University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Spring 2013

Final Exam

Instructions

1. Solve 7 of the problems below. All problems have equal weight.
2. Clearly mark your which problems you have chosen.
3. Do your work on separate sheets. Staple them to this exam paper.
4. You are allowed 2 sheet of standard 8.5x11 in paper and a calculator.

1. Determine the maximum scattering angle in a Compton experiment for which the scattered photon can produce a positron-electron pair. *Hint:* twice the electron's rest energy m_ec^2 is required of the incident photon for pair production.

2. In a crystal, the atoms are a distance L apart; that is, each atom must be localized to within a distance of at most L. **(a)** What is the minimum uncertainty in the momentum of the atoms of a solid that are 0.20 nm apart? **(b)** What is the average kinetic energy of such an atom of mass 65 u? (See formula sheet.) **(c)** What would a collection of such atoms contribute to the internal energy of a typical solid, such as copper? **(d)** Is this contribution important at room temperature?

3. A particle is represented by the following wave equation:

$$\psi(x) = \begin{cases} 0 & x < -L/2 \\ C\left(\frac{2x}{L}+1\right) & -L/2 < x < 0 \\ C\left(-\frac{2x}{L}+1\right) & 0 < x < +L/2 \\ 0 & x > +L/2 \end{cases} \tag{1}$$

(a) Use the normalization condition to find C. **(b)** Evaluate the probability to find the particle in an interval of width $0.010L$ at $x=L/4$ (that is, between $x=0.245L$ and $x=0.255L$. No integral is necessary for this calculation.) **(c)** Evaluate the probability to find the particle between $x=0$ and $x=+L/4$. **(d)** Find the average value of x and the rms value of x: $x_{\text{rms}}=\sqrt{\langle x^2\rangle}$.

4. If an elementary particle of mass m has a very short lifetime τ, it is found that repeated measurements of m give results that are spread out over a range Δm. One then calculates τ from the relationship

$$\tau\Delta\left(mc^2\right) \approx \hbar \qquad \text{or} \qquad \tau \approx \frac{\hbar}{\Delta\left(mc^2\right)} \tag{2}$$

so that the shorter the lifetime the more easily it is measured. **(a)** Why is this formula true? Justify it using ideas based on the uncertainty principle. **(b)** Delta particles, $m \approx 1232\,\mathrm{MeV}$, have $\Delta(mc^2) = 110\,\mathrm{MeV}$. What is their lifetime?

5. An electron is trapped in an infinitely deep one-dimensional well of width 0.251 nm. Initially, the electron occupies the $n = 4$ state. **(a)** Suppose the electron jumps to the ground state with the accompanying emission of a photon. What is the energy of the photon? **(b)** Find the energies of other photons that might be emitted if the electron takes other paths between $n = 4$ and the ground state.

6. An electron is in the $n = 5$ state of hydrogen. Within the Bohr model, to what states can the electron make transitions while emitting photons, and what are the energies of the emitted photons (starting from $n = 5$ only)?

7. Find the most probable radius **and** the expected value of the radial position $\langle r \rangle$ of an electron in the $2p$ state.

$$\psi_{2p} = \frac{1}{\sqrt{3}\,(2a_o)^{3/2}} \frac{r}{a_o} e^{-r/2a_o} \tag{3}$$

where $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 0.529 \times 10^{-10}$ m is the Bohr radius. Note the integrals on the formula sheet.

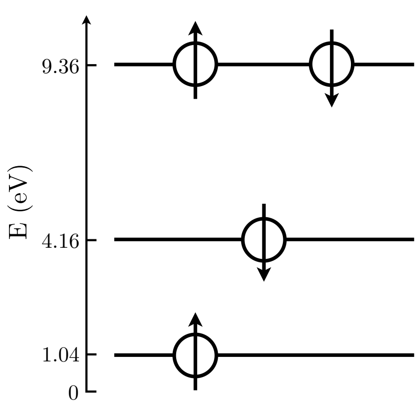
8. **(a)** List the 6 possible sets of quantum numbers (n, l, m_l, m_s) of a $2p$ electron. **(b)** Suppose we have an atom such as carbon, which has two $2p$ electrons. Ignoring the Pauli principle, how many different possible combinations of the two electrons are there? **(c)** How many of the possible combinations of part (b) are eliminated by applying the Pauli principle? **(d)** Suppose carbon is in an excited state with configuration $2p^1 3p^1$. Does the Pauli principle restrict the choice of quantum numbers for the electrons? How many different sets of quantum numbers are possible for the two electrons?

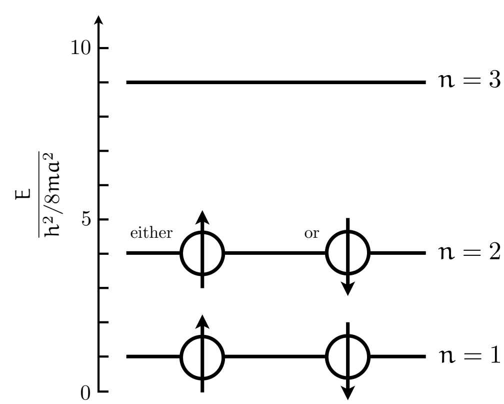
9. The wavelength of maximum intensity in the solar spectrum is about 500 nm, as some of you will verify in PH255. Assuming the sun radiates as a black body, compute its surface temperature.

10. *Pauli exclusion.* What are the energies of the photons that would be emitted when the four-electron system in the figure on the next page returns to its ground state?

11. Three non-interacting particles are in their ground state in an infinite square well;[i] see the figure on the next page. What happens when a magnetic field is turned on which interacts with the spins of the particles? Consider separately the two possible spins for the highest energy electron. Draw the new levels and particles (with spin).

[i] Recall the energies in an infinite square well are $E = n^2h^2/8ma^2$





Left, problem 10: *A system of four electrons with three energy levels.* **Right, problem 11:** *A system of three electrons in an infinite square well.*

Constants:

$$N_A = 6.022\times10^{23}\ \text{things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755\times10^{9}\ \text{N}\cdot\text{m}^2\cdot\text{C}^{-2}$$
$$\epsilon_o = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$$
$$\mu_0 \equiv 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$$
$$e = 1.60218\times10^{-19}\ \text{C}$$
$$h = 6.6261\times10^{-34}\ \text{J}\cdot\text{s} = 4.1357\times10^{-15}\ \text{eV}\cdot\text{s}$$
$$\hbar = \frac{h}{2\pi} \qquad hc = 1239.84\ \text{eV}\cdot\text{nm}$$
$$k_B = 1.38065\times10^{-23}\ \text{J}\cdot\text{K}^{-1} = 8.6173\times10^{-5}\ \text{eV}\cdot\text{K}^{-1}$$
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792\times10^{8}\ \text{m/s}$$
$$m_e = 9.10938\times10^{-31}\ \text{kg} \qquad m_e c^2 = 510.998\ \text{keV}$$
$$m_p = 1.67262\times10^{-27}\ \text{kg} \qquad m_p c^2 = 938.272\ \text{MeV}$$
$$m_n = 1.67493\times10^{-27}\ \text{kg} \qquad m_n c^2 = 939.565\ \text{MeV}$$
$$u = 1.66054\times10^{-27}\ \text{kg} \qquad uc^2 = 931.494\ \text{MeV}$$

Schrödinger

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V(x)\Psi \quad \text{time-dep, 1D}$$
$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi \quad \text{time-indep, 1D}$$
$$\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = 1 \qquad P(\text{in } [x, x+dx]) = |\psi(x)|^2 \qquad \text{1D}$$
$$\int_0^{\infty}|\psi(r)|^2\,4\pi r^2\,dr = 1 \qquad P(\text{in } [r, r+dr]) = 4\pi r^2|\psi(r)|^2 \qquad \text{3D}$$
$$\langle x^n\rangle = \int_{-\infty}^{\infty}x^n P(x)\,dx \qquad \text{1D} \qquad \langle r^n\rangle = \int_0^{\infty}r^n P(r)\,dr \qquad \text{3D}$$
$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

Basic Equations:

$$\vec{F}_{net} = m\vec{a} \text{ Newton's Second Law}$$
$$\vec{F}_{centr} = -\frac{mv^2}{r}\hat{r} \text{ Centripetal}$$
$$\vec{F}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Oscillators

$$E = \left(n+\frac{1}{2}\right)hf$$
$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA^2 = 2\pi^2 mf^2A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2 \qquad \tan x \approx x + \frac{1}{3}x^3$$
$$e^x \approx 1 + x + \frac{1}{2}x \qquad \sin x \approx x - \frac{1}{6}x^3 \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

E & M

$$\vec{F}_{12} = k_e\frac{q_1q_2}{r_{12}^2}\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

Misc Quantum/Relativity

$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$
$$E = hf \qquad p = h/\lambda = E/c \qquad \lambda f = c \qquad \text{photons}$$
$$\lambda_f - \lambda_i = \frac{h}{m_ec}(1-\cos\theta) \quad \text{Compton}$$
$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{\gamma mv} \approx \frac{h}{mv}$$
$$\Delta x\Delta p \geqslant \frac{h}{4\pi} \qquad \Delta E\Delta t \geqslant \frac{h}{4\pi}$$
$$eV_{stopping} = KE_{electron} = hf - \varphi = hf - W$$

Bohr

$$E_n = -13.6\,\text{eV}\left(Z^2/n^2\right) \qquad Z \text{ protons, 1 } e^-$$
$$\Delta E = -13.6\,\text{eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = hf$$
$$L = mvr = n\hbar$$
$$v^2 = \frac{n^2\hbar^2}{m_e^2r^2} = \frac{k_ee^2}{m_er}$$

Quantum Numbers

$$l = 0, 1, 2, \ldots, (n-1) \qquad L^2 = l(l+1)\hbar^2$$
$$m_l = -l, (-l+1), \ldots, l \qquad L_z = m_l\hbar$$
$$m_s = -\pm\frac{1}{2} \qquad S_z = m_s\hbar$$
$$\text{dipole transitions: } \Delta l = \pm1, \Delta m_l = 0, \pm1, \Delta m_s = 0$$

Calculus of possible utility:

$$\int u\,dv = uv - \int v\,du$$
$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$
$$\int_0^{\infty}x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$
$$\int_0^{\infty}x^2e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$
$$\int_{-\infty}^{\infty}x^3e^{-ax^2}\,dx = \int_{-\infty}^{\infty}xe^{-ax^2}\,dx = 0$$
$$\int_0^{\infty}x^4e^{-ax^2}\,dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

Blackbody

$$E_{tot} = \sigma T^4 \qquad \sigma = 5.672\times10^{-8}\ \text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$$
$$T\lambda_{max} = 0.29\times10^{-2}\ \text{m}\cdot\text{K} \qquad \text{Wien}$$
$$E_{quantum} = hf \quad E_{oscillator} = hf/\left(e^{hf/k_BT} - 1\right)$$
$$I(\lambda, T) = \frac{(\text{const})}{\lambda^5}\left[e^{\frac{hc}{\lambda k_bT}} - 1\right]^{-1}$$
$$I(f, t) = (\text{const})\,f^3\left[e^{\frac{hf}{k_bT}} - 1\right]^{-1}$$