Light as a wave again...

- light as particles worked GREAT for Compton!
- we still have diffraction/interference to explain

light = particle or wave?

neither ... failure is our imagination - unlike everyda[y]

roughly ... experiment probes scale $\lesssim \lambda \Rightarrow$ "wave"

scale $\gtrsim \lambda \Rightarrow$ "particle"

example w/ both properties: Gaussian wave packet

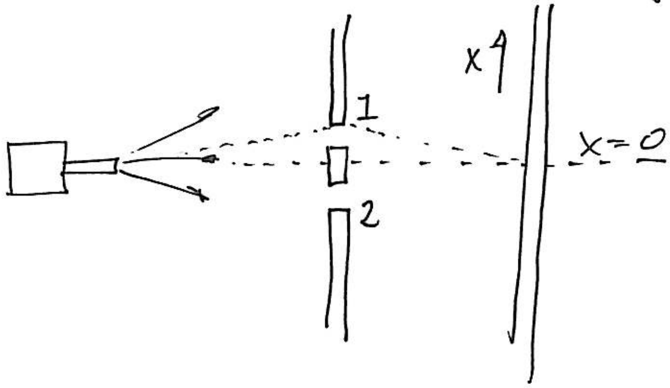
- Rather than play up the "spooky" wave-particle duality (TERRIBLE term which sells well)
  - let's think about how waves & particles differ

- remember the wave eqn from E&M?
  - Solutions were $E, B = f(x \pm vt)$
  - i.e. periodic, but NOT just sine/cosine

Be CAREFUL, wave $\not\Rightarrow$ sine ...

first: experiment with bullets

- gun sprays bullets randomly over a large angular spread
- Shoot @ wall with 2 bullet-sized slits
- detect bullets hitting far wall



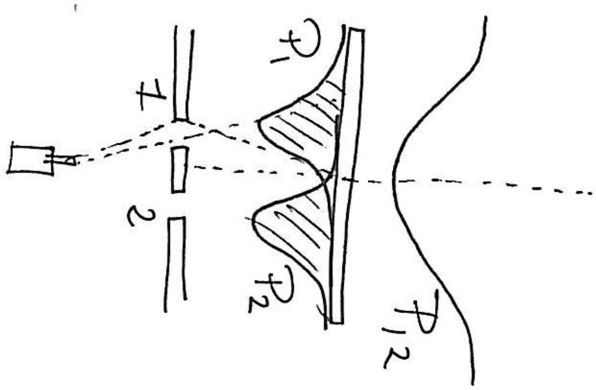
- probability that a bullet lands a dist x from center
- has to be probability - can't say where any particular bullet will go (just by distr)

- may bounce off of slit
- angular spread from source

} broad distr on wall

- presume rate of fire constant
- bullets all identical & can't split in two

what does distr look like?



P_1 = prob. bullet came thru 1
P_2 thru hole 2
P_{12} = either one

- if we only collect bullets at the screen, with both slits only P_{12} is meaningful

- close hole 2, get P_1
- close hole 1, get P_2

} add together ...

Clearly, $P_{12} = P_1 + P_2$

- probabilities add together – "no interference"
- because bullets come in lumps

? Same experiment w/ waves?

wave can prop around holes!

Same experiment with water waves?

- one slit open, or both
- first difference: <u>intensity</u> can have <u>any</u> value!

bullets: binary! discrete distr

intensity $\propto$ height^2
($E = \frac{1}{2} k A^2 \ldots$)

$$I_1 = |h_1|^2 \qquad I_2 = |h_2|^2 \qquad \underline{\underline{I_{12} = |h_1 + h_2|^2}}$$

- both slits: we add <u>waves</u> and get constr/destr interference!

$$I_{12} \neq I_1 + I_2$$

- depends on relative phase of waves at any point

Can write waves thru each slit as complex exp.

$$h_1(t) = h_1 e^{i\omega t} \qquad h_2(t) = h_2 e^{i\omega t}$$

$$\text{energy} \propto |h_1|^2 \qquad \text{energy} \propto |h_2|^2$$

square of complex number is just ampl squared both holes open?

intensity is both waves added together + interference

$h_{tot}(t) = (h_1 + h_2) e^{i\omega t}$

energy ~~= intensity~~ $\propto |h_1 + h_2|^2$

$|h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2| \cos\delta$

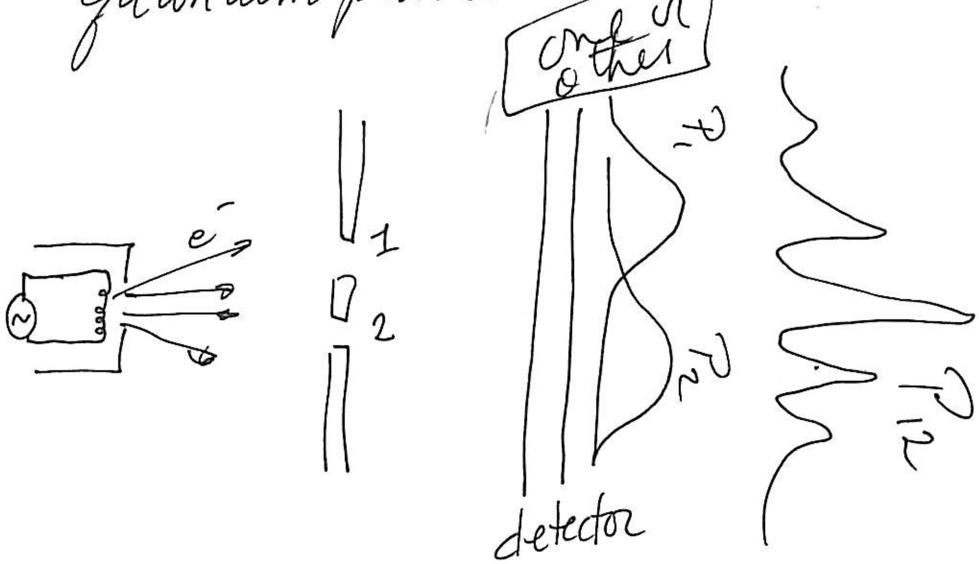
δ = phase difference

or $I_{12} = \underbrace{I_1 + I_2}_{\text{sum}} + \underbrace{2\sqrt{I_1 I_2}\cos\delta}_{\text{interference}}$

Since waves take on any height, interference shows up.

How about photons, or electrons?

quantum "particles"



- behaves like waves - interference
- but also like part.

$P_1 = |\varphi_1|^2$ $P_2 = |\varphi_2|^2$ $P_{12} = |\varphi_1 + \varphi_2|^2$

- hook the detector up to a speaker - CLICKS
  - either an e^- or photon hits or doesn't
  - no "half clicks"!
  - discrete events
  - rate is erratic, but has well-defined average
  - all "clicks" have the same intensity
    i.e. events = bundles of same size
    discrete
  - try 2 detectors: only 1 fires at a time

  really come through in clumps of definite size!
  i.e. like particles

- e^- and photons clearly discrete ...
- yet interference observed!
- can we construct probability that e^- or photon hits detector at posn x?

Proposition I: each e^- or photon goes through hole 1 or through hole 2
NOT both

True?

- if proposition 1 is true, detected particles in 2 classes:
  - those going thru hole 1
  - those going thru hole 2

if so, observed curve must be sum of curves from single slits!

CLOSE 1, measure P_1 }
2 P_2 }

P_1 or P_2

≠

P_{12}

<u>BOTH open ≠ (1 open) + (2 open)</u> !

e^- or photons: $P_{12} \neq P_1 + P_2$ like water waves

? how can our prop. be true then

- complex paths back & forth?
  - <u>NO</u> - some spots w/ few hits w/ both holes have <u>MORE</u> hits with one closed!
- split in half? <u>NO</u> - same "clicks"

worse: at center, $P_{12} > 2(P_1 + P_2)$

- as if closing one hole *decreased* # coming through other!

Very mysterious, moreso the closer you look!

- the [math] is just like water waves!
  surprising!
- can describe with 2 complex numbers

$\varphi_1(x) \quad \varphi_2(x)$

$P_1 = |\varphi_1|^2 \quad P_2 = |\varphi_2|^2$

$P_{12} = |\varphi_1 + \varphi_2|^2$

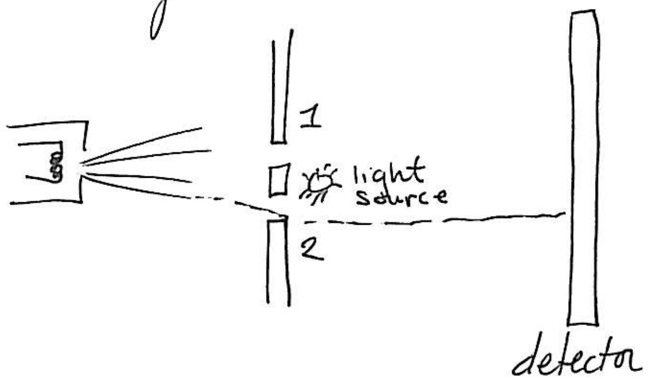
[Conclusion]: - e^- or photons arrive in lumps, like particles
- probability of lumps arriving ~ distr of wave

(Classically, complex #'s = simplifying math trick
Quantum, MUST use complex #'s)

[Further]: proposition I is FALSE!
NOT true that e^- or photon takes only 1 hole
like water wave, it takes BOTH

Watching the e^- or photons

- let's just look at one hole to see if e^- goes through!

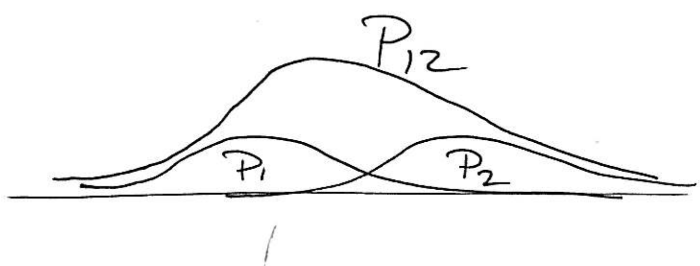


- light source near one slit
- if e^- goes thru slit 2, scatters a photon, we see it

- every time detector "clicks", see a photon from either 1 or 2, never both at once!

- proposition I now true! !?!?

measure w/ each slit alone...



now $P_{12} = P_1 + P_2$

! When we watch the e^-, interference dissapears!

what's going on?

photon scatters e^- ... we've altered the result by watching!

- e^- takes photon $\vec{p}$, E ... washes out interference
  randomizes one pattern...

? less bright source?

- NO! photons always have same energy!

- too dim, we only note some "clicks"
  w/o flashes - e^- snuck by... no collision

- if e^- are not "seen" interference comes back!

? use less momentum

$$p = h/\lambda$$

so $p\uparrow$ then $\lambda\downarrow$
$p\downarrow$ then $\lambda\uparrow$

"gentle" photons w/ low p?

$\Rightarrow$ large λ, resolution too low to tell!

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general principle impossible to design apparatus to tell which hole the e^- went through that won't disturb e^- enough to destroy interference

Proposition 1: — if we don't look, NOT true — takes both paths
Sometimes!
— if we look accurately enough to tell, then it goes through either 1 or 2!

basic idea: can't measure on this scale w/o affecting!

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back to bullets: turns out to be the SAME except "wavelength" is unobservably small!

general: scale of measurement relative to λ

next: what is "λ" for e^-?

Summary

(1) probability of an event in ideal expt is the absolute value of a complex # φ

$P = \text{prob} = \varphi^2$

ϕ = prob. amplitude

(2) when an event can occur in several alt ways prob. amplitudes add sep.

$$\phi = \varphi_1 + \varphi_2 \qquad P = |\varphi_1 + \varphi_2|^2$$

phase-dep interference

(3) if an experiment is capable of determining whether one or an alternative is actually taken the probability of the event is just a sum

$$P = P_1 + P_2$$

i.e., no interference, now independent ∵ meas

IMPLICATION – Determinism is lost, only probabilities can be predicted!

Next: matter waves

uncertainty

probability amplitudes revisited

Schrödinger equation