University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair Spring 2010

Problem Set 2: Black-Body Radiation

Instructions:

1. Answer all questions below.
2. Show your work for full credit.
3. All problems are due Thurs 28 January 2010 by the end of the day.
4. You may collaborate, but everyone must turn in their own work.

1. *Leighton, 2.4* As a function of wavelength, Planck's law states that the emitted power of a black body per unit area of emitting surface, per unit wavelength is

$$I(\lambda, T) = \frac{8\pi hc^2}{\lambda^5}\left[e^{\frac{hc}{\lambda k_b T}} - 1\right]^{-1} \tag{1}$$

That is, $I(\lambda, T)d\lambda$ gives the emitted power per unit area emitted between wavelengths λ and $\lambda + d\lambda$. Show by differentiation that the wavelength λ_m at which $I(\lambda, T)$ is maximum satisfies the relationship

$$\lambda_m T = b \tag{2}$$

where b is a constant. This result is known as *Wien's Displacement Law,* and can be used to determine the temperature of a black body radiator from only the peak emission wavelength. The constant above has a numerical value of $b = 2.9 \times 10^6$ nm-K. *Note: at some point you will need to solve an equation numerically. Advice will be given in lecture.*

2. As a function of *frequency*, Planck's law states that the spectral energy density of a black body, the energy per unit volume per unit frequency, is given by

$$u(f, T) = \frac{8\pi hf^3}{c^3}\left[e^{\frac{hf}{k_b T}} - 1\right]^{-1} \tag{3}$$

If you think of a black body as an insulated, perfectly mirrored box with a tiny hot object inside, $u(f, T)$ would give the energy per unit volume of radiation with frequencies between f and $f + df$.

Integrating this energy density over all frequencies, one obtains the *total* energy per unit volume V. Show that the total emitted power per unit volume is proportional to T^4. Specifically,

$$\frac{U(T)}{V} = \int_0^\infty u(f,T) = \sigma T^4 \tag{4}$$

Here σ is a constant. This result is essentially the *Stefan-Boltzmann law.* The following integral may be useful:

$$\int_0^\infty \frac{x^3}{e^x - 1}\,dx = \zeta(4)\Gamma(4) = \frac{\pi^4}{90} \times 6 = \frac{\pi^4}{15} \tag{5}$$

A clever substitution might be to define a variable $x = hf/k_bT$.

3. *Leighton, 2.8* The wavelength of maximum intensity in the solar spectrum is about 500 nm, as some of you will verify in PH255. Assuming the sun radiates as a black body, compute its surface temperature.

4. In the figure below, the measured intensity as a function of wavelength is shown for a 60 W incandescent bulb at various supply voltages V. (You may ignore the smaller secondary peaks at higher wavelengths, they are due to a phosphorescent coating on the inside of the bulb.) A larger version of this plot is on the last page; raw data is available upon request.

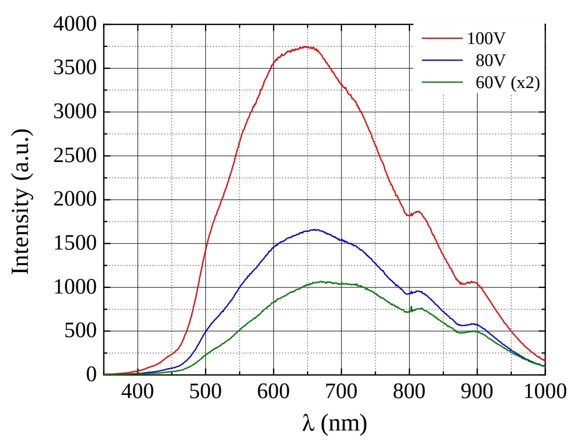

Figure 1: *Spectrum of a* 60 *W soft white incandescent bulb at three different supply voltages, measured in the PH255 lab. Note that the* 60 *V curve has been multiplied by a constant factor!*

(a) Assuming the bulb filament radiates as a perfect black body, the wavelength at which peak intensity occurs should be inversely proportional to temperature, $\lambda_m = b/T$ with $b = 2.9 \times 10^6$ nm-K. Estimate the peak position for each curve. Plot the resulting estimated filament temperature versus the *relative* electrical power supplied to the filament. You may assume the bulb has constant resistance, such that the power supplied to the bulb is proportional to V^2. Do the results make sense?

(b) The total emitted power is proportional to the area under the intensity-wavelength graph. *Roughly estimate* the area under the curves for each voltage. This in turn should be proportional to the bulb temperature to the fourth power, T^4. Plot the estimated area versus for each curve versus T^4 using your temperature estimates from part a. Is the Stefan-Boltzmann law obeyed, within your margin of error?

(c) Is the bulb a reasonable approximation of a black body? You may want to check the melting point of the tungsten filament.

(something to think about, not to turn in) Compare your spectra qualitatively to the solar spectrum, e.g., `http://en.wikipedia.org/wiki/Sunlight`. Can you understand why incandescent bulbs at particular powers are favored for indoor lighting? Why is "color temperature" used to characterize such lighting sources?

5. *Frank, 20.16* Compute the ratio of the increase of intensity of black-body radiation at a wavelength of 641 nm for an increase of temperature from 1200 to 1500 K.

6. An accelerating charge loses electromagnetic energy at a rate of

$$\mathscr{P} = \frac{\Delta E}{\Delta t} = -\frac{2k_e q^2 a^2}{3c^3}$$

where k_e is Coulomb's constant, q is the charge of the particle, c is the speed of light, and a is the acceleration of the charge. Assume that an electron is one Bohr radius ($a_0 = 0.053\,\text{nm}$) from the center of a Hydrogen atom, with the proton stationary. **(a)** Find the acceleration of the electron (hint: circular path). **(b)** Calculate the kinetic energy of the electron and determine within an order of magnitude how long it will take the electron to loose all of its energy, assuming a constant acceleration as found in part a. Be sure to point out whether you need to consider relativistic effects or not (hint: how big is v/c if you ignore relativity?).

7. Assuming that the human body has a surface area of 2 square meters and radiates like a black body at a temperature of 35°C, calculate the rate at which it loses heat in surroundings that have a temperature of 15°C.

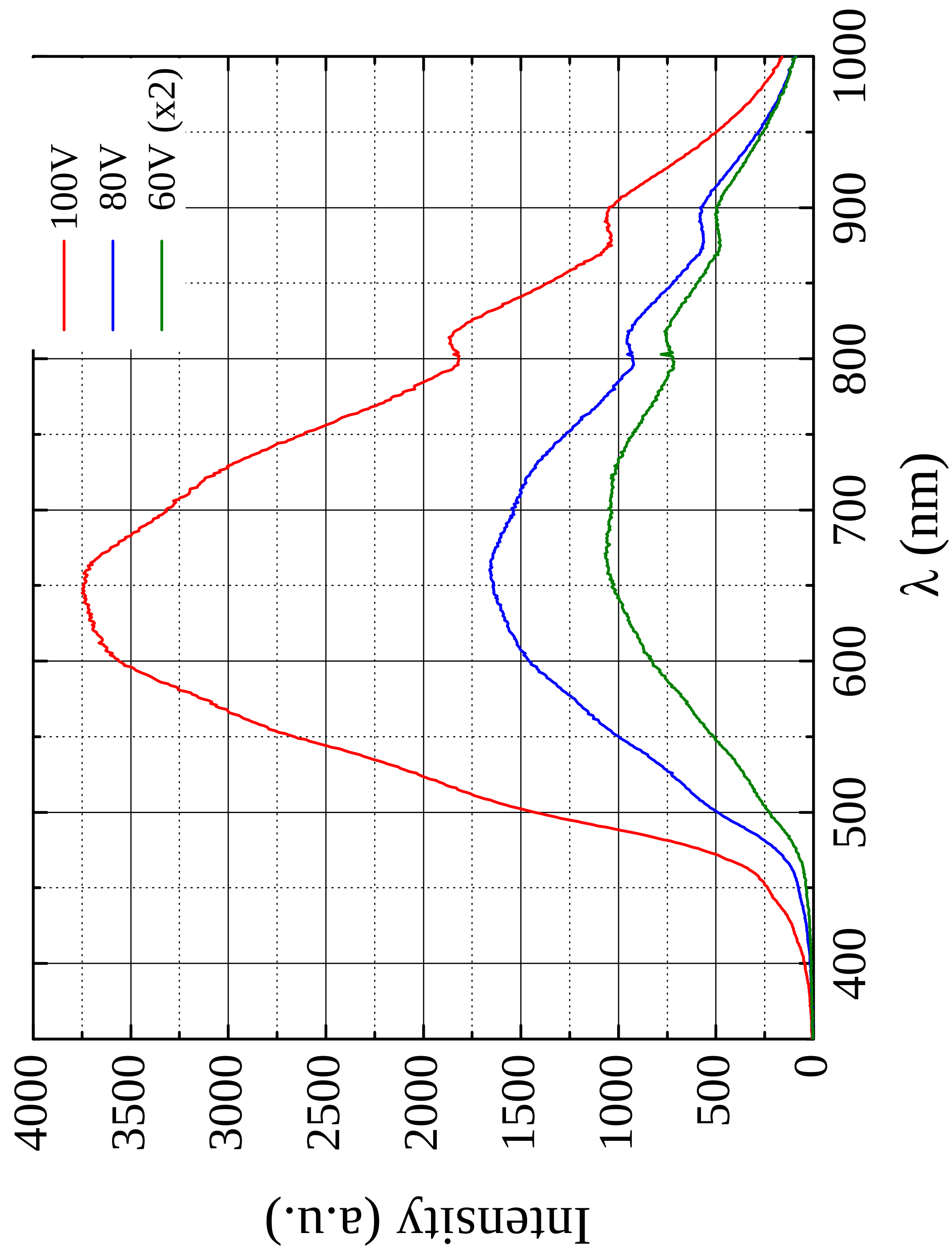

Figure 2: *Spectrum of a* 60 *W soft white incandescent bulb at various supply voltages. Note that some curves have been multiplied by a constant factor to make all curves visible!*