note: need $4m^2 < m_K^2$ or $m_K > 2m_\pi$

to create 2 identical* π decay products

and have E-p conservation

(* in terms of mass anyway)

③ $I(\lambda, T) = \left(\frac{\text{const}}{\lambda^5}\right) \frac{1}{e^{hc/\lambda kT} - 1}$

long λ, small $\frac{hc}{\lambda kT} << 1$

then $e^{hc/\lambda kT} \approx 1 + \left(\frac{hc}{\lambda kT}\right) + \frac{1}{2}\left(\frac{hc}{\lambda kT}\right)^2 + \cdots = \sum_{n=0}^{\infty} \left(\frac{hc}{\lambda kT}\right)^n \cdot \frac{1}{n!}$

take 1^{st} term for large λ a small $\frac{hc}{\lambda kT} << 1$

$$\Rightarrow I(\lambda, T) \cong \left(\frac{\text{const}}{\lambda^5}\right) \frac{1}{1 + \frac{hc}{\lambda kT} - 1} = \left(\frac{\text{const}}{\lambda^5}\right) \frac{\lambda kT}{hc}$$

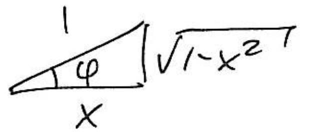
$$\simeq \left(\frac{kT}{hc}\right)(\text{const}) \frac{1}{\lambda^4}$$

or $I(\lambda, T) = (\text{const}') / \lambda^4$ ✓

Recall HW3: $\tan\varphi\left(1+\frac{hf_i}{mc^2}\right) = \frac{1}{\tan\frac{\theta}{2}} = \frac{1}{\tan\varphi}$

$\Rightarrow \frac{1}{\tan^2\varphi} = 1+\frac{hf_i}{mc^2}$ or $\boxed{\tan^2\varphi = \frac{1}{1+\frac{hf_i}{mc^2}}}$

Consistent?



$\Rightarrow \tan\varphi = \frac{\sqrt{1-x^2}}{x}$

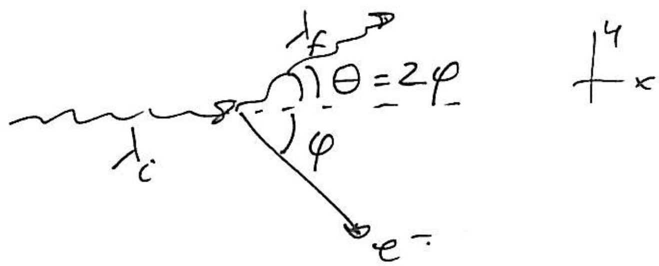
$\cos\varphi = x$

$$\tan\varphi = \frac{\sqrt{1-\frac{mc^2+E_0}{2mc^2+E_0}}}{\sqrt{\frac{mc^2+E_0}{2mc^2+E_0}}} = \sqrt{\frac{mc^2}{mc^2+E_0}} = \sqrt{\frac{1}{1+\frac{E_0}{mc^2}}}$$

or $\frac{1}{\tan^2\varphi} = 1+\frac{hf_i}{mc^2}$ ✓

④

a)



Compton: $\lambda_f - \lambda_i = \frac{h}{mc}(1-\cos\theta)$ (1)   note: $\frac{hc}{\lambda_i} = hf_i = E_i$

energy: $\frac{hc}{\lambda_i} = \frac{hc}{\lambda_f} + (\gamma - 1)mc^2$ (2)   note $\begin{cases} \sin 2\varphi = 2\sin\varphi\cos\varphi \\ \cos 2\varphi = 2\cos^2\varphi - 1 \end{cases}$

P_x: $\frac{h}{\lambda_i} = \frac{h}{\lambda_f}\cos\theta + \gamma mv\cos\varphi$ (3)

P_y: $\frac{h}{\lambda_f}\sin\theta = \gamma mv\sin\varphi$ (4)

(4) + trig $\quad \frac{h}{\lambda_f}\sin\theta = \frac{h}{\lambda_f}\sin 2\varphi = \frac{2h}{\lambda_f}\sin\varphi\cos\varphi = \gamma mv\sin\varphi$

$\Rightarrow \gamma mv = \frac{2h}{\lambda_f}\cos\varphi$ put in (3)

(3) $\frac{h}{\lambda_i} = \frac{h}{\lambda_f}\cos\theta + \left(\frac{2h}{\lambda_f}\cos\varphi\right)\cos\varphi = \frac{h}{\lambda_f}(2\cos^2\varphi - 1) + \frac{2h}{\lambda_f}\cos^2\varphi$

$\Rightarrow \frac{h}{\lambda_i} = \frac{4h}{\lambda_f}\cos^2\varphi - \frac{h}{\lambda_f}$

$\frac{1}{\lambda_i} = \frac{4}{\lambda_f}\cos^2\varphi - \frac{1}{\lambda_f} \Rightarrow \lambda_f = 4\lambda_i\cos^2\varphi - \lambda_i$

put in Compton

$\lambda_f - \lambda_i = 4\lambda_i\cos^2\varphi - 2\lambda_i = \frac{2h}{mc}(1-\cos^2\varphi)$

$\left(4\lambda_i + \frac{2h}{mc}\right)\cos^2\varphi = 2\lambda_i + \frac{2h}{mc}$ note $\frac{h}{mc} = \lambda_c$

$$\boxed{\cos^2\varphi = \frac{\lambda_i + \lambda_c}{2\lambda_i + \lambda_c} = \frac{mc^2 + E_0}{2mc^2 + E_0}} \Rightarrow \boxed{\varphi \approx 33°}$$

w/ $E_0 = hf_i = \frac{hc}{\lambda_i}$

(5)

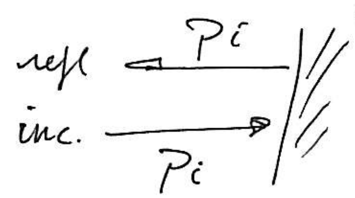
a) $P_{\substack{net \\ per\,sec}} = (10^{18}\,\text{photons/sec})\left(\frac{h}{\lambda}\right) = 4.3\times10^{-6}\ \frac{kg\cdot m}{s}$ per sec

or $\frac{\Delta P_{net}}{\Delta t} = 4.3\times10^{-6}\ \frac{kg\, m}{s^2} = \underline{\underline{4.3\times10^{-6}\,N}}$

b) $E_{\substack{tot \\ per\,sec}} = (10^{18}\,\text{photons/sec})\left(\frac{hc}{\lambda}\right) = cP_{net} = 1.29\times10^{3}\ J/sec$

$= \underline{\underline{1.29\times10^{3}\,W}} = \frac{\Delta E}{\Delta t}$

c) momentum change on reflection is twice incident p!



$\Delta P_{tot} = 2p_i$

$\Rightarrow \quad F_{net} = 2\frac{\Delta P_{net}}{\Delta t} = 2(4.3\times10^{-6}N) = \underline{\underline{8.6\times10^{-6}\,N}}$

d) $\frac{dp/dt}{dE/dt} = \frac{4.3\times10^{-6}\,N}{1.29\times10^{3}\,W} \simeq \frac{1}{3\cdot10^{8}} \simeq \frac{1}{c}$

$\frac{h/\lambda}{hc/\lambda} = \frac{1}{c}$ as expected

(6)

$V_2 = V_1 + 1.48$

at threshold, $KE = V = 0$

$KE = V_{stop} = hf - \varphi = 0$

$\Rightarrow$ at threshold, $hf_{th} = \varphi$

if $f_{th,1} = 0.6 f_{th,2}$ then $\varphi_1 = 0.6 \varphi_2$

$eV_1 = hf - \varphi_1$

$eV_2 = hf - \varphi_2 = eV_1 - 1.48$

$\Rightarrow 1.48 = \varphi_2 - \varphi_1 = \varphi_2 - 0.6\varphi_2 = 0.4\varphi_2$

$$\Rightarrow \boxed{\begin{aligned} \varphi_2 &= \frac{1.48}{0.4} = 3.7\,eV \\ \varphi_1 &= 0.6\,\varphi_2 = 2.2\,eV \end{aligned}}$$