University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Fall 2010

Problem Set 7: Atoms

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Fri 15 October 2010 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. (a) How many different photons can be emitted by hydrogen atoms that undergo transitions from the ground state from the $n=4$ state? **(b)** Enumerate their energies, in electron volts.

2. The wave function for the 3s state of hydrogen ($n=3, l=0, m_l=0$) is

$$\psi_{300} = \frac{1}{81\sqrt{3\pi}a_o^{3/2}}\left(27 - 18\frac{r}{a_o} + 2\frac{r^2}{a_o^2}\right)e^{-r/3a_o} \tag{1}$$

where a_o is the Bohr radius.

(a) What is the *most probable* value of r?
(b) What is $\langle r\rangle$?
(c) What is the total probability of finding the electron at a distance greater than this radius?

3. Schrödinger's equation for a simple harmonic oscillator reads

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi \tag{2}$$

The ground state wave function has the form

$$\psi_o = ae^{-\alpha^2x^2} \tag{3}$$

(a) Determine the value of the constant α and the energy of the state.
(b) Find $\langle r\rangle$, $\langle r^2\rangle$, and Δr.

4. By considering the visible spectrum of hydrogen and He^+, show how you could determine spectroscopically if a sample of hydrogen was contaminated with helium. (Hint: look for differences in the visible emission lines, $\lambda\approx 390\sim 750\,\text{nm}$. A difference of 10 nm is easily measured.)

5. Show that whenever a solution $\Psi(x,t)$ of the time-dependent Schrödinger equation separates into a product $\Psi(x,t) = F(x) \cdot G(t)$ then $F(x)$ *must* satisfy the corresponding time-independent Schrödinger equation and $G(t)$ *must* be proportional to $e^{-iEt/\hbar}$.

6. An experimenter asks for funds from a foundation to observe visually through a microscope the quantum behavior of a small harmonic oscillator. According to his proposal, the oscillator consists of an object 10^{-6} m in diameter and estimated mass of 10^{-15} kg. It vibrates on the end of a thin fiber with a maximum amplitude of 10^{-5} m and frequency 1000 Hz. You are the referee for the proposal.

(a) What is the approximate quantum number for the system in the state described?
(b) What would its energy be in electron volts if it were in its lowest energy state? Compare with the average thermal energy at room temperature, ~1/40 eV.
(c) What would its classical amplitude of vibration be if it were in its lowest energy state? Compare with the wavelength of visible light, about 500 nm, with which it is presumably observed.
(d) Would you, as a referee of this proposal, recommend award of a grant to carry out this research?

7. The molecular bonding in the compound NaCl is predominantly ionic, and to a good approximation we can consider a sodium chloride molecule as consisting of two units – an Na^+ ion and a Cl^- ion – bound together. Assuming an electrostatic attraction and a power-law repulsion between the ions, their potential energy as a function of ion spacing has the form

$$V(r) = -\frac{ke^2}{r} + \frac{A}{r^n} \tag{4}$$

(a) Find the equilibrium spacing r_o.
(b) Find the potential energy at this separation, V_{min}.
(c) Find the effective "spring constant" for the molecule, assuming small deviations from r_o. One way to do this is to find the second derivative of $V(r)$ at $r = r_o$.