University of Alabama
Department of Physics and Astronomy
PH 253-002 Spring 2019

Homework 4

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due by 4:45pm on Fri 8 Mar as a hard copy, or by 11:59pm on Fri 8 Mar via Blackboard
3. You may collaborate, but everyone must turn in their own work.

1. *Can you see me now?* Using the Bohr model for the hydrogen atom, which possible emitted or absorbed wavelengths fall in the visible region of the spectrum ($380-770\,\mathrm{nm}$)? Include transitions that involve the "level" $n=\infty$, e.g., an electron absorbing a photon and subsequently escaping the proton to a state with $E=0$.

2. *Spatial distribution of probability for a H state.* What is the probability of finding an $n=3$, $l=2$ electron between $5a_o$ and $6a_o$? *Hint: you need only use the radial wave function* $R(r)$*, see section 7.4 in your text.*

3. *Angular distribution of probability for a H state.* Find the directions in space where the angular probability density for the $l=3$, $m_l=0$ electron in hydrogen has its maxima and minima. *Hint: you only need* $P(\theta,\varphi)$*. See section 7.5 in your text.*

4. *Most probable radius for a H state.* Find the most probable radius of an electron in the $3p$ state. Note

$$R_{3p}(r)=\frac{8}{9\sqrt{2}\,(3a_o)^{3/2}}\left(\frac{r}{a_o}-\frac{r^2}{6a_o^2}\right)e^{-r/3a_o} \tag{1}$$

where $a_0=\frac{4\pi\epsilon_0\hbar^2}{m_e e^2}=0.529\times10^{-10}\,\mathrm{m}$ is the Bohr radius.

5. *Expectation value of the radius for a H state.* Using the radial wave function in the previous problem, find the expected value of the radial position $\langle r\rangle$ of an electron in the $3p$ state. Is this position the same you found in the previous question? Why or why not?

6. *Quantum numbers.* Explain why each of the following sets of quantum numbers (n,l,m_l,m_s) is not permitted for hydrogen:

$$(3,3,-1,+\frac{1}{2})\qquad(2,1,+2,-\frac{1}{2})\qquad(2,1,+1,-\frac{3}{2})\qquad(3,-1,+1,+\frac{1}{2})$$

7. *Multiplicity of atomic magnetic moments.* Calculate the magnetic moments that are possible for the $n=4$ level of Hydrogen, making use of the quantization of angular momentum. You may neglect spin. Compare this with the Bohr prediction for $n=4$.

8. *Transitions in a magnetic field.* Transitions occur in an atom between $l=2$ and $l=1$ states in a magnetic field of 3.5000 T, obeying the selection rules $\Delta m_l = 0, \pm 1$. If the wavelength before the field was turned on was 543.00 nm, determine the wavelengths that are observed. You may find the following relationship useful:

$$|\Delta\lambda| = \left|\frac{d\lambda}{dE}\right| \Delta E = \frac{hc}{E^2}\Delta E = \frac{\lambda^2}{hc}\Delta E \tag{2}$$

Recall that the Zeeman effect changes the energy of a single-electron atom in a magnetic field by

$$\Delta E = m_l \left(\frac{e\hbar}{2m_e}\right) B \qquad \text{with} \qquad m_l = -l, -(l-1), \ldots, 0, \ldots, l-1, l \tag{3}$$

For convenience, note that $e\hbar/2m_e = \mu_B \approx 57.9\,\mu\text{eV/T}$, and neglect the existence of spin.

9. *Gaussian Wave Packets and minimum uncertainty.* A particle of mass m is in the state

$$\psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]} \tag{4}$$

where $\{A, a\} \in \mathbb{R}$ and $\{A, a\} > 0$. **(a)** Find A. **(b)** For what potential energy function $V(x)$ does ψ satisfy the Schrödinger equation? **(c)** Calculate the expected values of x, x^2, p, and p^2. **(d)** Find Δx and Δp. Is their product consistent with the uncertainty principle?

10. *Is zero energy still a free choice?* Suppose you add a constant V_o to the potential energy (by "constant" we mean independent of both x and t). In classical mechanics, this doesn't change anything, but what about quantum mechanics? **(a)** Show that the wave function picks up a time-dependent phase factor: $\exp(-iV_o t/\hbar)$. **(b)** What effect does this have on the expectation value of a dynamical variable like x or p?