University of Alabama
Department of Physics and Astronomy

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Problem Set 2: Random Hints

A little something to get you started ...

1. (a) Writing down the total energy is easy enough:

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_o r} \tag{1}$$

How to get it in terms of radius and frequency? Use the force ... balance. The electric force between the proton and neutron has to equal the centripetal force to maintain circular motion:

$$\frac{e^2}{4\pi\epsilon_o r^2} = \frac{mv^2}{r} \tag{2}$$

Solve for v, plug in the first equation. Orbital (angular) frequency is just $\omega = v/r$.

(b) Energy radiated per unit time can be found from the Larmor formula (see the notes) once you've got the acceleration (v^2/r).

(c) You just found dE/dt in part (b). You now need dE/dr, which you can get from the formula you derived in part (a) that has E in terms of r and constants only.

2. First, check that you know what is meant by "angular diameter."

`http://en.wikipedia.org/wiki/Angular_diameter`

Thus, if the angular diameter is δ, then $\tan\delta = \frac{R_s}{2D}$, where R_s is the sun's radius and D the sun-moon distance.

In equilibrium, the power emitted by the sun at temperature T_S and absorbed by the moon must be the same as that re-emitted by the moon at temperature T. The power emitted by a body at temperature T over area A is

$$P = \sigma A T^4 \tag{3}$$

where σ is a constant you won't need. Using the sun's surface area, you can figure out the power it emits. The moon receives a fraction of this power: at the moon's distance, the sun's emitted power is spread out over a sphere of radius D, and the moon intercepts an area πR_m^2 of that sphere. Multiplying that geometric ratio by the sun's emitted power gives you the power received by the moon. Equate that to a blackbody at the moon's temperature T_m an you've got it, once you see how to put the geometric factor in terms of the angular diameter δ.

Note that you *don't* need to know any constants other than π, or any of the distances, just the angular diameter and the sun's temperature. You should find the moon's temperature to be about $T_m \approx 390\,\mathrm{K}$, which is about right for the maximum temperature at the lunar equator.

See also:
`http://en.wikipedia.org/wiki/Black_body#Temperature_relation_between_a_planet_and_its_star`
`http://www.oberlin.edu/physics/Scofield/p268/library/Ch-03%20Sunlight.pdf`

3. Funny, I feel like I asked this one last year.

4. This one's just math ... life will be easier if you do a change of variables to $u = kx - \omega t'$.

To find the average for $T \gg \tau$, try taking the limit of the expression you get after integration as $T \to \infty$. One part will be at max 2 or 3, involving only sin and cos, the other will tend toward zero.

5. The emitted power by a blackbody is given. The power emitted over a range of wavelengths λ_1 to λ_2 is found by integrating $I(\lambda, T)$ over those limits. The fraction is then this divided by the total power:

$$\text{(fraction)} = \frac{\int\limits_{\lambda_1}^{\lambda_2} I(\lambda, T)\, d\lambda}{\int\limits_{0}^{\infty} I(\lambda, T)\, d\lambda} \tag{4}$$

If you read your notes on blackbody radiation, the integral in the denominator is given if you make a change of variables to $u = \frac{hc}{\lambda kT}$. That leaves the numerator, which has no closed form. You can approximate it, however. Using the preceding change of variables, you will end up with an integral like

$$\int\limits_{u_1}^{u_2} \frac{u^3}{e^u - 1}\, du \tag{5}$$

If the limits are 0 and ∞, the integral evaluates to $\pi^4/15$. For the range of wavelengths you're interested in, the limits amount to $e^u \gg 1$, so you can approximate the denominator as $\frac{1}{e^u-1} \approx 1/e^u = e^{-u}$. The integral is then analytically solvable, and you can proceed.

The numerical answer I get is about 35% using this approximation.

6. Plug in the trial solutions for x and E. This actually gives you two equations: one equating the real parts on each side, one part equating the imaginary parts on each side. It is the same thing as an LCR circuit, if you've done those before ...

7. We'll do this one in class, or at least set it up. Start by plugging in the expressions for x and E, expand $\cos \omega t + \delta$ using the rule for $\cos A + B$. $\cos \delta$ can be found exactly using

$$\cos \delta = \cos\left(\tan^{-1} X\right) = \frac{1}{\sqrt{1+X^2}} \tag{6}$$

For small damping, use $\sin \delta \sim 0$. Grind through the algebra from there ....