University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair | Spring 2010

Problem Set 1: Relativity

Instructions:

1. Answer all questions below.
2. Show your work for full credit.
3. All problems are due Thurs 21 January 2010 by the end of the day.
4. You may collaborate, but everyone must turn in their own work.

1. *Pfeffer & Nir, Prob. 5* The radius of the circular path of an electron moving with a velocity v at right angles to a magnetic field B is given classically by

$$r = \frac{mv}{eB} \tag{1}$$

This equation is valid for $v \ll c$.

(a) What does the relativistic version of this formula look like, valid for all speeds.
(b) Calculate the radius of the path of an electron with an energy of 10 MeV moving at right angles to a magnetic field strength of $B = 2\,\mathrm{T}$.

2. A classic "paradox" involving length contraction and the relativity of simultaneity is as follows: Suppose a runner moving at $0.75c$ carries a horizontal pole 15 m long toward a barn that is 10 m long. The barn has front and rear doors. An observer on the ground can instantly and simultaneously open and close the two doors by remote control. When the runner and the pole are inside the barn, the ground observer closes and then opens both doors so that the runner and pole are momentarily captured inside the barn and then proceed to exit the barn from the back door. Do both the runner and the ground observer agree that the runner makes it safely through the barn?

3. An astronaut takes a trip to Sirius, which is located a distance of 8 lightyears from the Earth. The astronaut measures the time of the one-way journey to be 6 yr. If the spaceship moves at a constant speed of $0.8c$, how can the 8-ly distance be reconciled with the 6-yr trip time measured by the astronaut?

4. *The red shift.* A light source recedes from an observer with a speed $v_{\text{source}} << c$.
(a) Show that the fractional shift in the measured wavelength is given by the approximate expression

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v_{\text{source}}}{c} \tag{2}$$

This phenomenon is known as the red shift, because the visible light is shifted toward the red.
(b) Spectroscopic measurements of light at $\lambda = 397\,\mathrm{nm}$ coming from a galaxy in Ursa Major reveal a red shift of $20.0\,\mathrm{nm}$. What is the recessional speed of the galaxy?

5. A particle with electric charge q moves along a straight line in a uniform electric field $\vec{\mathbf{E}}$ with a speed of u. The electric force exerted on the charge is $q\vec{\mathbf{E}}$. The motion and the electric field are both in the x direction. Show that the acceleration of the particle in the x direction is given by

$$a = \frac{du}{dt} = \frac{qE}{m}\left(1 - \frac{u^2}{c^2}\right)^{3/2} \tag{3}$$

6. A stick of length L is at rest on one system and is oriented at an angle θ with respect to the x axis. What are the apparent length and orientation angle of this stick as viewed by an observer moving at a speed v with respect to the first system?

7. A particle appears to move with speed u at an angle θ with respect to the x axis in a certain system. At what speed and angle will this particle appear to move in a second system moving with speed v with respect to the first? Why does the answer differ from that of the previous problem?

8. The acceleration of a particle in one reference frame is $a_x = dv_x/dt$, where the particle has an instantaneous velocity v_x in that frame. Consider a reference frame moving with speed V parallel to the positive x axis of the first frame. Show that the acceleration in the second frame is given by

$$a'_x = \frac{dv'_x}{dt} = a_x \frac{\left(1 - V^2/c^2\right)^{3/2}}{\left(1 - v_x V/c^2\right)^3}$$

9. A particle of mass m is subject to a constant force F along the x axis. If it starts from rest at the origin at time $t = 0$, find its position x as a function of time, using relativistic dynamics. Recall that Newton's second law in relativistic form is

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt} \qquad \text{with} \qquad \vec{\mathbf{p}} \equiv \frac{m\vec{\mathbf{v}}}{\sqrt{1 - v^2/c^2}} \tag{4}$$

Note the following useful integral:

$$\int \frac{x}{\sqrt{1 + ax^2}}\,dx = \frac{1}{a}\sqrt{1 + ax^2} + C \tag{5}$$