UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 253 / LeClair Fall 2010

Problem Set 4 Hints

1. **(a)** The electron acquires kinetic energy $p^2/2m$ from the electrical potential difference it moves through, so $p^2/2m = e\Delta V$. From momentum p, you can get wavelength. **(b)** Imaging at 0.25 nm requires light with a comparable wavelength. From wavelength, you can get photon energy.

2. Note the typo (now fixed), it should read $\Delta p \Delta x \geqslant h/4\pi$. The length traveled along a circle is radius times angular displacement, so moving through a distance Δx along a circle of radius r implies one has moved through an angle $\Delta\theta$, and $\Delta x = \theta r$. The same relationship must hold between linear and angular uncertainties.

Momentum is $p = mv = mr\omega$, since $v = r\omega$. If we have an uncertainty in momentum Δp, it must come from the uncertainty in v, and thus ω since r (and m) are constant: $\Delta p = mr\Delta\omega$.

Now, angular momentum is $\vec{L} = \vec{r} \times \vec{p}$. In circular motion, $\vec{r}$ and $\vec{p}$ are perpendicular, so we have $L = rp = mvr = mr^2\omega$. An uncertainty in ω of $\Delta\omega$ thus gives an uncertainty in angular momentum of $\Delta L = mr^2\Delta\omega$ since m and r are constant.

Put the pieces together, and you should have the desired result.

3. We basically did this one in lecture. The $\sin\alpha = \lambda/d$ is a result fro wave optics that you can take as a given; see your PH106 book if you're curious about its derivation, or look here (clickable link):[i]

`http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/sinslit.html#c1`.

4. If the photon were a massive particle, then you could write $hf = \gamma mc^2$. Solve that for v/c, and use the approximation $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ when $x \ll 1$. Then note that $(c - v)/c = 1 - v/c$, and you should have a nice tidy expression involving mc^2 and hf.

For the second part of the question (note that there are two things asked!), you use the given data ($\lambda = 30\,\text{m}$, $(c - v)/c = 0.01$) and solve for m. De Broglie was a bit overzealous in his estimate, it seems, but not by much.

[i] Basically, the condition for destructive interference, and thus a minimum in the intensity profile, is that the path difference between beams going through the top and bottom of the slit to the same point on the screen is exactly an integer number of wavelengths, so $d \sin\alpha = n\lambda$.

5. The probability that the bob is in a tiny region dx in the interval $[x, x+dx]$ is

$$P(\text{in } [x, x+dx])\, dx = \frac{\text{time to move } dx}{\text{half the period}} \tag{1}$$

Here the speed is a function of position, $v(x)$, s to travel a distance dx requires a time given by

$$dx = v(x)\, dt \qquad \text{or} \qquad dt = \frac{dt}{v(x)} \tag{2}$$

The period of the pendulum is T, you should remember how to relate this to its length and the gravitational acceleration. With these two facts, we have

$$P(x)\, dx = \frac{dt}{\frac{1}{2}T} = \frac{2}{T}\frac{dx}{v(x)} \qquad \text{or} \qquad P(x) = \frac{2}{T}\frac{1}{v(x)} \tag{3}$$

Noting that $K = \frac{1}{2}mv^2 = E_{tot} - U$ you should be able to come up with an expression for $v(x)$, the speed of the pendulum as a function of its lateral position. Your PH105 book has such an expression. You should find a probability that diverges at lateral positions $x=A$ and $x=-A$.

There is an interesting article on this problem in the American Journal of Physics (vol. 63, page 823, 1995), a physics education journal. It is available online from campus (subscription screened by IP address).

6. For the minimum energy, we presume that the momentum and position are near their minimum possible values, $x \sim \Delta x$, $p \sim \Delta p$, so plug those in. You want to get E as a function of Δx so you can minimize it, so substitute $\Delta p = \hbar/2\Delta x$. Take the derivative of E with respect to Δx, find the minimum, plug it back in the energy equation. Noting $2\pi f = \sqrt{k/m}$, after carefully canceling all the constants you should find $E = \frac{1}{2}hf$.

7. Did this one class, for the most part. The electron will recoil after colliding with the photon, and we only know that the collision was such that the photon came off at an angle somewhere between α and $-\alpha$, but we can't say any more than that. The horizontal component of the electron's recoil momentum can be anywhere from $p \sin\alpha$ to $-p \sin\alpha$, where p is the photon momentum. This gives an uncertainty in the electron's x momentum, and the microscope lens has its own uncertainty (resolution) given in the problem.

8. Its really just math. What you're doing is Fourier transforming the momentum wave function to find the position wave function, and demonstrating that they obey an uncertainty relationship, but for the moment you can just treat it as a mathematical exercise meant to introduce you to a few things we'll need next week.

Let's do part (a) just to get you started.

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \varphi(k) e^{ikx}\, dk = \frac{1}{\sqrt{2\pi}} \int_{-K}^{K} N e^{ikx}\, dk = \frac{N}{ix\sqrt{2\pi}} e^{ikx} \Big|_{-K}^{K} = \frac{N}{ix\sqrt{2\pi}} \left(e^{iKx} - e^{-iKx} \right)$$

$$= \frac{N}{ix\sqrt{2\pi}} (\cos Kx + i \sin Kx - \cos -Kx - i \sin -Kx) \tag{4}$$

For the last line, we used the Euler identity $e^{\theta} = \cos\theta + i\sin\theta$. Now note that $\cos x = \cos -x$ and $\sin x = -\sin -x$,

$$\psi(x) = \frac{N}{ix\sqrt{2\pi}} 2i \sin Kx = N\sqrt{\frac{2}{\pi}} \frac{\sin Kx}{x} \tag{5}$$

For the next part, square $\psi(x)$, integrate over $\pm\infty$, and set that equal to 1. Solve for N. For part (c), do the same with $\varphi(k)$, but set the integral equal to $1/\sqrt{2\pi}$. For part (d), the width of $\varphi(k)$ should be obvious; the width of $\psi(x)$ is a bit trickier. If you plot $\psi(x)$, you'll notice that most of the intensity comes in the region between the first two zeros, so the distance between the zeros would be a reasonable estimate of the "width."