University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Fall 2010

Problem Set 1: Relativity

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Fri 27 August 2010 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. A classic "paradox" involving length contraction and the relativity of simultaneity is as follows: Suppose a runner moving at $0.75c$ carries a horizontal pole 15 m long toward a barn that is 10 m long. The barn has front and rear doors. An observer on the ground can instantly and simultaneously open and close the two doors by remote control. When the runner and the pole are inside the barn, the ground observer closes and then opens both doors so that the runner and pole are momentarily captured inside the barn and then proceed to exit the barn from the back door. Do both the runner and the ground observer agree that the runner makes it safely through the barn?

2. A pilot is supposed to fly due east from A to B and then back again to A due west. The velocity of the plane in air is u' and the velocity of the air with respect to the ground is v. The distance between A and B is l and the plane's air speed u' is constant.

(a) If $v=0$ (still air) show that the time for the round trip is $t_o=2l/u'$.
(b) Suppose that the air velocity is due east (or west). Show that the time for a round trip is then

$$t_E = \frac{t_o}{1-v^2/\left(u'\right)^2}$$

(c) Suppose the air velocity is due north (or south). Show that the time for a round trip is then

$$t_N = \frac{t_o}{\sqrt{1-v^2/\left(u'\right)^2}}$$

(d) In parts (b) and (c) we must assume that $v<u'$. Why?

3. The length of a spaceship is measured to be exactly half its proper length. **(a)** What is the speed of the spaceship relative to the observer's frame? **(b)** What is the dilation of the spaceship's unit time?

4. Derive the relativistic acceleration transformation

$$a'_x = \frac{a_x \left(1 - \frac{v^2}{c^2}\right)^{3/2}}{\left(1 - \frac{u_x v}{c^2}\right)^3}$$

in which $a_x = du_x/dt$ and $a'_x = du'_x/dt'$. *Hint:* $du'_x/dt' = (du'_x/dt)\,(dt/dt')$

5. A charge q at $x = 0$ accelerates from rest in a uniform electric field $\vec{E}$ which is directed along the positive x axis.
(a) Show that the acceleration of the charge is given by

$$a_x = \frac{qE}{m}\left(1 - \frac{u^2}{c^2}\right)^{3/2}$$

(b) Show that the velocity of the charge at any time t is given by

$$u_x = \frac{qEt/m}{\sqrt{1 + (qEt/mc)^2}}$$

(c) Find the distance the charge moves in a time t. *Hint:* *http://integrals.wolfram.com*

6. Show that the angular frequency of a charge moving in a uniform magnetic field B is given by

$$\omega = \frac{qB}{m}\sqrt{1 - u^2/c^2}$$

7. The "effective mass" of a photon (bundle of electromagnetic radiation of zero rest mass and energy hf) can be determined from the relationship $m = E/c^2$. Compute the "effective mass" for a photon of wavelength 500 nm (visible) and for a photon of wavelength 0.1 nm (X-ray).

8. A meterstick makes an angle of 30° with respect to the x'-axis of O'. What must be the value of v if the meterstick makes an angle of 45° with respect to the x-axis of O?

9. A particle appears to move with speed u at an angle θ with respect to the x axis in a certain system. At what speed and angle will this particle appear to move in a second system moving with speed v with respect to the first? Why does the answer differ from that of the previous problem?

10. Consider a charged parallel-plate capacitor, whose electric field (in its rest frame) is uniform (neglecting edge effects) between the plates and zero outside. Find the electric field according to an observer in motion at constant velocity v **(a)** along a line running through the center of the capacitor between the plates, and **(b)** along a line perpendicular to the plates.