Name & CWID

PH 253 Exam II Makeup

Instructions

1. Solve three of the six problems below. All problems have equal weight.
2. Clearly mark your which problems you have chosen.
3. Do your work on separate sheets. Staple them to this exam paper when you are finished.
4. You are allowed 1 sheet of standard 8.5x11 in paper and a calculator.

1. The wavefunction of a particle in a double slit experiment with slit spacing d and slit width $w < d$ (w,d both positive quantities) in the plane of the slits is described by

$$\psi(x) = \begin{cases} C & -\frac{d}{2} - \frac{w}{2} \leqslant x \leqslant -\frac{d}{2} + \frac{w}{2} \qquad \text{(slit a)} \\ C & \frac{d}{2} - \frac{w}{2} \leqslant x \leqslant \frac{d}{2} + \frac{w}{2} \qquad \text{(slit b)} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(a) Determine the normalization constant C.
(b) Determine $\langle x \rangle$ and $\langle x^2 \rangle$ in the limit $w \ll d$, i.e., ignore any terms of order w/d and higher in the end result.
(c) Again for $w \ll d$, find the uncertainty in position $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

2. The Schrödinger equation for a simple harmonic oscillator of mass m can be written

$$-a^4 \frac{d^2\psi}{dx^2} + x^2\psi = \frac{2E}{C}\psi \tag{2}$$

where $a^4 = \hbar^2/mC$, C is the force constant, and E the energy. [i]
(a) Below are the wave functions for the first two states; find their energies in terms of $\hbar\omega_o$.
(b) Suggest a general formula for energy the n^{th} state. How does it differ from Planck's hypothesis for the energy of his oscillators?

$$\psi_0 = \left(\frac{1}{a\sqrt{\pi}}\right)^{1/2} e^{-x^2/2a^2}$$

$$\psi_1 = \left(\frac{1}{2a\sqrt{\pi}}\right)^{1/2} 2\left(\frac{x}{a}\right) e^{-x^2/2a^2}$$

To save you some time, we note $\frac{d}{dx}(e^{-x^2/2a^2}) = -\frac{x}{a^2}e^{-x^2/2a^2}$ and $\frac{d^2}{dx^2}(e^{-x^2/2a^2}) = \frac{x^2-a^2}{a^4}e^{-x^2/2a^2}$

3. The molecular bonding in the compound NaCl is predominantly ionic, and to a good approximation we can consider a sodium chloride molecule as consisting of two units – an Na^+ ion and a Cl^- ion – bound together. Assuming an electrostatic attraction and a power-law repulsion between the ions, their potential energy as a function of ion spacing has the form

[i] Note $\omega_o = 2\pi f_o = \sqrt{C/m}$, $a = (\hbar/\sqrt{mC})^{1/2} = \sqrt{\hbar/m\omega_o}$.

Name & CWID

$$V(r) = -\frac{ke^2}{r} + \frac{A}{r^n} \tag{3}$$

(a) Find the equilibrium spacing r_o.
(b) Find the potential energy at this separation, V_{min}.
(c) Find the effective "spring constant" for the molecule, assuming small deviations from r_o. One way to do this is to find the second derivative of $V(r)$ at $r=r_o$. $[(n-1)ke^2/r_o^3]$

4. **(a)** Using the Bohr model, what wavelength of photon is emitted when an electron in a hydrogen atom makes a transition from the 3d to 2p state?
(b) Show that in the presence of a magnetic field, the $3d \to 2p$ transition in hydrogen appears as three spectral lines. You may ignore spin, and assume only dipole transitions will occur (see formula sheet).

5. By considering the visible spectrum of hydrogen and He^+, show how you could determine spectroscopically if a sample of hydrogen was contaminated with helium. (Hint: look for differences in the visible emission lines, $\lambda \approx 390 \sim 750$ nm. A difference of 10 nm is easily measured.)

6. Find the most probable radius and the expected value of the radial position $\langle r \rangle$ of an electron in the 2p state.

$$\psi_{2p} = \frac{1}{\sqrt{3}\,(2a_o)^{3/2}} \frac{r}{a_o} e^{-r/2a_o} \tag{4}$$

where a_o is the Bohr radius, $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 0.529 \times 10^{-10}$ m. Make use of the integrals given on the formula sheet.

Constants:

$$N_A = 6.022 \times 10^{23}\text{ things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^9\ \mathrm{N\cdot m^2\cdot C^{-2}}$$
$$\epsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2}$$
$$\mu_o \equiv 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}$$
$$e = 1.60218 \times 10^{-19}\ \mathrm{C}$$
$$h = 6.6261 \times 10^{-34}\ \mathrm{J\cdot s} = 4.1357 \times 10^{-15}\ \mathrm{eV\cdot s}$$
$$\hbar = \frac{h}{2\pi} \qquad hc = 1239.84\ \mathrm{eV\cdot nm}$$
$$k_B = 1.38065 \times 10^{-23}\ \mathrm{J\cdot K^{-1}} = 8.6173 \times 10^{-5}\ \mathrm{eV\cdot K^{-1}}$$
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792 \times 10^8\ \mathrm{m/s}$$
$$m_e = 9.10938 \times 10^{-31}\ \mathrm{kg} \qquad m_e c^2 = 510.998\ \mathrm{keV}$$
$$m_p = 1.67262 \times 10^{-27}\ \mathrm{kg} \qquad m_p c^2 = 938.272\ \mathrm{MeV}$$
$$m_n = 1.67493 \times 10^{-27}\ \mathrm{kg} \qquad m_n c^2 = 939.565\ \mathrm{MeV}$$

Schrödinger

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V(x)\Psi$$
$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi$$
$$\int_{-\infty}^{\infty} |\psi(x)|^2\,dx = 1 \qquad P(\text{in }[x, x+dx]) = |\psi(x)|^2 \qquad \text{1D}$$
$$\int_0^{\infty} |\psi(r)|^2\,4\pi r^2\,dr = 1 \qquad P(\text{in }[r, r+dr]) = 4\pi r^2|\psi(r)|^2 \qquad \text{3D}$$
$$\langle x^n\rangle = \int_{-\infty}^{\infty} x^n P(x)\,dx \qquad \text{1D} \qquad \langle r^n\rangle = \int_0^{\infty} r^n P(r)\,dr \qquad \text{3D}$$
$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

Basic Equations:

$$\vec{F}_{net} = m\vec{a} \quad \text{Newton's Second Law}$$
$$\vec{F}_{centr} = -\frac{mv^2}{r}\,\hat{r} \quad \text{Centripetal}$$
$$\vec{F}_{12} = k_e\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\,\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Oscillators

$$E = \left(n + \frac{1}{2}\right)hf$$
$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA^2 = 2\pi^2 mf^2A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2 \qquad \tan x \approx x + \frac{1}{3}x^3$$
$$e^x \approx 1 + x + \frac{1}{2}x \qquad \sin x \approx x - \frac{1}{6}x^3 \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

Misc Quantum

$$E = hf \qquad p = h/\lambda = E/c \qquad \lambda f = c \qquad \text{photons}$$
$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$
$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{\gamma m v} \approx \frac{h}{mv}$$
$$\Delta x\Delta p \geqslant \frac{h}{4\pi} \qquad \Delta E\Delta t \geqslant \frac{h}{4\pi}$$
$$eV_{\text{stopping}} = KE_{\text{electron}} = hf - \varphi = hf - W$$

Bohr

$$E_n = -13.6\,\mathrm{eV}/n^2 \qquad \text{Hydrogen}$$
$$E_n = -13.6\,\mathrm{eV}\left(Z^2/n^2\right) \qquad Z\text{ protons, 1 }e^-$$
$$E_i - E_f = -13.6\,\mathrm{eV}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = hf$$
$$L = mvr = n\hbar$$
$$v^2 = \frac{n^2\hbar^2}{m_e^2 r^2} = \frac{k_e e^2}{m_e r}$$

Quantum Numbers

$$l = 0, 1, 2, \ldots, (n-1) \qquad L^2 = l(l+1)\hbar^2$$
$$m_l = -l, (-l+1), \ldots, l \qquad L_z = m_l\hbar$$
$$m_s = -\pm\frac{1}{2} \qquad S_z = m_s\hbar \qquad S^2 = s(s+1)\hbar^2$$
$$\text{dipole transitions: } \Delta l = \pm 1, \Delta m_l = 0, \pm 1, \Delta m_s = 0$$
$$\mu_{sz} = \pm\mu_B$$
$$\vec{\mu}_s = 2\vec{S}\,\mu_B$$
$$E_\mu = -\vec{\mu}\cdot\vec{B}$$
$$J^2 = j(j+1)\hbar^2 \qquad j = l \pm \frac{1}{2}$$
$$J_z = m_j\hbar \qquad m_j = -j, (-j+1), \ldots, j$$

Calculus of possible utility:

$$\int\frac{1}{x}\,dx = \ln x + c$$
$$\int u\,dv = uv - \int v\,du$$
$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$
$$\frac{d}{dx}\tan x = \sec^2 x = \frac{1}{\cos^2 x}$$
$$\int_0^{\infty} x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$
$$\int_0^{\infty} x^2 e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$
$$\int_{-\infty}^{\infty} x^3 e^{-ax^2}\,dx = \int_{-\infty}^{\infty} x e^{-ax^2}\,dx = 0$$
$$\int_0^{\infty} x^4 e^{-ax^2}\,dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$