Problem Set 4: Whither, thou turbid wave?

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Tues 26 February 2013 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. The wavefunction of a transverse wave on a string is

$$\psi(x,t) = (30.0\,\text{cm})\cos\left[(6.28\,\text{rad/m})\,x - (20.0\,\text{rad/s})\,t\right] \tag{1}$$

Compute the frequency, wavelength, period, amplitude, phase velocity, and direction of motion.

2. What is the uncertainty in the location of a photon of wavelength 300 nm if this wavelength is known to an accuracy of one part in a million?

3. Find the potential difference through which electrons must be accelerated (as in an electron microscope, for example) if we wish to resolve: (a) a virus of diameter 12 nm, (b) an atom of diameter 0.12 nm, (c) a proton of diameter 1.2 fm. Show your work, and do not forget about relativity.

4. In order to study the atomic nucleus, we would like to observe the diffraction of particles whose de Broglie wavelength is about the same size as the nuclear diameter, about 14 fm for a heavy nucleus such as lead. What kinetic energy should we use if the diffracted particles are (a) electrons? (b) Neutrons? (c) Alpha particles ($m = 4\,\text{u}$)?

5. The speed of an electron is measured to within an uncertainty of 2.0×10^4 m/s. What is the size of the smallest region of space in which the electron can be confined?

6. Use the distribution of wave numbers

$$A(K) = A_o e^{-(k-k_o)^2/2(\Delta k)^2} \qquad k \in \mathbb{R} \tag{2}$$

and equation 4.23 from your text, viz.,

$$y(x) = \int A(k)\cos kx\,dk \tag{3}$$

to obtain equation 4.25 from your text, viz.,

$$y(x) = A_o \Delta k \sqrt{2\pi} e^{-(\Delta k\, x)^2/2} \cos k_o x \tag{4}$$

7. A particle of mass m is confined to a one-dimensional line of length L. From arguments based on the uncertainty principle, estimate the value of the smallest energy that the body can have.

8. (a) Find the de Broglie wavelength of a nitrogen molecule in air at room temperature (293 K). (b) The density of air at room temperature and atmospheric pressure is $1.292\,\mathrm{kg/m^3}$. Find the average distance between air molecules at this temperature and compare with the de Broglie wavelength. What do you conclude about the importance of quantum effects in air at room temperature? (c) Estimate the temperature at which quantum effects might become important.

9. Refer to the video lecture by Feynman you watched. (a) In the two slit experiment, is it possible to design an experiment which allows you to tell which slit an electron goes through without spoiling the interference pattern? (b) A particle can proceed from point A to point B along two indistinguishable paths. Path A has an amplitude of a, path B has an amplitude of b. What is the overall probability for the particle to go from point A to point B?

10. X-ray photons of wavelength 0.154 nm are produced by a copper source. Suppose that 1.00×10^{18} of these photons are absorbed by the target each second.

(a) What is the total momentum p transferred to the target each second? (b) What is the total energy E of the photons absorbed by the target each second? (c) If the beam shines perpendicularly onto a perfectly reflecting surface, what force does it exert on the surface? Recall $F = \Delta p/\Delta t$. (d) For these values, verify that the force on the target is related to the rate of energy transfer by

$$\frac{dp}{dt} = \frac{1}{c}\frac{dE}{dt}$$