University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Fall 2010

Problem Set 10: the last one.

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Friday 3 December 2010 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. For a one-dimensional infinite square well of length l the allowed energies for noninteracting particles of mass m were found to be $E_n = n^2 E_o$, where n is a positive integer and $E_o = h^2/8ml^2$. The generalization to a three-dimensional infinite well of side l is

$$E = \left(n_x^2 + n_y^2 + n_z^2\right) E_o \qquad E_o = \frac{h^2}{8ml^2} \tag{1}$$

where n_x, n_y, and n_z are positive integers. It is seen that a number of different states (n_x, n_y, n_z) may have the same energy, a situation called degeneracy. For the first 6 energy levels, list the energy of the level and the order of degeneracy, i.e., the number of states having the given energy.

2. For electrons in a metal or gas molecules in a container, the value of l in the previous problem is so large that the energy levels can be regarded as forming a continuous spectrum. For this case determine the number of states (n_x, n_y, n_z) with energies in the interval between E and $E+dE$.

3. **(a)** Obtain an expression for the Fermi energy at $T=0\,\mathrm{K}$ for an electron gas in a metal in terms of the total number of electrons, the volume, and fundamental constants. **(b)** At $T=0\,\mathrm{K}$, what is the rms speed, in terms of the Fermi energy, of an electron gas in a metal?

4. Show that the average kinetic energy of a conduction electron in a metal at $0\,\mathrm{K}$ is $E_{\text{av}} = \frac{3}{5}E_F$. *Hint: in general, the average kinetic energy is*

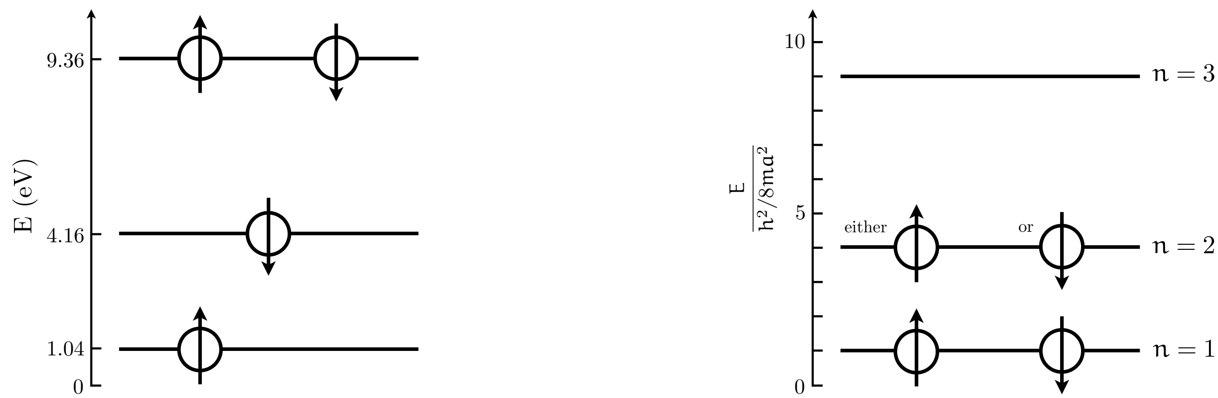
$$E_{\text{av}} = \frac{1}{n_e}\int E\,N(E)\,dE \tag{2}$$

where n_e is the density of particles, $N(E)\,dE$ is the number of electrons per unit volume that have energies in $[E, E+dE]$, and the integral is over all possible values of energy.

5. Explain the basic operation of a field-effect transistor (FET). How does a field effect transistor uses a small gate voltage to produce large modulations in source to drain current?

6. Explain, appealing to band theory, why conductors tend to be opaque and insulators transparent.

7. *Pauli exclusion.* What are the energies of the photons that would be emitted when the four-electron system in the figure below returns to its ground state? See also: Pfeffer & Nir 3.4.5



Left, problem 7: *A system of four electrons with three energy levels.* **Right, problem 8:** *A system of three electrons in an infinite square well.*

8. Three non-interacting particles are in their ground state in an infinite square well;[i] see the figure above. What happens when a magnetic field is turned on which interacts with the spins of the particles? Draw the new levels and particles (with spin).

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[i] Recall the energies in an infinite square well are $E = n^2h^2/8ma^2$