Solution

PH253 S19
Exam 2

① $\lambda = 350\,nm = 350\times10^{-9}\,m \qquad K_{max} = 1.3\,eV$

photoelectric : $K_{max} = hf - \varphi = \frac{hc}{\lambda} - \varphi$

$$1.3\,eV = \frac{1240\,eV\cdot nm}{350\,nm} - \varphi \Rightarrow \boxed{\varphi \simeq 2.24\,eV}$$

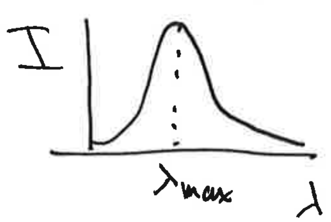
② given $\lambda_f = 3.5\,pm = 3.5\times10^{-12}\,m$ and $\theta = 60°$, want $E_i \leftrightarrow \lambda_i$

Compton effect: $\lambda_f - \lambda_i = \frac{h}{mc}(1-\cos\theta)$

$$\Rightarrow \lambda_i = \lambda_f - \frac{h}{mc}(1-\cos\theta) \simeq 2.29\,pm$$

$$\boxed{E_i = \frac{hc}{\lambda_i} \simeq 543\,keV} \text{ or } 8.6\times10^{-14}\,J$$

③ Wien displacement : $\lambda_{max} T = const = b = 2.9\times10^{-3}\,m\cdot K$



$$\Rightarrow T = \frac{b}{\lambda_{max}} = \frac{2.9\times10^{-3}\,m\cdot K}{500\times10^{-9}\,m} \simeq 5800K$$

④ $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad L = 0.251\,nm = 2.51\times10^{-10}\,m$

a) from $n=4$ to ground state $(n=1)$, photon will have difference in energy between these levels

$$\Delta E = E_4 - E_1 = (4^2-1^2)\frac{\pi^2\hbar^2}{2mL^2} = 89.6\,eV$$

b) starting from $n=4$, enumerate all paths to $n=1$

$4\to3 \quad 3\to2 \quad 3\to1$

$4\to2 \quad 2\to1$

$4\to1$

$\Rightarrow$ 6 distinct ΔE values

For each transition, find $E_i - E_f = \Delta E$

e.g. from $3 \to 1$, $\Delta E = E_3 - E_1 = (3^2 - 1^2)\frac{\pi^2 \hbar^2}{2mL^2} \simeq 47.8\,eV$

transition	energy (eV)
$4 \to 1$	89.6
$4 \to 2$	71.6
$4 \to 3$	41.8
$3 \to 2$	29.9
$3 \to 1$	47.8
$2 \to 1$	17.9

• In general, given n levels there will be $\binom{n}{2}$ transitions

$$\binom{n}{2} = \frac{n!}{2!(n-2)!}$$

(5.) a) to normalize, enforce $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$, note ψ is nonzero only over interval $[-b, 5b]$

$$1 = \int_{-\infty}^{\infty} |\psi|^2 dx = \int_{-b}^{5b} |A|^2 dx = A^2 x \Big|_{-b}^{5b} = 6bA^2$$

$$\Rightarrow A^2 = \frac{1}{6b} \quad \text{or} \quad A = \frac{1}{\sqrt{6b}}$$

b) $$P(\text{in } [0,b]) = \int_0^b |\psi|^2 dx = \int_0^b A^2 dx = A^2 b = \frac{1}{6b} \cdot b = \frac{1}{6}$$

• ψ is constant and nonzero over an interval of width $6b$, so finding it in any subinterval of width 6 should be $\frac{1}{6}$

c) $$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi|^2 dx = \int_{-b}^{5b} A^2 x \, dx = A^2 \frac{x^2}{2}\Big|_{-b}^{5b} = \frac{25b^2A^2}{2} - \frac{b^2A^2}{2} = 12b^2A^2$$

$$\langle x \rangle = 12b^2\left(\frac{1}{6b}\right) = 2b$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi|^2 dx = \int_{-b}^{5b} A^2 x^2 dx = \frac{1}{3} A^2 x^3 \Big|_{-b}^{5b} = 42A^2b^3 = 7b^2$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{7b^2 - 4b^2} = b\sqrt{3}$$