[quantum after photoel, Compton, slits]

- Already: once light is discretized (NO way out!) we are forced to discretize the energy of atoms
- it follows then that any <u>observable</u> energy difference is discretized (for better or worse)
- Slit experiments: waves and particles give very different results!

<u>electrons and photons</u> don't look like either... some characteristics of <u>BOTH</u>

- now we have a new problem then:

if photons are wave-particle, discrete energy is anything else any different?

What makes MATTER so special?

Relativity says *nothing* !

<u>Back to EM waves</u>: have E, $\bar{p}$

- can model as stream of particles (Newton)
- dynamical properties well described

only energy quantization is really *NEW*

Worse: consider conservation laws in more detail

e.g. $\vec{p}, E$ ... mechanisms for transfer? μscopic?

(1) e^- emits | absorbs light

(2) e^- scatters off another particle

(1) is much of everyday life. Quantization implies ...?

e.g. photon emitted by atom

- polarization?
- angular momentum?

2 photons emitted?
photon absorbed?

... all must obey same CONSV laws
and symmetry principles!

Back to relativity: what makes photons so special?

- Only a lack of rest mass!
- modern view: matter acquires mass by interactions

photon has zero rest mass

↳ require $v = c$

- since photons cannot have $v < c$ "rest mass" is anyway meaningless

general: $E = \sqrt{p^2c^2 + m^2c^4}$ $\xrightarrow{e^-}$ if $p = 0$, $E = mc^2$; $pc \gg mc^2$, $E \approx pc$

$\xrightarrow{\text{photon}}$ never have $p = 0$; $m = 0 \Rightarrow E = pc$

- if only rest mass distinguishes electrons (for now) why should it also not have wave characteristics?

- dynamical properties still explainable
- problem? by analogy w/ photon, $|\bar{p}|$ sets scale

photon: $\lambda = \frac{h}{p}$ analogy, e^-: $\lambda = \frac{h}{p} = \frac{h}{\gamma m v}$

? what is the scale

- from E&M, everyday we interact with e^- on relatively large scales
  e.g. even circuit features ~ 40nm
  in "quantum era" ~ 1900 ... many μm

- on these scales, we should not notice.

  let's say <u>100 nm</u> is our experimental probe

photon : $E = pc = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{h}{p} = \frac{hc}{E} \simeq 100\,\text{nm}$

$\Rightarrow E \simeq 12.4\,\text{eV}$ hard UV, easy to see wave beh

electron : $\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}} \simeq 100\,\text{nm}$

$\Rightarrow v \sim 7000\,\text{m/s}$ or $\left(\frac{h}{\sqrt{2Km}} = \lambda\right)$

! actually, hard to slow an electron down this much!

$\boxed{\text{how about } 0.1\,\text{nm} \sim \text{atom spacing}}$

$E_{photon} \sim 10\,\text{keV}$ X-rays ... easy

$v_{e^-} \sim 10^7\,\text{m/s}$ or $KE \sim 150\,\text{eV}$ <u>doable</u>.

$\boxed{\text{Put another way}}$ electron wavelengths are *tiny* at everyday energies!

- this was de Broglie's idea: matter *also* has wave-particle
- Borne out by experiments like double slits

1924: de Broglie   1927: expt confirm of e^- waves!

why hard: e^- beams need to be in vacuum
"lenses" harder – $\vec{B}$ fields
still need atomic-scale diffraction
$\Rightarrow$ perfect crystal & surface

anyway: de Broglie $\longrightarrow$ matter waves

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{\gamma m v} \simeq \frac{h}{mv} \quad (v \ll c)}$$

again: $\boxed{\lambda \ll \text{probe size}}$ – wave behavior *cannot* be seen
our analogies
acts as lumps/particles

$\boxed{\lambda \gg \text{probe size}}$ – possible to see wave effects
e.g. diffraction/interference

- even for e^-, m is large enough for λ to be tiny!
- Never see this in every day: 100mph baseball
$\lambda \sim 10^{-35}\,m$!
- Electron Microscope!

· bad news: matter treated like photons – (wave-like or particle like)

good news: math is the same
· Scale is unobservably small for anything beyond a few atoms! everyday life is safe!!
· interesting new effects to exploit...

bad news: we know enough about waves to expect very unsavory new things

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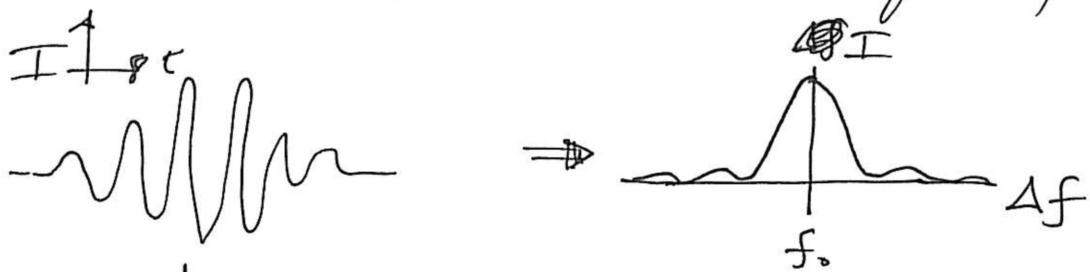
Uncertainty waves have a limited spatial resolution

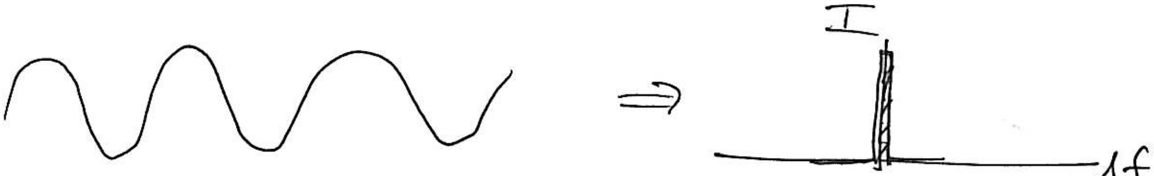
° if "waves" are the right math tool... follow consequences

e.g. signal processing.
– if you make a pulse too short, its frequency is ill-defined
– exploit this for spectroscopy!

- as the time spread shrinks, frequency spread increases



pulse



sine

(spread in time dom)(Spread in freq) $\geq$ const

$$\Delta f \Delta t \geq 1/4\pi$$

(bandwidth)(duration) $\geq 1/2\pi$ (Related to Shannon-Nyquist)

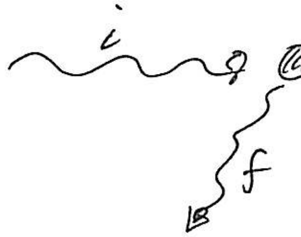
A basic property of waves, formalized by Fourier analysis

$\boxed{\text{Optics}}$: diffraction limit of a microscope

$$\Delta x \sim \lambda$$

how does this apply to quantum particles?

Consider measuring an electron's position by photon scattering



the photon has a resolution limit

$\Delta x \sim \lambda$

- make λ smaller! ... but now it has higher $\vec{p}$
- photon's momentum kicks the e^-, alters its posn!

e^- acquires momentum proportional to photons!

$\Delta p_{e^-} \sim p_i = \frac{h}{\lambda}$

So: as $\lambda \downarrow$ we have better spatial resolution

<u>but</u> we've messed up the electrons position more
<u>and</u> given it random motion...

Δx & Δp work against each other!

$$\boxed{\Delta x \Delta p \gtrsim \hbar}$$

- <u>minimum exists</u>
- <u>tiny</u> due to $\hbar$ being tiny

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more formally: comes out of ANY wave mechanics
optics, signals ... <u>useful</u> e.g. FTIR
NOT spooky!

$\boxed{\text{Signals:}}$ shorter pulse ⟷ ill-defined frequency

long | continuous ⟷ well-defined freq

wave needs to "hang around" long enough for an accurate freq. det.

$\boxed{\text{Stuff}}$ more localized position ⟷ ill-defined momentum

uncertain posn ⟷ well-defined $\vec{p}$

$\Delta x \, \Delta p_x \geq \hbar/2$

$\Delta y \, \Delta p_y \geq \hbar/2$

etc

$\Delta E \, \Delta t \geq \hbar/2$ energy-time

$\Delta \theta \, \Delta L \gtrsim \hbar/2$ angle-angular mom

and more

Again: NOT spooky! only on *tiny* scales

e.g. 10gram ball at 100m/s? know Δv to $\pm 0.01\,\frac{m}{s}$

$$\Delta x \, \Delta p = \Delta x \, \Delta(mv) = m \, \Delta x \, \Delta v \geq \hbar/2$$

$$\Delta x \geq \frac{\hbar}{2m\Delta v} \sim 10^{-30}\,m$$ ... not a problem!

clearly, particle model is safe!

Now: electron at $100.00 \pm 0.01 \frac{m}{s}$?

$\underline{\Delta x \gtrsim 0.01\,m}$ clearly, fuzzy & wave-like!

electron at 10^7 m/s, 1% uncertainty?

$\Delta x \gtrsim 6 \times 10^{-10}$ m ~ 2-3 atoms!

- clearly, particle-like for most cases
- microscopy now apparent: tiny wavelength gives huge resolution!

$\boxed{\text{Example:}}$ how big is an atom?

- from crystal diffraction & periodic table, good guesses
- can do it just from uncertainty!

$\boxed{\text{Size of an atom}}$ just an idea, not an accurate analysis after all ... what is "size" now anyway?

$\boxed{\text{Classical:}}$ death spiral of e^- into proton, bound @ same place

$\boxed{\text{Quantum:}}$ if so, we'd know ΔX too well!

electron must be "spread out" around the proton to satisfy

$$\Delta x \Delta p \geq \hbar/2$$

i.e., minimum approach, max extent

Say the e^- is spread over distance $\boxed{a} = \Delta X$

then $\Delta p \simeq \frac{\hbar}{2\Delta X}$ or just $p_{min} \sim \hbar/2a$

i.e., momentum is set by spread in posn!

$$KE = \frac{1}{2}mv^2 = p^2/2m = \frac{\hbar^2}{8ma^2} \sim \hbar^2/a^2$$

total energy of atom? KE - potential if proton sits still

$$E = p^2/2m - ke^2/a = \frac{\hbar^2}{8ma^2} - \frac{ke^2}{a}$$

atom will minimize energy!

$$\frac{\partial E}{\partial a} = -\frac{\hbar^2}{4ma^3} + \frac{ke^2}{a^2} = 0 \Rightarrow \boxed{a \simeq \frac{\hbar^2}{4kme^2} \sim 10^{-11}\,m}$$

implies $E_{min} \sim -10\,eV$ $\Big\}$ NEG = BOUND
IONIZATION energy
ATOMS ARE STABLE!

- So!: atoms are stable
- we don't fall through the floor!

Next: matter waves / wave mechanics
probabilistic interpretation
more on LOCALITY and measurement

on our way to an atomic model!