PH 253 Exam I

Instructions

1. Solve 6 of the 8 problems below. All problems have equal weight.
2. Do your work on separate sheets.
3. You are allowed 1 sheet of standard 8.5x11 in paper and a calculator.

1. An electron and a proton are each accelerated starting from rest through a potential difference of 10.0 million volts (10^7 V). Find the momentum (in MeV/c) and kinetic energy (in MeV) of each, and compare the results with the classical expectation. Recall $\mathsf{PE} = q\Delta V$.

2. An electron is released from rest and falls under the influence of gravity. **(a)** How much power does it radiate? **(b)** How much energy is lost after it falls 1 m? (*Hint:* $P = \Delta K/\Delta t$, $y = \frac{1}{2}gt^2$.)

3. An electron initially moving at constant speed v is brought to rest with uniform deceleration a lasting for a time $t = v/a$. Compare the electromagnetic energy radiated during this deceleration with the electron's initial kinetic energy. Express the ratio in terms of two lengths, the distance light travels in time t and the classical electron radius $r_e = e^2/4\pi\epsilon_o mc^2$.

4. Observer O notes that two events are separated in space and time by 600 m and 8×10^{-7} s. How fast must observer O' be moving relative to O in order that the events seem simultaneous?

5. A bassist taps the lowest E on her bass at 140 beats per minute during one portion of a song. What tempo would an observer on a ship moving *toward* the bassist at 0.70c hear?

6. The proper lifetime of a certain particle is 100.0 ns. **(a)** How long does it live in the laboratory frame if it moves at $v = 0.960c$? **(b)** How far does it travel in the laboratory during that time? **(c)** What is the distance traveled in the lab according to an observer moving with the particle?

7. Two electrons leave a radioactive sample in opposite directions, each having a speed 0.67c with respect to the sample. What is the speed of one electron relative to the other? That is, what would one electron say the other's speed is?

8. A capacitor consists of two parallel rectangular plates with a vertical separation of 0.02 m. The east-west dimension of the plates is 0.2 m, the north-south dimension is 10 cm. The capacitor has been charged by connecting it temporarily to a battery of 300 V.

(a) How many excess electrons are on the negative plate?
(b) What is the electric field strength between the plates?

Now, give the quantities as they would be measured in a frame of reference which is moving eastward, relative to the laboratory in which the plates are at rest, with speed 0.6c.

(c) The dimensions of the capacitor,
(d) The number of excess electrons on the negative plate,
(e) The electric field strength between the plates.

Constants:

$$g \approx 9.81\,\mathrm{m/s^2}$$
$$N_A = 6.022 \times 10^{23}\,\text{things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^9\,\mathrm{N \cdot m^2 \cdot C^{-2}}$$
$$\epsilon_o = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$$
$$\mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$$
$$e = 1.60218 \times 10^{-19}\,\mathrm{C}$$
$$h = 6.6261 \times 10^{-34}\,\mathrm{J \cdot s} = 4.1357 \times 10^{-15}\,\mathrm{eV \cdot s}$$
$$\hbar = \frac{h}{2\pi}$$
$$k_B = 1.38065 \times 10^{-23}\,\mathrm{J \cdot K^{-1}} = 8.6173 \times 10^{-5}\,\mathrm{eV \cdot K^{-1}}$$
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792 \times 10^8\,\mathrm{m/s}$$
$$hc = 1240\,\mathrm{eV \cdot nm}$$
$$m_e = 9.10938 \times 10^{-31}\,\mathrm{kg} \qquad m_e c^2 = 510.998\,\mathrm{keV}$$
$$m_p = 1.67262 \times 10^{-27}\,\mathrm{kg} \qquad m_p c^2 = 938.272\,\mathrm{MeV}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Basic Equations:

$$\vec{\mathbf{E}} = \sigma/\epsilon_0 \text{ capacitor}$$
$$C = \epsilon_0 A/d$$
$$\vec{\mathbf{F}}_{\mathrm{net}} = m\vec{a} \text{ Newton's Second Law}$$
$$\vec{\mathbf{F}}_{\mathrm{centr}} = -\frac{mv^2}{r}\hat{\mathbf{r}} \text{ Centripetal}$$
$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} = q_2 \vec{\mathbf{E}}_1 \qquad \vec{\mathbf{r}}_{12} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$$
$$\vec{\mathbf{E}}_1 = \vec{\mathbf{F}}_{12}/q_2 = k_e \frac{q_1}{r_{12}^2}\,\hat{\mathbf{r}}_{12}$$
$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

E & M

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} = q_2 \vec{\mathbf{E}}_1 \qquad \vec{\mathbf{r}}_{12} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$$
$$\vec{\mathbf{E}}_1 = \vec{\mathbf{F}}_{12}/q_2 = k_e \frac{q_1}{r_{12}^2}\,\hat{\mathbf{r}}_{12}$$
$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Oscillators & waves

$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 m A^2 = 2\pi^2 m f^2 A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$
$$c = \lambda f$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2 \qquad e^x \approx 1 + x + \frac{1}{2}x$$
$$\sin x \approx x - \frac{1}{6}x^3 \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

Radiation

$$P_{\mathrm{rad}} = \frac{q^2 a^2}{6\pi\epsilon_o c^3} \qquad \text{total emitted power, E and B fields}$$

Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t'_{\mathrm{moving}} = \gamma \Delta t_{\mathrm{stationary}} = \gamma \Delta t_p$$
$$L'_{\mathrm{moving}} = \frac{L_{\mathrm{stationary}}}{\gamma} = \frac{L_p}{\gamma}$$
$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$
$$v_{\mathrm{obj}} = \frac{v + v'_{\mathrm{obj}}}{1 + \frac{vv'_{\mathrm{obj}}}{c^2}} \qquad v'_{\mathrm{obj}} = \frac{v_{\mathrm{obj}} - v}{1 - \frac{vv_{\mathrm{obj}}}{c^2}}$$
$$KE = (\gamma - 1)mc^2 = \sqrt{m^2c^4 + c^2p^2} - mc^2$$
$$E_{\mathrm{rest}} = mc^2$$
$$p = \gamma m v$$
$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$

Calculus of possible utility:

$$\int \frac{1}{x}dx = \ln x + c$$
$$\int u\,dv = uv - \int v\,du$$

Vectors:

$$|\vec{\mathbf{F}}| = \sqrt{F_x^2 + F_y^2} \quad \text{magn} \qquad \theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{dir}$$