Problem Set 1: Relativity

Instructions:

1. Answer all questions below. Show your work for full credit.
2. The first problem is due at the start of class on 23 Aug 2013
3. The second problem is due at the start of class on 26 Aug 2013
4. The remaining problems are due by the end of the day on 28 Aug 2013
5. You may collaborate, but everyone must turn in their own work.

1. Daily problem due 23 Aug 2013: How fast must a rocket travel relative to the earth so that time in the rocket "slows down" to half its rate as measured by earth-based observers? Do present-day jet planes approach such speeds?

2. Daily problem due 26 Aug 2013: A cube of metal with sides of length a sits at rest in frame S with one edge parallel to the x-axis. Therefore, in S the cube has volume a^3. Frame S' moves along the x-axis with speed u. As measured by an observer in frame S', what is the volume of the metal cube?

The problems below are due by the end of the day on 28 Aug 2013.

3. One of the wavelengths of light emitted by hydrogen atoms under normal laboratory conditions is $\lambda = 656.3\,\text{nm}$, in the red portion of the electromagnetic spectrum. In the light emitted from a distant galaxy this same spectral line is observed to be Doppler-shifted to $\lambda = 953.4\,\text{nm}$, in the infrared portion of the spectrum. How fast are the emitting electrons moving relative to the earth? Are they approaching the earth or receding from it?

4. Two particles in a high-energy accelerator experiment approach each other head-on with a relative speed of $0.890c$. Both particles travel at the same speed as measured in the laboratory. What its he magnitude of the velocity of one particle relative to the other?

5. (a) Through what potential difference does an electron have to be accelerated, starting from rest, to achieve a speed of $0.980c$? **(b)** What is the kinetic energy of the electron at this speed? Express your answer in both joules and electron volts.

6. Use the following two equations:

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}} \tag{1}$$

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \tag{2}$$

to derive the following relationship:

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

7. A charge q at $x = 0$ accelerates from rest in a uniform electric field $\vec{\mathbf{E}}$ which is directed along the positive x axis.

(a) Show that the acceleration of the charge is given by

$$a = \frac{qE}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

(b) Show that the velocity of the charge at any time t is given by

$$v = \frac{qEt/m}{\sqrt{1 + (qEt/mc)^2}}$$

(c) Find the distance the charge moves in a time t. *Hint:* ***http://integrals.wolfram.com***

8. Show that for the preceding question the particle's speed approaches c as $t \to \infty$.

9. At what speed is the momentum of a particle twice as great as the result obtained from the non-relativistic expression mv? Express your answer in terms of the speed of light.

10. Light travels with respect to earth at $3 \times 10^8 \ \frac{m}{s}$. A rocket travels at $2.5 \times 10^8 \ \frac{m}{s}$ with respect to earth in opposite direction of the light. What is the speed of light as viewed from the rocket?