Problem Set 2: Special Relativity

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Thurs 24 January 2013 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. One of the strongest emission lines observed from distant galaxies comes from hydrogen and has a wavelength of 122 nm (in the ultraviolet region). **(a)** How fast must a galaxy be moving away from us in order for that line to be observed in the visible region at 366 nm? **(b)** What would be the wavelength of the line if that galaxy were moving toward us at the same speed?

2. Two spaceships approach the Earth from opposite directions. According to an observer on the Earth, ship A is moving at a speed of $0.753c$ and ship B at a speed of $0.851c$. What is the velocity of ship A observed from ship B? Of ship B observed from ship A?

3. Derive the Lorentz velocity transformations for v'_x and v'_z.

4. According to observer O, two events occur separated by a time interval of $\Delta t = +0.465\,\mu s$ at at locations separated by $\Delta x = +53.4\,m$. **(a)** According to observer O', who is in motion relative to O at a speed of $0.762c$ in the positive x direction, what is the time interval between the two events? **(b)** What is the spatial separation between the two events according to O'?.

5. The work-energy theorem relates the change in kinetic energy of a particle to the work done on it by an external force: $\Delta K = W = \int F\,dx$. Writing Newton's second law as $F = dp/dt$, show that $W = \int v\,dp$ and integrate by parts to obtain the result

$$K = \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$$

6. Electrons are accelerated to high speeds by a two-stage machine. The first stage accelerates the electrons from rest to $v = 0.99c$. The second stage accelerates the electrons from $0.99c$ to $0.999c$. **(a)** How much energy does the first stage add to the electrons? **(b)** How much energy does the second stage add in increasing the velocity by only 0.9%?

7. A charge q at $x = 0$ accelerates from rest in a uniform electric field $\vec{\mathbf{E}}$ which is directed along the positive x axis.

(a) Show that the acceleration of the charge is given by

$$a = \frac{qE}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

(b) Show that the velocity of the charge at any time t is given by

$$v = \frac{qEt/m}{\sqrt{1 + (qEt/mc)^2}}$$

(c) Find the distance the charge moves in a time t. *Hint:* *`http://integrals.wolfram.com`*

8. An astronaut takes a trip to Sirius, which is located a distance of 8 lightyears from the Earth. The astronaut measures the time of the one-way journey to be 6 yr. If the spaceship moves at a constant speed of $0.8c$, how can the 8-ly distance be reconciled with the 6-yr trip time measured by the astronaut?

9. A proton is accelerated to a velocity $v = 0.999c$ and sent down an evacuated metal tube 100 m long. Take the speed of light as $c = 3.0 \times 10^8$ m/s.

(a) In the protons reference frame, how long is the tube? **(b)** In the protons frame, how long does it take to traverse the length of the tube? **(c)** In the laboratory frame, how long does it take for the proton to traverse the length of the tube?

10. An atomic clock aboard a spaceship runs slow compared to an Earth-based atomic clock at a rate of 2.0 seconds per day. What is the speed of the spaceship?