Problem Set 5: Mostly Wavefunctions

"I don't like it, and I'm sorry I ever had anything to do with it."
– Erwin Schrödinger about the probability interpretation of quantum mechanics

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Thurs 7 March 2013 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. The state of a free particle is described by the following wave function

$$\psi(x) = \begin{cases} 0 & x < -b \\ A & -b \leqslant x \leqslant 7b \\ 0 & x > 7b \end{cases} \tag{1}$$

(a) Determine the normalization constant A.
(b) What is the probability of finding the particle in the interval $[0, b]$?
(c) Determine $\langle x \rangle$ and $\langle x^2 \rangle$ for this state.
(d) Find the uncertainty in position $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$.

2. An electron in a helium atom is in a state described by the (normalized) wave function

$$\psi = \frac{4}{\sqrt{2\pi}\,(a_o)^{3/2}} e^{-2r/a_o} \tag{2}$$

where a_o is the Bohr radius.

(a) What is the *most probable* value of r?
(b) What is the energy E of the electron in this state? *Hint: use Schrödinger's equation.*

3. The wave function for the ground state of hydrogen ($n=1$) is

$$\psi_1 = \frac{1}{\sqrt{\pi a_o^3}} e^{-r/a_o} \tag{3}$$

where a_o is the Bohr radius.

(a) What is the *most probable* value of r for the ground state?
(b) What is the total probability of finding the electron at a distance greater than this radius?

4. Schrödinger's equation for a simple harmonic oscillator reads

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}m\omega^2 x^2\psi = E\psi \tag{4}$$

The ground state wave function has the form

$$\psi_o = a e^{-\alpha^2 x^2} \tag{5}$$

Determine the value of the constant α and the energy of the state.

5. A phenomenological expression for the potential energy of a bond as a function of spacing is given by

$$U(r) = \frac{A}{r^n} - \frac{B}{r^m} \tag{6}$$

For a stable bond, $m < n$. Show that the molecule will break up when the atoms are pulled apart to a distance

$$r_b = \left(\frac{n+1}{m+1}\right)^{1/(n-m)} r_o \tag{7}$$

where r_o is the equilibrium spacing between the atoms. Be sure to note your criteria for breaking used to derive the above result.