Problem Set 8

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due by the end of the day on 26 Nov 2013.
3. You may collaborate, but everyone must turn in their own work.

1. Suppose we have a gas in thermal equilibrium at temperature T. Each molecule of the gas has mass m. **(a)** What is the ratio of the number of molecules at the Earth's surface to the number at height h (with potential energy mgh)? **(b)** What is the ratio of the density of the gas at height h to the density ρ_o at the surface? **(c)** Would you expect this simple model to give an adequate description of the Earth's atmosphere? Why?

2. A collection of noninteracting hydrogen atoms is maintained in the $2p$ state in a magnetic field of strength 5.0 T. **(a)** At room temperature (293 K), find the fraction of the atoms in the $m_l = +1, 0,$ and -1 states. **(b)** If the $2p$ state made a transition to the $1s$ state, what would be the relative intensities of the three normal Zeeman components? Ignore any effects of electron spin.

3. Calculate the de Broglie wavelength of an electron with energy E_F in copper, and compare the value with the atomic separation in Copper.

4. The electrical conductivity of copper at room temperature is $5.96 \times 10^7\,\Omega^{-1}\mathrm{m}^{-1}$. Evaluate the average distance between electron scatterings. How many lattice spacings does this amount to?

5. Gallium phosphide ($E_g = 2.26\,\mathrm{eV}$) and zinc selenide ($E_g = 2.87\,\mathrm{eV}$) are commonly used to make LEDs. What is the most prominent emission wavelength of these devices, and what color is the corresponding light?

6. Consider a system of N free electrons within a volume V. Even at absolute zero, such a system exerts a pressure p on its surroundings due to the motion of the electrons (much like an ideal gas). To calculate this pressure, imagine that the volume increases by a small amount dV. The electrons will do an amount tot work $p\,dV$ on their surroundings, which means that the total energy E_{tot} of the electrons will change by an amount $dE_{\mathrm{tot}} = -p\,dV$. Hence, $p = -dE_{\mathrm{tot}}/dV$. **(a)** Find the pressure of the electrons at absolute zero for free electrons in *both* 2D and 3D in terms of the number of electrons, volume (or area), and fundamental constants. **(b)** Evaluate this pressure for copper, which has a free-electron concentration of $8.45 \times 10^{28}\,\mathrm{m}^3$. **(c)** The pressure you found is extremely high. Why don't electrons then simply explode out of the copper?