University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Fall 2010

Problem Set 3 Hints

1. Recall the photoelectric equation

$$e\Delta V_{\text{stop}} = hf - W \tag{1}$$

where ΔV_{stop} is the stopping potential, f the frequency of incident light, and W the binding energy (work function) of the material. The "threshold" wavelength is the maximum wavelength (and thus minimum energy or frequency) of incident light that is capable of ejecting electrons. Thus, at this threshold wavelength, the incident photon energy must be equal to the work function, so a measurement of the critical wavelength gives you the work function.

If the stopping potential for a particular wavelength is ΔV, that means an electron of charge e moving through that potential difference acquires an energy of $e\Delta V$. Knowing that and the work function allows you to deduce the incident frequency or wavelength.

Note that an electron moving through a 2.5 V potential difference acquires (or loses) an energy of 2.5 eV. Using electron volts as a measure of energy is very handy, but you will need to use Planck's constant in eV·s rather than J·s to make everything come out correctly . . . it is also handy to know that $hc \approx 1240\,\text{eV·nm}$.

You should find a wavelength of about 271 nm

2. By 'circle of 20 cm' we mean a radius of 20 cm. The fastest (highest energy) electrons will have the largest radius of curvature, so if you can fit those inside a circle of radius 20 cm, everything is fine. The radius of curvature of an electron traveling with momentum p perpendicular to a magnetic field B is

$$r = \frac{p}{qB} \tag{2}$$

The kinetic energy of the electrons is $p^2/2m$ and can be found from the photoelectric equation. Given an incident wavelength and work function, the photoelectric effect equation tells you the maximum possible electron kinetic energy. This relates momentum to the wavelength of incident light and the barium work function, use that momentum in the equation above.

You should find a field of about 13 μT.

3. The momentum equations for Compton scattering in terms of the incident photon momentum p_i, scattered photon momentum p_f and electron momentum p_e read

$$p_e \cos\varphi = p_i - p_f \cos\theta \tag{3}$$
$$p_e \sin\varphi = p_f \sin\theta \tag{4}$$

Divide them to get an expression for $\tan\varphi$. Rearrange until you can make the equation look like

$$\tan\varphi = (\text{stuff}) \frac{\sin\theta}{1-\cos\theta} \tag{5}$$

and use the identity

$$\frac{1-\cos\theta}{\sin\theta} = \tan\left(\frac{\theta}{2}\right) \tag{6}$$

I asked this problem last year.

4. The incident photon energy ends up as the electron's kinetic energy *plus* the energy of the exiting photon. In order to conserve momentum and energy, there must still be an exiting photon that takes some of the energy. Thus, you can't simply equate the electron's energy to the incident photon energy, in doing that you've not accounted for the exiting photon's energy.

In class I wrote down a formula for the electron's kinetic energy in terms of the dimensionless photon energy $\alpha_i = E_{\text{photon}}/mc^2 = hf_i/mc^2$ for the incident photon:

$$\mathsf{KE} = mc^2 \left(\frac{\alpha_i^2 (1-\cos\theta)}{1+\alpha_i(1-\cos\theta)} \right) \tag{7}$$

Here m is the electron mass, of course, and θ is the angle that the exiting photon makes with respect to the incident photon energy. The maximum electron energy is when $\theta = 180$, or $\cos\theta = -1$, giving the maximum kinetic energy as

$$E_{\text{max}} = mc^2 \left(\frac{2\alpha_i^2}{1+2\alpha_i} \right) \tag{8}$$

Since you are given $E_{\text{max}} = 45\,\text{keV}$, you can solve for α_i which gives you the incident photon energy (and thus wavelength). You'll end up with a quadratic; the root with negative α can be discarded as unphysical.

Again, if you're using energy in eV, then you do need to use Planck's constant in eV·s when necessary. It may also be convenient in that case to note that the electron's rest energy is $mc^2 = 511\,\text{keV} = 5.11 \times 10^5\,\text{eV}$.

You should find a wavelength of about $9 \times 10^{-12}\,\text{m}$.

5. The electron's energy is, noting that $\alpha_i mc^2 = hf_i$,

$$\mathsf{KE} = mc^2 \left(\frac{\alpha_i^2 \,(1 - \cos\theta)}{1 + \alpha_i \,(1 - \cos\theta)} \right) = hf_i \left(\frac{\alpha_i \,(1 - \cos\theta)}{1 + \alpha_i \,(1 - \cos\theta)} \right) \tag{9}$$

If the photon is absorbed by the electron, that means the electron's kinetic energy must equal the photon's initial energy. Try setting $\mathsf{KE} = hf_i$ and show that a logical and/or mathematical fallacy results.

I asked this last year in a somewhat different form.

6. All this means is that the exiting (scattered) photon must have an energy of at least $2mc^2$. In terms of the dimensionless photon energies $\alpha_i = hf_i/mc^2$, $\alpha_f = hf_f/mc^2$, the Compton equation reads

$$\frac{1}{\alpha_i} = \frac{1}{\alpha_f} - (1 - \cos\theta) \tag{10}$$

If the exiting photon energy is $hf_f = 2mc^2$, this means $\alpha_f = 2$. Solve the Compton equation for α_i. Enforce the condition that $\alpha_i > 0$.
You should find $\theta = 60°$.

7. I've given you the answer above (e.g., problem 5), you just need to derive it . . . asked this last year.

8. Asked this one last year. The power is the number of photons per second times the energy per photon. If the frequency is 1 MHz, this is 10^6 cycles per second. If you know the number of photons per second, then there must be a factor 10^6 less per cycle . . .

9. Find the power per unit area in the beam. The power the atom absorbs is that power per unit area times the area of the atom using the given geometry. That power is how many joules per second are absorbed. How many seconds does it take to come up with $5\,\text{eV} = 5 \times 1.6 \times 10^{-19}\,\text{J}$ worth of energy?

The only thing that changes for part b is to replace the area by $\lambda^2 = (hc/E)^2$.

For part c, the power is the photon energy (given) times the number of photons per second. Using the known power, find the number of photons per second. Inverting that gives you the average time between the arrival of individual photons.