University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair — Spring 2019

Exam 2

Instructions

1. **I will grade your best four problems.** All problems have equal weight.
2. **Show your work** for full credit. Significant partial credit will be given.
3. Answer on separate sheets of paper, turn in all work.

1. Ultraviolet light of wavelength 350 nm falls on a potassium surface. The maximum energy of the photoelectrons is 1.3 eV. What is the work function of potassium?

2. A beam of X rays is scattered by electrons at rest. What is the energy of the incident X rays if the wavelength of the X rays scattered at 60° relative to the incident beam is 3.5 pm?

3. The wavelength of maximum intensity in the solar spectrum is about 500 nm, as some of you will verify in PH255. Assuming the sun radiates as a black body, compute its surface temperature.

4. An electron is trapped in an infinitely deep one-dimensional well of width 0.251 nm. Initially, the electron occupies the $n=4$ state. **(a)** Suppose the electron jumps to the ground state with the accompanying emission of a photon. What is the energy of the photon? **(b)** Find the energies of other photons that might be emitted if the electron takes other paths between $n=4$ and the ground state. Recall that for a particle in an infinitely deep one-dimensional well of width L the energy levels are $E_n = n^2h^2/8mL^2$.

5. The state of a free particle is described by the following wave function

$$\psi(x) = \begin{cases} 0 & x < -b \\ A & -b \leqslant x \leqslant 5b \\ 0 & x > 5b \end{cases} \tag{1}$$

(a) Determine the normalization constant A.
(b) What is the probability of finding the particle in the interval $[0, b]$?
(c) Determine $\langle x \rangle$ and $\langle x^2 \rangle$ for this state.
(d) Find the uncertainty in position $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

Constants:

$$N_A = 6.022 \times 10^{23}\,\text{things/mol}$$
$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^{9}\,\text{N}\cdot\text{m}^2\cdot\text{C}^{-2}$$
$$\epsilon_o = 8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2$$
$$\mu_0 \equiv 4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$$
$$e = 1.60218 \times 10^{-19}\,\text{C}$$
$$h = 6.6261 \times 10^{-34}\,\text{J}\cdot\text{s} = 4.1357 \times 10^{-15}\,\text{eV}\cdot\text{s}$$
$$\hbar = \frac{h}{2\pi} \qquad hc = 1239.84\,\text{eV}\cdot\text{nm}$$
$$k_B = 1.38065 \times 10^{-23}\,\text{J}\cdot\text{K}^{-1} = 8.6173 \times 10^{-5}\,\text{eV}\cdot\text{K}^{-1}$$
$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792 \times 10^{8}\,\text{m/s}$$
$$m_e = 9.10938 \times 10^{-31}\,\text{kg} \qquad m_e c^2 = 510.998\,\text{keV}$$
$$m_p = 1.67262 \times 10^{-27}\,\text{kg} \qquad m_p c^2 = 938.272\,\text{MeV}$$
$$m_n = 1.67493 \times 10^{-27}\,\text{kg} \qquad m_n c^2 = 939.565\,\text{MeV}$$
$$u = 1.66054 \times 10^{-27}\,\text{kg} \qquad uc^2 = 931.494\,\text{MeV}$$

Schrödinger

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V(x)\Psi \quad \text{time-dep, 1D}$$
$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi \quad \text{time-indep, 1D}$$
$$\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = 1 \qquad P(\text{in } [x, x+dx]) = |\psi(x)|^2 \qquad \text{1D}$$
$$\int_0^{\infty}|\psi(r)|^2\,4\pi r^2\,dr = 1 \qquad P(\text{in } [r, r+dr]) = 4\pi r^2|\psi(r)|^2 \qquad \text{3D}$$
$$\langle x^n\rangle = \int_{-\infty}^{\infty} x^n P(x)\,dx \qquad \text{1D} \qquad \langle r^n\rangle = \int_0^{\infty} r^n P(r)\,dr \qquad \text{3D}$$
$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

Basic Equations:

$$\vec{F}_{net} = m\vec{a} \text{ Newton's Second Law}$$
$$\vec{F}_{centr} = -\frac{mv^2}{r}\hat{r} \text{ Centripetal}$$
$$\vec{F}_{12} = k_e\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$
$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Oscillators

$$E = \left(n + \frac{1}{2}\right)hf$$
$$E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA^2 = 2\pi^2 mf^2A^2$$
$$\omega = 2\pi f = \sqrt{k/m}$$

Approximations, $x \ll 1$

$$(1+x)^n \approx 1 + nx + \frac{1}{2}n(n+1)x^2 \qquad \tan x \approx x + \frac{1}{3}x^3$$
$$e^x \approx 1 + x + \frac{1}{2}x \qquad \sin x \approx x - \frac{1}{6}x^3 \qquad \cos x \approx 1 - \frac{1}{2}x^2$$

E & M

$$\vec{F}_{12} = k_e\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} = q_2\vec{E}_1 \qquad \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$
$$\vec{E}_1 = \vec{F}_{12}/q_2 = k_e\frac{q_1}{r_{12}^2}\hat{r}_{12}$$
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

Misc Quantum/Relativity

$$E^2 = p^2c^2 + m^2c^4 = \left(\gamma mc^2\right)^2$$
$$E = hf \qquad p = h/\lambda = E/c \qquad \lambda f = c \qquad \text{photons}$$
$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta) \quad \text{Compton}$$
$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{\gamma mv} \approx \frac{h}{mv}$$
$$\Delta x\Delta p \geqslant \frac{h}{4\pi} \qquad \Delta E\Delta t \geqslant \frac{h}{4\pi}$$
$$eV_{stopping} = KE_{electron} = hf - \varphi = hf - W$$
$$E_n = n^2h^2/8mL^2 = n^2\pi^2\hbar^2/2mL^2 \qquad \text{particle in a box}$$

Calculus of possible utility:

$$\int u\,dv = uv - \int v\,du$$
$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$
$$\int_0^{\infty} x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}$$
$$\int_0^{\infty} x^2 e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$
$$\int_{-\infty}^{\infty} x^3 e^{-ax^2}\,dx = \int_{-\infty}^{\infty} x e^{-ax^2}\,dx = 0$$
$$\int_0^{\infty} x^4 e^{-ax^2}\,dx = \frac{3}{8}\sqrt{\frac{\pi}{a^5}}$$

Blackbody

$$E_{tot} = \sigma T^4 \qquad \sigma = 5.672 \times 10^{-8}\,\text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$$
$$T\lambda_{max} = 0.29 \times 10^{-2}\,\text{m}\cdot\text{K} \qquad \text{Wien}$$
$$E_{quantum} = hf \quad E_{oscillator} = hf/\left(e^{hf/k_B T} - 1\right)$$
$$I(\lambda, T) = \frac{(\text{const})}{\lambda^5}\left[e^{\frac{hc}{\lambda k_b T}} - 1\right]^{-1}$$
$$I(f, t) = (\text{const})\,f^3\left[e^{\frac{hf}{k_b T}} - 1\right]^{-1}$$