Problem Set 2: Radiation

Instructions:

1. Answer all questions below. Show your work for full credit.
2. The first problem is due at the start of class on 4 Sept 2013
3. The second problem is due at the start of class on 6 Sept 2013
4. The remaining problems are due by the end of the day on 9 Sept 2013
5. You may collaborate, but everyone must turn in their own work.

Daily problem due 4 Sept 2013: Recall the formula we developed for the electric field of a charge in motion at constant velocity. Calculate the field strength using that expression for the limiting cases of **(a)**: $\theta = 0$, **(b)** $\theta = 90°$, **(c)** $v = 0$.

Daily problem due 6 Sept 2013: Which of the following expressions correspond to traveling waves? For each of those, what is the speed of the wave? The quantities A, a, b, c are positive real constants.

$$\psi(x,t) = (ax - bt)^2 \tag{1}$$

$$\psi(x,t) = A\sin\left(ax^2 - bt^2\right) \tag{2}$$

$$\psi(x,t) = \frac{1}{ax^2 + b} \tag{3}$$

$$\psi(x,t) = A\sin 2\pi\left(\frac{x}{a} + \frac{t}{b}\right) \tag{4}$$

The problems below are due by the end of the day on 9 Sept 2013.

1. (a) Charge q_a is at rest at the origin in system S; charge q_b flies by at speed v on a trajectory parallel to the x axis, but at $y = d$. What is the electromagnetic force on q_b as it crosses the y axis?
(b) Now study the same problem from system S', which moves to the right with speed v. What is the force on q_b when q_a passes the y' axis? You can either use your previous answer and transform the force, or compute the fields in S' using the Lorentz force law.

2. A proton is uniformly accelerated in a van de Graaff accelerator through a potential difference of 700 kV. The length of the linear accelerating region is 3 m. **(a)** Compute the ratio of the radiated energy to the final kinetic energy. **(b)** Show that for a particle moving in a linear accelerator the rate of radiation of energy is

$$\frac{dU}{dt} = \frac{q^2}{6\pi\epsilon_o m^2 c^3}\left(\frac{dK}{dx}\right)^2 \tag{5}$$

where K is the kinetic energy.

3. Assume the sun radiates like a black body at 5500 K. Assume the moon absorbs all the radiation it receives from the sun and reradiates an equal amount of energy like a black body at temperature T. The angular diameter of the sun seen from the moon is about 0.01 rad. What is the equilibrium temperature T of the moon's surface? (Note: you do not need any other data than what is contained in the statement above.

4. Presume the surface temperature of the sun to be 5500 K, and that it radiates approximately as a blackbody. What fraction of the sun's energy is radiated in the visible range of $\lambda = 400 - 700$ nm? One valid solution is to plot the energy density on graph paper and find the result numerically.

5. An electron is released from rest and falls under the influence of gravity. **(a)** How much power does it radiate? **(b)** How much energy is lost after it falls 1 m? (*Hint:* $P = \Delta K / \Delta t$, $y = \frac{1}{2} g t^2$.)

6. An electron initially moving at constant speed v is brought to rest with uniform deceleration a lasting for a time $t = v/a$. Compare the electromagnetic energy radiated during this deceleration with the electron's initial kinetic energy. Express the ratio in terms of two lengths, the distance light travels in time t and the classical electron radius $r_e = e^2 / 4\pi\epsilon_o m c^2$.

7. A capacitor consists of two parallel rectangular plates with a vertical separation of 0.02 m. The east-west dimension of the plates is 0.2 m, the north-south dimension is 10 cm. The capacitor has been charged by connecting it temporarily to a battery of 300 V.

(a) How many excess electrons are on the negative plate?
(b) What is the electric field strength between the plates?

Now, give the quantities as they would be measured in a frame of reference which is moving eastward, relative to the laboratory in which the plates are at rest, with speed $0.6c$.

(c) The dimensions of the capacitor,
(d) The number of excess electrons on the negative plate,
(e) The electric field strength between the plates.

8. *Hecht 2.38* Show that the imaginary part of a complex number z is given by

$$\frac{z - z^*}{2i} \tag{6}$$

9. The equation for a driven damped oscillator is

$$\frac{d^2x}{dt^2} + 2\gamma\omega_o \frac{dx}{dt} + \omega_o^2 x = \frac{q}{m} E(t) \tag{7}$$

(a) Explain the significance of each term.

(b) Let $E = E_o e^{i\omega t}$ and $x = x_o e^{i(\omega t - \alpha)}$ where E_o and x_o are real quantities. Substitute into the above expression and show that

$$x_o = \frac{qE_o/m}{\sqrt{(\omega_o^2 - \omega^2)^2 + (2\gamma\omega\omega_o)^2}} \tag{8}$$

(c) Derive an expression for the phase lag α, and sketch it as a function of ω, indicating ω_o on the sketch.