University of Alabama
Department of Physics and Astronomy

PH 253 / LeClair   Spring 2010

Problem Set 8

Instructions:

1. Answer all questions below.
2. Show your work for full credit.
3. All problems are due Fri 26 March 2010 by the end of the day.
4. You may collaborate, but everyone must turn in their own work.

1. *Multiplicity of atomic magnetic moments.* Calculate the magnetic moments that are possible for the $n=3$ level of Hydrogen, making use of the quantization of angular momentum. You may neglect the existence of spin. Compare this with the Bohr prediction for $n=3$.

2. *Transitions in a magnetic field.* Transitions occur in an atom between $l=2$ and $l=1$ states in a magnetic field of 0.6 T, obeying the selection rules $\Delta m_l = 0, \pm 1$. If the wavelength before the field was turned on was 500.0 nm, determine the wavelengths that are observed. You may find the following relationship from last week's homework useful:

$$|\Delta\lambda| = \frac{\lambda^2 \Delta E}{hc} \tag{1}$$

Recall that the Zeeman effect changes the energy of a single-electron atom in a magnetic field by

$$\Delta E = m_l \left(\frac{e\hbar}{2m_e}\right) B \qquad \text{with} \qquad m_l = -l, -(l-1), \ldots, 0, \ldots, l-1, l \tag{2}$$

For convenience, note that $e\hbar/2m_e = \mu_B \approx 57.9\,\mu\text{eV/T}$, and neglect the existence of spin. See also: Pfeffer & Nir 3.2.2

3. *Stern-Gerlach experiment.* A beam of *free electrons* moves perpendicularly through a uniform magnetic field of 0.8 T. What is the energy difference between the electrons whose spins are "aligned" and "anti-aligned" with the magnetic field? See also: Pfeffer & Nir 3.4.1-2

4. *Radio astronomy.* The hydrogen $\lambda = 21$ cm line is used in radio astronomy to map the galaxy. The line arises from the emission of a photon when the electron in a galactic hydrogen atom "flips" its spin from being aligned to being anti-aligned with the spin of the proton in the hydrogen atom.[i] What is the magnetic field the electron experiences to induce this spin flip?

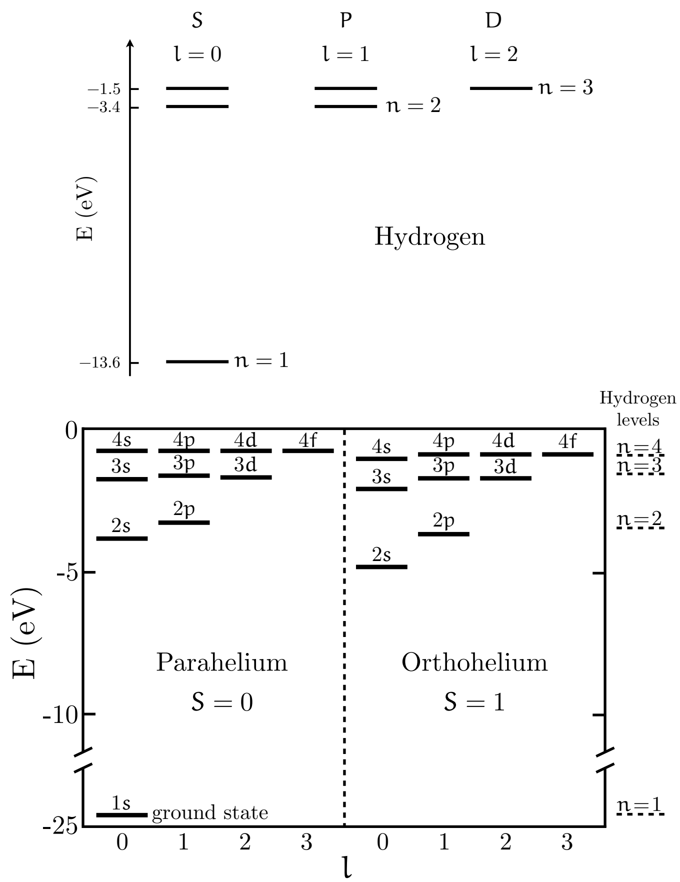
[i] Even though this process is strongly forbidden, the abundance of hydrogen in the galaxy is sufficiently enormous to make observation practical.

5. *Dipole selection rules.*

(a) For hydrogen, the energy levels through $n=3$ are shown below. What are the possible electric dipole transitions for these states? It may be convenient to simply draw arrows in the diagram. Recall the "selection rules" for electric dipole transitions, $\Delta l = \pm 1$. Spin may be ignored.

(b) Repeat for para- and ortho-helium, also shown below, treating both as distinct atoms.[ii]

See also: Pfeffer & Nir 3.3.2



Problem 5. **(upper)** *Energy levels of H through* $n=3$*, neglecting spin.* **(lower)** *Energy levels of para- and ortho- He through* $n=4$.

[ii] Two types of helium: para-helium, with the two electron spins parallel ($S=0$), and ortho-helium, with the two electron spins antiparallel ($S=1$). According to the dipole selection rules, helium atoms cannot change by a radiative process from one to the other, as this would not conserve angular momentum, so ortho- and para-helium behave largely as distinct atoms. (Forbidden transitions are not strictly forbidden, but violating the selection rules incurs a cost of $\sim 10^5$ in transition probability).

As the energy level diagram shows, the lowest state corresponds to para-helium, and the next highest excited state ortho-helium. The ortho-helium excited state can be reached by electrical discharge excitation, a non-radiative process (i.e., not obeying the same selection rules.) This excited state is very long lived (~ 10 ms) because returning to the ground state would violate selection rules.

6. *Splitting of Sodium D lines.* The electron's intrinsic magnetic moment $\vec{\mu}_s$ and intrinsic spin angular momentum $\vec{S}$ are proportional to each other; their relationship can be written as

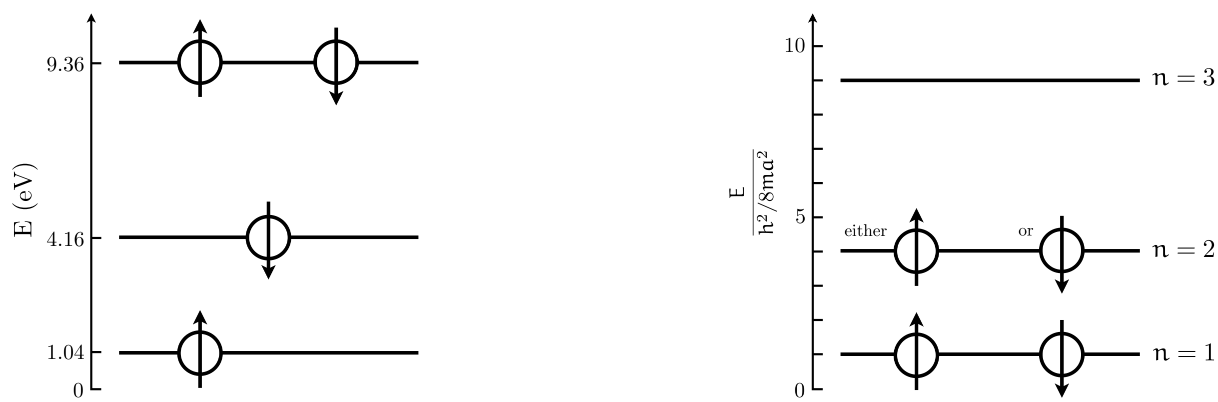
$$\vec{\mu}_s = -g_s \frac{e}{2m}\vec{S} = -g\mu_b\vec{S} \tag{3}$$

with $g_s \approx 2$. The energy of the electron in a effective magnetic field $\vec{B}$ is $E = -\vec{\mu}_s \cdot \vec{B}$.

In Sodium, transitions occur between two spin-orbit-split $L = 1$ states and a single $L = 0$ state, leading to emission lines at 588.995 nm and 589.592 nm. Estimate the strength of the effective magnetic field produced by the electron's orbital motion (i.e., the effective field due to the spin-orbit interaction) which results in this wavelength difference. You may wish to make use of the relationship given in problem 2. See also: Pfeffer & Nir 3.4.3,

`http://hyperphysics.phy-astr.gsu.edu/HBASE/quantum/sodium.html#c2`

7. *Pauli exclusion.* What are the energies of the photons that would be emitted when the four-electron system in the figure below returns to its ground state? See also: Pfeffer & Nir 3.4.5



Left, problem 7: *A system of four electrons with three energy levels.* **Right, problem 8:** *A system of three electrons in an infinite square well.*

8. Three non-interacting particles are in their ground state in an infinite square well;[iii] see the figure above. What happens when a magnetic field is turned on which interacts with the spins of the particles? Draw the new levels and particles (with spin).

[iii] Recall the energies in an infinite square well are $E = n^2h^2/8ma^2$