Problem Set 7

Instructions:

1. Answer all questions below. Show your work for full credit.
2. Read partial derivative symbols, e.g., $\partial/\partial x$, as d/dx if you are unfamiliar.
3. All problems are due by the end of the day on 18 Nov 2013
4. You may collaborate, but everyone must turn in their own work.

1. The specific heat at constant volume *per electron* is defined as

$$c_V = \frac{d\langle E\rangle}{dT} \tag{1}$$

where $\langle E\rangle$ is the average energy of electrons. A classical electron would have an average energy of $\frac{3}{2}k_BT$. Presuming the electrons follow a Fermi-Dirac distribution, estimate the specific heat for a collection of free electrons. Presume low temperatures, i.e., $k_BT \ll E_F$. *Hint: electrons that have been thermally excited above the Fermi energy behave basically as free electrons. What fraction of electrons are excited above E_F, roughly?*

2. Following section 10.6 in your textbook (find one), you can find the total energy of a photon gas U as well as the total number of photons in the gas N.[i] Recall that a photon gas was our model for blackbody radiation! **(a)** What is the *total* specific heat C_V for the photon gas, noting

$$C_V = \frac{\partial U}{\partial T} \tag{2}$$

(b) How does the average energy per photon (U/N) vary with temperature? **(c)** Given this average energy per photon, estimate the specific heat *per photon* using the expression given in problem 1.

3. Use the free electron theory to determine the Fermi energy and density of states vs. energy for a *two-dimensional* metal. Take N as the average number of electrons per unit area.

4. **(a)** Obtain an expression for the Fermi energy at $T = 0\,\mathrm{K}$ for an electron gas in a three dimensional metal in terms of the total number of electrons, the volume, and fundamental constants. **(b)** At $T = 0\,\mathrm{K}$, what is the average speed, in terms of the Fermi energy, of a three-dimensional electron gas in a metal?[ii]

5. Show that the average kinetic energy of an electron in a three-dimensional electron gas at $0\,\mathrm{K}$ is $E_{\text{av}} = \frac{3}{5}E_F$.[iii]

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[i] For N, use the expression for dN in Eq. 10.38 and integrate it over all energies. Or look here: `http://en.wikipedia.org/wiki/Photon_gas`. Always read the footnotes.

[ii] Open your textbook to 10.7.

[iii] Open your textbook to 10.7.

6. Now that you've followed the derivation for the three dimensional case from the textbook, repeat the previous **two** problems for a two dimensional electron gas, using your results from problem 3.