UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 253 / LeClair Spring 2019

Exam 1

Instructions

1. **I will grade your best four problems.** All problems have equal weight.
2. **Show your work** for full credit. Significant partial credit will be given.
3. Answer on separate sheets of paper, turn in all work.

1. A stick of length L is at rest in a system O and is oriented at an angle θ with respect to the x axis. An observer in system O' travels at velocity v with respect to the system O along the x axis. On a recent homework problem, you were asked for the apparent angle θ' that the stick makes with the x' axis according to the observer in O'. What is the apparent *length* L' that the observer in O' sees, in terms of given quantities? The x and x' axes are parallel.

2. An atomic clock aboard a spaceship runs slow compared with an earth-based atomic clock at a rate of 1.0 second per day. What is the speed of the spaceship? Note for convenience $60 \times 60 \times 24 = 8.64 \times 10^4$.

3. At time $t_1 = 0$, a boiler explodes in town A. At time $t_2 = 0.0003\,\mathrm{s}$ a similar boiler explodes in town B, 150 km away from the first explosion. Show that, in a reference frame of a spaceship moving at speed greater than 0.60 c from town A toward town B, the first explosion occurs *after* the second.

4. An electron (charge e and mass m_e) moves in a solid at a thermal speed of $v_o = 10^5\,\mathrm{m/s}$ ($v \ll c$), and after colliding with an atom it experiences a constant deceleration such that it comes to rest after traveling $d = 3 \times 10^{-9}\,\mathrm{m}$. What fraction of the electron's initial kinetic energy is lost as radiation during the deceleration? *Hint: recall from kinematics how to relate time, distance, and acceleration.*

5. **(a)** Show that the momentum of a particle can be expressed in the concise form $p = Ev/c^2$. **(b)** Given the following:

$$p = \gamma m v = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

$$E = \gamma m c^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

demonstrate that $E^2 = p^2c^2 + m^2c^4$.

Formula Sheet

Constants:

$$g \approx 9.81\,\mathrm{m/s^2}$$
$$N_A = 6.022 \times 10^{23}\,\text{things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^9\,\mathrm{N \cdot m^2 \cdot C^{-2}}$$
$$\epsilon_o = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$$
$$\mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$$
$$k_B = 1.38065 \times 10^{-23}\,\mathrm{J \cdot K^{-1}}$$
$$e = 1.60218 \times 10^{-19}\,\mathrm{C}$$
$$h = 6.6261 \times 10^{-34}\,\mathrm{J \cdot s}$$
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 2.99792 \times 10^8\,\mathrm{m/s}$$
$$hc = 1240\,\mathrm{eV \cdot nm}$$
$$m_e = 9.10938 \times 10^{-31}\,\mathrm{kg}$$
$$m_p = 1.67262 \times 10^{-27}\,\mathrm{kg}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Basic Equations:

$$\vec{\mathbf{E}} = \sigma/\epsilon_0 \text{ capacitor}$$
$$C = \epsilon_0 A/d$$
$$\vec{\mathbf{F}}_{\text{net}} = m\vec{a} \text{ Newton's Second Law}$$
$$\vec{\mathbf{F}}_{\text{centr}} = -\frac{mv^2}{r}\hat{\mathbf{r}} \text{ Centripetal}$$
$$K_{\text{classical}} = \frac{1}{2}mv^2$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1-D classical motion:

$$v(t) = \frac{d}{dt}x(t) \qquad a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)$$
$$v(t) = \int_0^t a\,dt \qquad x(t) = \int_0^t v\,dt$$

$\downarrow$ const. acc.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$v_{xf}^2 = v_{xi}^2 + 2a_x\Delta x$$
$$v_f = v_i + at$$

E & M

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = q_2\vec{\mathbf{E}}_1 \qquad \vec{\mathbf{r}}_{12} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$$
$$\vec{\mathbf{E}}_1 = \vec{\mathbf{F}}_{12}/q_2 = k_e \frac{q_1}{r_{12}^2}\hat{\mathbf{r}}_{12}$$
$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$
$$U_{12} = k_e \frac{q_1 q_2}{r_{12}} \quad \text{potential E, 2 charges}$$

Oscillators & waves

$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA^2 = 2\pi^2 mf^2 A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$
$$c = \lambda f$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx \qquad e^x \approx 1 + x$$
$$\sin x \approx x \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

Radiation

$$P_{\text{rad}} = \frac{q^2 a^2}{6\pi\epsilon_o c^3} \quad \text{total emitted power, E and B fields}$$
$$E = \int P\,dt \quad \text{constant power} \Longrightarrow \quad E = Pt$$

Special Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$\Delta t'_{\text{moving}} = \gamma\Delta t_{\text{stationary}} = \gamma\Delta t_p$$
$$L'_{\text{moving}} = \frac{L_{\text{stationary}}}{\gamma} = \frac{L_p}{\gamma}$$
$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$
$$v_{\text{obj}} = \frac{v + v'_{\text{obj}}}{1 + \frac{vv'_{\text{obj}}}{c^2}} \qquad v'_{\text{obj}} = \frac{v_{\text{obj}} - v}{1 - \frac{vv_{\text{obj}}}{c^2}}$$
$$KE = (\gamma - 1)mc^2 = \sqrt{m^2c^4 + c^2p^2} - mc^2$$
$$E_{\text{rest}} = mc^2$$
$$p = \gamma mv$$
$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$