PH253 Lecture 12: its waves all the way down

de Broglie waves

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Outline

Last time:

1. Double slit experiment - waves or particles?
2. Yes.
3. Depending on scale and details of experiment, e^- and photons can look like either
4. Because they are neither!
5. Arrive as particles, distribution of particles is wave-like
6. Can have interference, but not if you watch ...
7. Next: how to explain waviness of an electron?

Outline

Nature is discretized

1. Photons (light) and electrons are discrete
2. Energy states of atoms must also be discrete
3. Follows that any observable energy difference will be
4. Slit experiments: waves and particles behave very differently
5. Photons and electrons look a bit like both (but are neither)
6. But how does this work for matter like electrons?

What makes photons so special?

1. Relativity: nothing, just lack of mass.
2. Modern view: matter acquires mass by interactions
3. Photon happens to have zero rest mass, requiring $v = c$ always
4. General case: $E = \sqrt{p^2c^2 + m^2c^4}$
5. Photon: $v = c, m = 0, \implies E = pc = hf$
6. e^-: if $p = 0$, $E_{\text{rest}} = mc^2$; if $p \gg mc, E \approx pc$
7. Only rest mass distinguishes electron.
8. High enough energies: $KE \gg E_{\text{rest}}$ - photon-like

What makes photons so special?

1. If only rest mass distinguishes e^- (for now) ...
2. Why should it not also have wave properties?
3. Dynamical properties still explainable
4. By analogy with photon, p sets length scale
5. Photon: $\lambda = h/p$, p related to E
6. e^-: why not $\lambda = h/p = h/\gamma m v$?
7. What is the scale? Must be tiny to escape notice so long

What is the length scale?

1. Calibrating ourselves first …
2. Visible light: $\lambda \sim 400 - 700\,\text{nm}$
3. Circuit features: $\sim 10\,\text{nm}$
4. Atoms: $\sim 0.1\,\text{nm}$
5. Clearly we can't see the waviness ordinarily.
6. Let's say our scale is 100 nm. For light, $\lambda = hc/E$
7. This gives $E \sim 12\,\text{eV}$, hard UV light

What is the length scale?

1. For e^-, if $\lambda = h/p \approx h/mv \approx 100\,\text{nm}$, $v \sim 7000\,\text{m/s}$
2. Thermal speed at RT? $\frac{1}{2}mv^2 = \frac{3}{2}k_bT$, $v \sim 10^5\,\text{m/s}$
3. Actually hard to slow down the electron enough to observe!
4. At atom spacing? $v \sim 10^7\,\text{m/s}$, $K \sim 150\,\text{eV}$ - doable
5. Electron wavelengths are *tiny* at everyday energies
6. This was de Broglie's big idea: treat matter like photons
7. Borne out by experiments like double slits
8. 1924: de Broglie publishes PhD thesis. 1927: experimental confirmation.
9. 1929: Nobel.

Why was it hard to figure out?

1. e^- beams need to be in vacuum
2. "Lenses" are harder - E and B fields
3. Still need regular atomic scale features to see
4. E.g., a perfect crystal and surface
5. Long story short:

$$\lambda = \frac{h}{p} = \frac{h}{\gamma m v} \approx \frac{h}{mv} \quad (v \ll c) \tag{1}$$

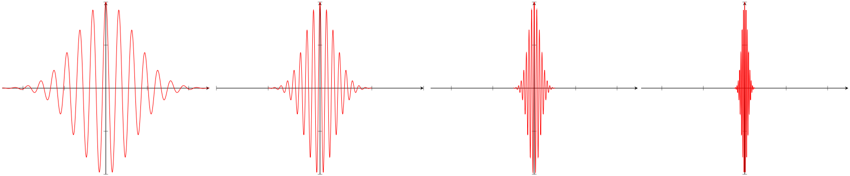
Wave-particle?

1. As with photons, probe size matters!
2. $\lambda \ll$ probe size: wave behavior can't bee seen. Lumps/particles
3. $\lambda \gg$ probe size: can see wave effects, e.g., interference
4. Basically: m is tiny for e^-, and so is λ
5. Never see this in everyday life.
6. 100 mph baseball, $\lambda \sim 10^{-35}\,\text{m}$
7. Proton diameter $\sim 10^{-15}\,\text{m} \ldots$
8. This is what allows electron microscopes.
9. (There are several right above us.)

Visualizing

Same Gaussian wave packet ($y \sim e^{-x^2} \cos x$).

Just zooming out on length (x) axis.

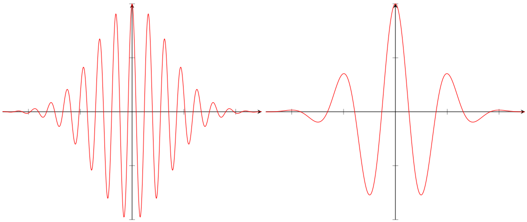


Uncertainty?

1. Bad news: this is weird. Matter has to be treated like photons
2. Both wave and particle aspects
3. Good news: we already figured out the math
4. Scale is unobserveably small most of the time
5. Interesting new effects to exploit
6. We need this for cell phones and computers
7. Bad news: we know enough now to expect unsavory new things

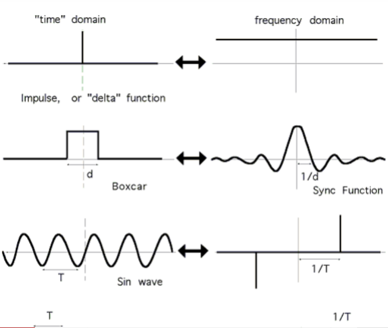
Time and frequency

1. If waves are the right mathematical tool, consequences?
2. Forget spookiness, think more like signal processing
3. Measure frequencies? Need to watch wave fronts go by
4. Longer you measure, more accurate. Shorter? Less accurate
5. Short pulse? Only a few wave fronts to measure, not accurate
6. As time spread ↓, frequency spread ↑



Time and frequency

1. This is a general thing and has nothing to do with quantum
2. "Benedicks's theorem" - cannot be both time & band limited
3. Can't sharpen in both time and frequency - dual variables
4. Narrow in time = broad in frequency
5. Perfectly periodic in time = single frequency
6. Pulse: too short to measure f very well, spread out
7. $\Delta f \Delta t = (\text{bandwidth})(\text{duration}) \geq 1/4\pi$



Time and frequency

1. Time and frequency pictures related by *Fourier transformation*
2. Basic property of waves: trade off in resolution
3. Optics: diffraction limit of microscope $\Delta x \sim \lambda$
4. How does this apply to quantum particles?
5. Let's think about measuring an e^- position with a photon
6. Better photon resolution = smaller λ, but then higher p
7. Better resolution = more invasive experiment

Measurement

1. Making photon λ smaller makes p higher
2. Photon momentum kicks the e^-, alters its position
3. e^- acquires p proportional to what photon has
4. $\Delta p_{e^-} \sim p_{\text{photon,i}} = h/\lambda$
5. So as $\lambda \downarrow$, better resolution ...
6. ...but in the process we messed up e^- position more, randomly
7. Uncertainty in resolution and position are antagonistic

Measurement

1. resolution uncertainty means momentum uncertainty
2. Works against position resolution/uncertainty
3. In the end: $\Delta x \Delta p \gtrsim \hbar/2$
4. There is a limit to how well you can measure p or x
5. Minimum exists, but tiny due to size of $\hbar$
6. Comes out of any wave mechanics (e.g. signal processing, optics)
7. If you know where you are, you don't know how fast you're going

Measurement

1. Shorter pulse = ill-defined frequency (FTIR FTW)
2. Long/continuous signal = well defined frequency
3. Wave needs to "hang around" long enough to measure well
4. e^- and photons: more localized x = ill-defined p
5. Uncertain x = well-defined p
6. Along each axis separately x, y, z
7. Similar: $\Delta E \Delta t \geq \hbar/2$, $\Delta\theta \Delta L \geq \hbar/2$

Uncertainty

1. Only on tiny scales!
2. 10 g ball at 100 m/s, know Δv to ± 0.01 m/s?
3. $\Delta x \Delta p = \Delta x \Delta(mv) = m \Delta x \Delta v \geq \hbar/2$
4. $\Delta x \geq \hbar/2m\Delta v \sim 10^{-30}$ m - not a problem!
5. e^- at 100.00 ± 0.01 m/s? $\Delta x \geq 1$ cm - fuzzy!
6. e^- at 10^7 m/s, 1% uncertainty? $\Delta x \geq 6 \times 10^{-10}$ m - 2-3 atoms!
7. Clearly particle-like for most cases. But tiny λ = electron microscopy!

Size of an atom

1. Can get a ballpark estimate from uncertainty.
2. But what does size really mean now?
3. Classical orbiting charge model doesn't work.
4. Quantum: if we know position too well, don't know speed
5. e^- must be "spread out" around proton to satisfy $\Delta x \Delta p \geq \hbar/2$
6. I.e., minimum approach, maximum extent for e^-
7. From x-ray diffraction, know rough size of atom $\Delta x = a$

Size of an atom

1. Then $\Delta p \sim \hbar/2\Delta x$
2. Or, minimum p must be $p_{\text{min}} \sim \hbar/2a$
3. p spread is set by size of atom!
4. $K = \frac{1}{2}mv^2 = p^2/2m = \hbar^2/8ma^2 \sim h^2/a^2$
5. Total energy? $E = K + U = p^2/2m - ke^2/a = \hbar^2/8ma^2 - ke^2/a$
6. Atom will minimize its energy. PE wants closer, uncertainty limits
7. Need $\partial E/\partial a = 0$

Size of an atom

$$\frac{\partial E}{\partial a} = -\frac{\hbar^2}{4ma^3} + \frac{ke^2}{a^2} = 0 \tag{2}$$

1. $a \sim \frac{\hbar^2}{4kme^2} \sim 10^{-11}\,\mathrm{m}$
2. Basically right (from experiments)!
3. Implies $E_{\mathrm{min}} \approx -10\,\mathrm{eV}$
4. Negative = bound state, stable
5. Implies ionization energy $\sim 10\,\mathrm{eV}$ - about right!
6. (For H: $-13.6\,\mathrm{eV}$)
7. Atoms are stable! But still hand-wavy … more details yet