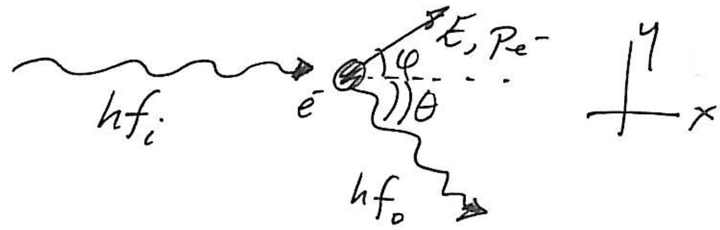
Compton scattering

C1



incident photon

$E_i = hf_i \qquad p_i = h/\lambda_i$

scattered photon

angle θ

$E_f = hf_f \quad p_f = h/\lambda_f$

Scattered electron

angle φ

$E_{e^-} = (\gamma - 1)mc^2 = \sqrt{p_e^2 c^2 + m^2 c^4} - mc^2$

$p_e = \gamma m v$

Consv Energy

$$hf_i = hf_f + \sqrt{p_e^2 c^2 + m^2 c^4} - mc^2$$

Consv $\vec{p}$

(x) $p_i = p_e \cos\varphi + p_f \cos\theta$

(y) $p_e \sin\varphi = p_f \sin\theta$

Convenient:

define $\alpha_i = \dfrac{hf_i}{mc^2} = p_i/mc$

$\alpha = \dfrac{\text{photon energy}}{e^- \text{ rest energy}}$

$$\Rightarrow \quad \alpha_i = \alpha_f + \sqrt{\frac{p_e^2}{m^2c^2} + 1} - 1$$

$$\alpha_i = \alpha_f \cos\theta + \left(\frac{p_e}{mc}\right)\cos\varphi$$

$$\alpha_f \sin\theta = \left(\frac{p_e}{mc}\right)\sin\varphi$$

- dimensionless E scale
- easier algebra (HW)
- important eqns simple

Big deal? Classical scattering of ~~[illegible]~~ light by e^- cannot explain! Need particle-like light rearrange E eqn

$$(\alpha_i - \alpha_f) + 1 = \sqrt{\frac{p_e^2}{m^2c^2} + 1}$$

square & solve for p_e^2

$$p_e^2 = m^2c^2\left[(\alpha_i - \alpha_f)^2 + 2(\alpha_i - \alpha_f)\right]$$

square and add $\vec{p}$ eqns

$$\left.\begin{aligned} p_e \sin\varphi &= mc\alpha_f \sin\theta \\ p_e \cos\varphi &= mc\left[\alpha_i - \alpha_f\cos\theta\right] \end{aligned}\right\} p_e^2\sin^2\varphi + p_e^2\cos^2\varphi = p_e^2 = \ldots$$

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now 2 eqns in p_e^2 ... equate!

$$\Rightarrow \boxed{\alpha_i - \alpha_f = \alpha_i\alpha_f(1-\cos\theta)} \quad \text{-or-} \quad \boxed{\frac{1}{\alpha_f} - \frac{1}{\alpha_i} = 1 - \cos\theta}$$

Resulting electron energy?

$$E_{e^-} = hf_i - hf_f = \alpha_i mc^2 - \alpha_f mc^2 = (\alpha_i - \alpha_f)mc^2$$

solve compton for α_f: $\quad \alpha_f = \dfrac{\alpha_i}{1+\alpha_i(1-\cos\theta)}$

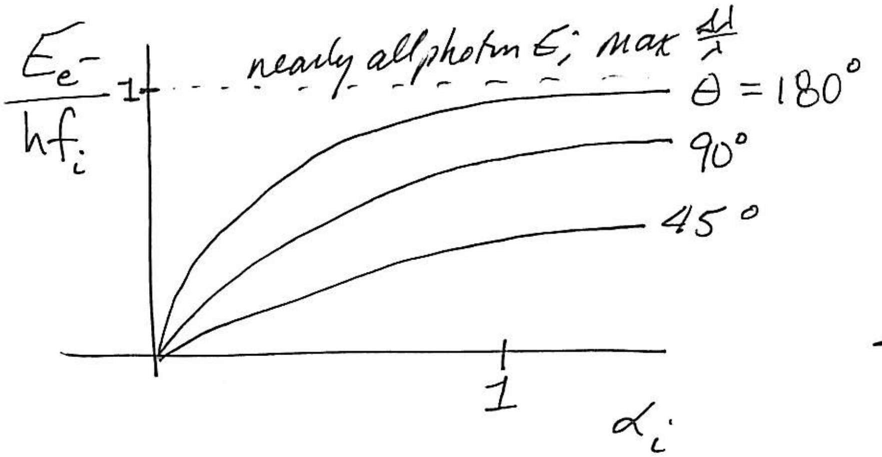
$$E_{e^-} = mc^2\left[\alpha_i - \frac{\alpha_i}{1+\alpha_i(1-\cos\theta)}\right] = mc^2\left[\frac{\alpha_i^2(1-\cos\theta)}{1+\alpha_i(1-\cos\theta)}\right]$$

$$E = mc^2\left[\frac{\alpha_i^2(1-\cos\theta)}{1+\alpha_i(1-\cos\theta)}\right] = hf_i\left[\frac{\alpha_i(1-\cos\theta)}{1+\alpha_i(1-\cos\theta)}\right] \qquad \alpha_i = \frac{hf_i}{mc^2}$$

max e^- energy? when $\underline{\cos\theta = -1}$ ($\frac{\partial E}{\partial\theta} = 0 \ldots$ careful)

$$\Rightarrow E_{max} = mc^2\left[\frac{2\alpha_i^2}{1+2\alpha_i}\right] = hf_i\left(\frac{2\alpha_i}{1+2\alpha_i}\right)$$

- max is a fraction of incident photon E
- cannot absorb photon & conserve $\vec{p}$ & E (HW)

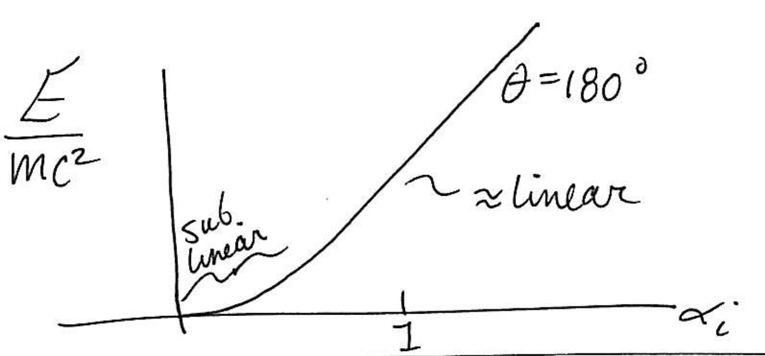


photon transfers most of its energy:

- $\theta \sim 180°$ (head-on)
- $hf_i > mc^2$

or $\underline{\alpha_i > 1}$

($\approx$ free e^-)



important point: α sets the energy range

characterizes physical regimes

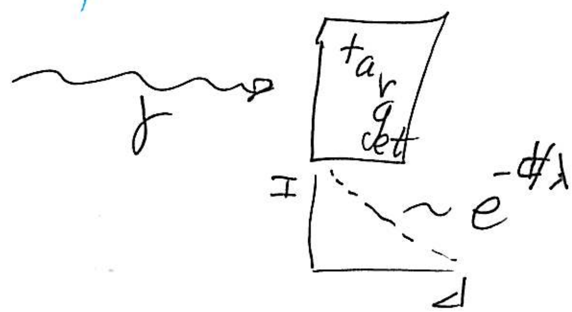
$\boxed{\text{Misc}}$ HW #6 : divide $\vec{p}$ eqns to get $\tan\varphi$

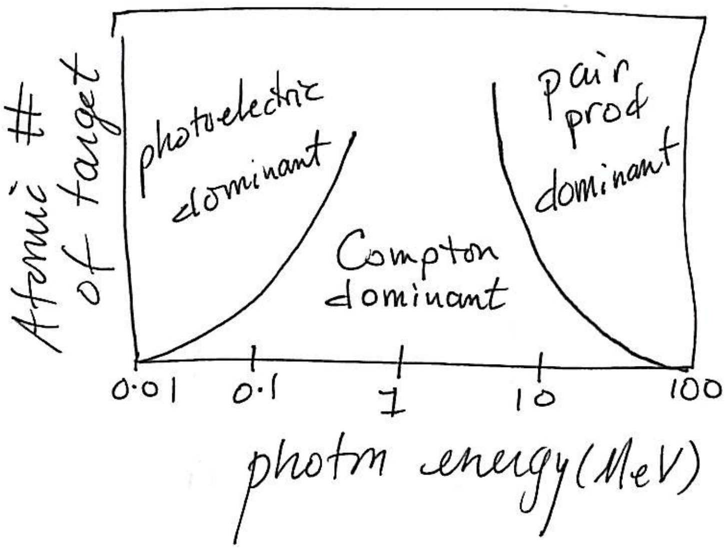
: substitute Compton eqn for α_f (or p_f)

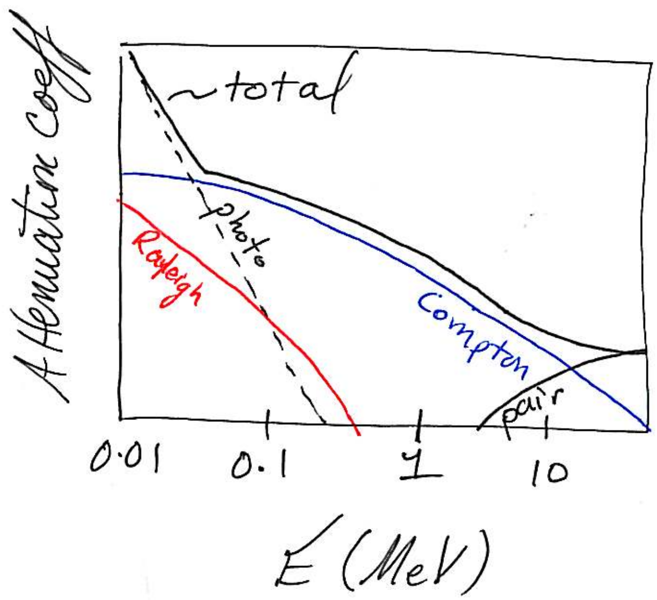
: note $\dfrac{1-\cos\theta}{\sin\theta} = \tan\left(\dfrac{\theta}{2}\right)$

$$\Rightarrow \tan\varphi = \left(\frac{1}{1+\alpha}\right)\cot\left(\frac{\theta}{2}\right)$$

$\boxed{\text{Interaction of photons w/matter}}$



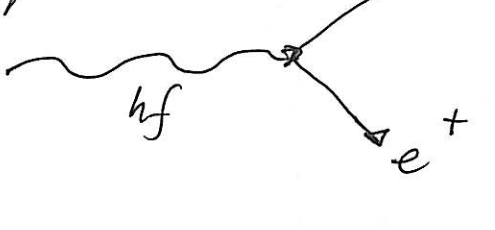




<u>low E / high mass</u> : photo

<u>mid</u> : compton

<u>high E</u> : <u>pair production</u>



} more on this later