Problem Set 6: spin and so forth

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due 9 April 2013 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. Explain why each of the following sets of quantum numbers (n, l, m_l, m_s) is not permitted for hydrogen:

$$(2, 2, -1, +\frac{1}{2})$$
$$(3, 1, +2, -\frac{1}{2})$$
$$(4, 1, +1, -\frac{3}{2})$$
$$(2, -1, +1, +\frac{1}{2})$$

2. List the excited states (in spectroscopic notation) to which the $4p$ state can make downward transitions.

3. *Splitting of Hydrogen lines.* The electron's intrinsic magnetic moment $\vec{\mu}_s$ and intrinsic spin angular momentum $\vec{\mathbf{S}}$ are proportional to each other; their relationship can be written as

$$\vec{\mu}_s = -g_s \frac{e}{2m} \vec{\mathbf{S}} = -g \mu_b \vec{\mathbf{S}} \tag{1}$$

with $g_s \approx 2$. The energy of the electron in a effective magnetic field $\vec{\mathbf{B}}$ is $E = -\vec{\mu}_s \cdot \vec{\mathbf{B}}$.

In hydrogen, transitions occur between two spin-orbit-split $2p$ states and a single $1s$ state, leading to two emission lines. If the emission wavelength in the absence of spin-orbit coupling is 656.47 nm, and the spin-orbit splitting is 0.016 nm, estimate the strength of the effective magnetic field produced by the electron's orbital motion (i.e., the effective field due to the spin-orbit interaction) which results in this wavelength difference.

4. *Multiplicity of atomic magnetic moments.* Calculate the magnetic moments that are possible for the $n = 4$ level of Hydrogen, making use of the quantization of angular momentum. You may neglect the existence of spin. Compare this with the Bohr prediction for $n = 4$.

5. *Transitions in a magnetic field.* Transitions occur in an atom between $l = 2$ and $l = 1$ states in a magnetic field of 2.0 T, obeying the selection rules $\Delta m_l = 0, \pm 1$. If the wavelength before the field was turned on was 680.0 nm, determine the wavelengths that are observed. You may find the following relationship useful:

$$|\Delta\lambda| = \left|\frac{d\lambda}{dE}\right| \Delta E = \frac{hc}{E^2}\Delta E = \frac{\lambda^2}{hc}\Delta E \tag{2}$$

Recall that the Zeeman effect changes the energy of a single-electron atom in a magnetic field by

$$\Delta E = m_l \left(\frac{e\hbar}{2m_e}\right) B \qquad \text{with} \qquad m_l = -l, -(l-1), \ldots, 0, \ldots, l-1, l \tag{3}$$

For convenience, note that $e\hbar/2m_e = \mu_B \approx 57.9\,\mu\text{eV/T}$, and neglect the existence of spin.

6. By considering the visible spectrum of hydrogen and He^+, show how you could determine spectroscopically if a sample of hydrogen was contaminated with helium. (Hint: look for differences in the visible emission lines, $\lambda \approx 390 \sim 750$ nm. A difference of 10 nm is easily measured.)