PH253 Lecture 2: relativity

Time and length in different reference frames

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Spring 2020

Outline

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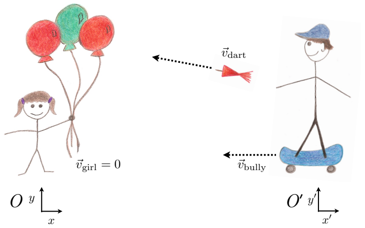
Observers in relative motion

O frame: girl on ground, O′ frame: bully on skateboard
relative velocity between the two is v_{bully}



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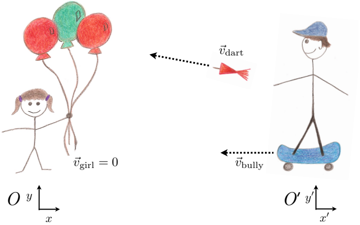

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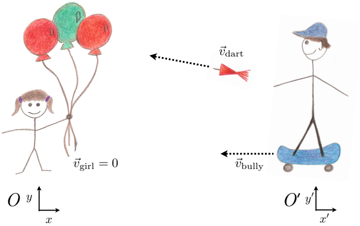

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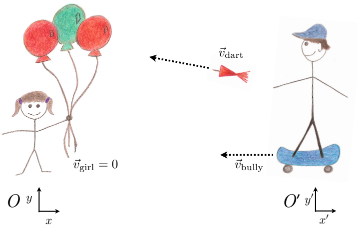

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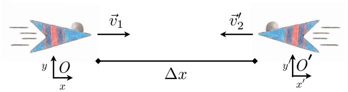

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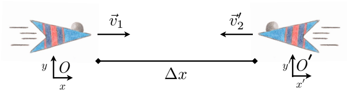

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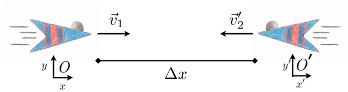

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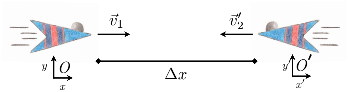

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"Who is moving" is a relative notion!

Outline

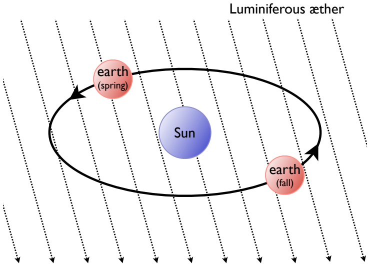
No absolute frame of reference

Principle of relativity:

All laws of nature are the same in all uniformly moving (non-accelerating) frames of reference. No frame is preferred or special.

No absolute frame of reference

If there were some preferred frame in space, the speed of light should vary seasonally.



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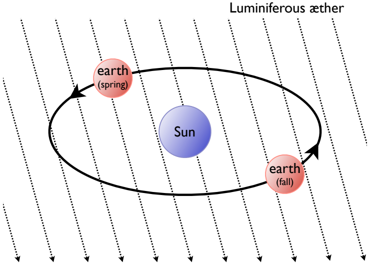
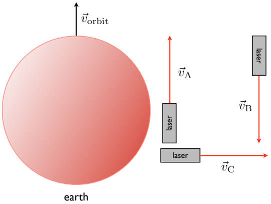


It doesn't.

No absolute frame of reference

More to the point: if the speed of light was relative to some background medium, what happens when we change direction?



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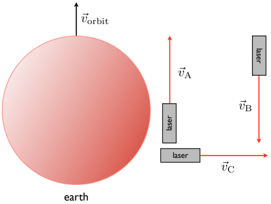

Experiments: same velocity in all directions - there is no medium, and no preferred frame.

Invariance of the speed of light

Invariance of the speed of light:

The speed of light in free space c is *independent* of the motion of the source or observer. It is an invariant constant.

$$c \equiv 299,792,458\,\mathrm{m/s} \approx 3 \times 10^8\,\mathrm{m/s} \text{ (fixed value in the SI system).}$$

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Why? It is experimentally so.
If it *weren't*, causality would be violated. More on this later.

Summary: Principles of Special Relativity

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1. **Special principle of relativity:** Laws of physics look the same in all inertial (non-accelerating) reference frames. There are no preferred inertial frames of reference.
2. **Invariance of c:** The speed of light in a vacuum is a universal constant, c, independent of the motion of the source or observer.

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Failure to agree on simultaneity

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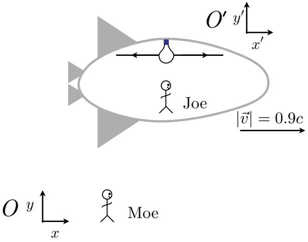
Everyday analogy (inexact): see lightening before thunder due to propagation delay

Failure of simultaneity example



- Moe on ground (reference frame O)

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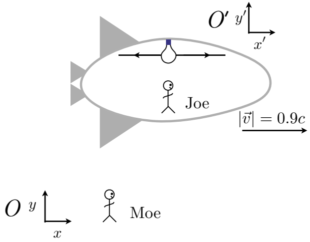

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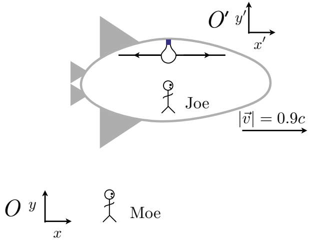

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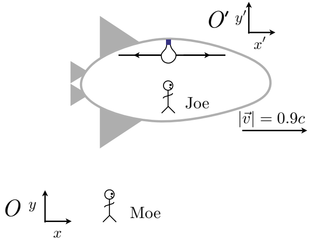

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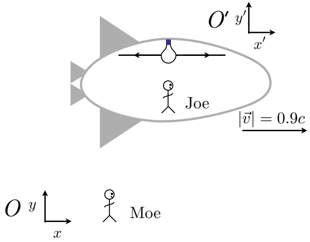

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What does Moe see?

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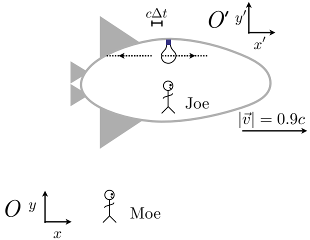

- Moe: ship moved forward after light was created!

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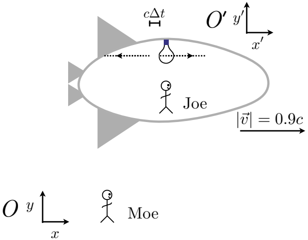

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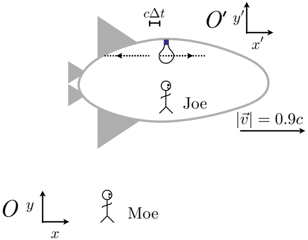

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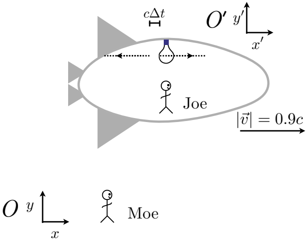

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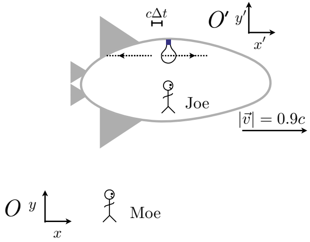

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Simultaneous events for Joe but not Moe - relative!
Spatially separated events + relative motion key.

Consequence of an invariant speed of light

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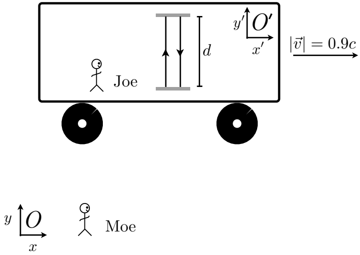
If the events are happing in your frame, stationary relative to you, there is no problem. Spatial separation and relative motion are crucial.

Time dilation



Next thought experiment: *passage of time* is also relative!

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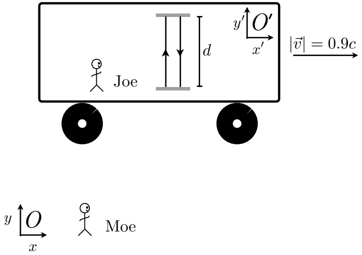

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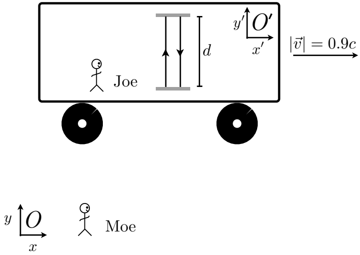

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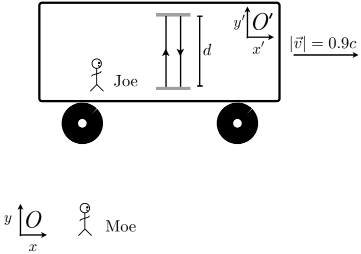

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- Bounce light beam between mirrors separated by d.
- One round trip is one 'tick' of our clock.

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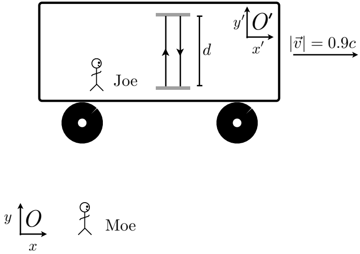

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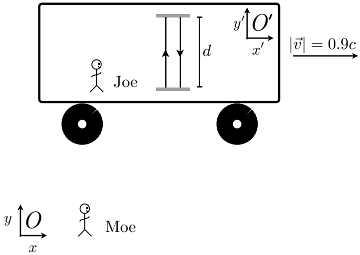

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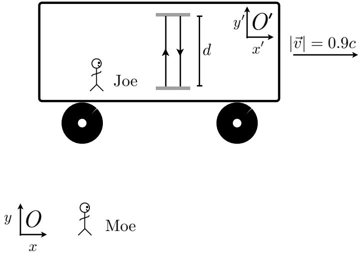

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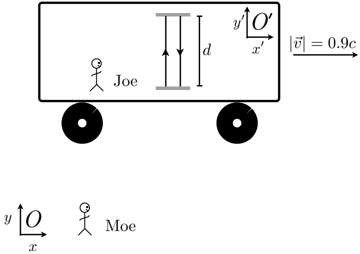

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What's the difference?

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No problem. How about Moe?

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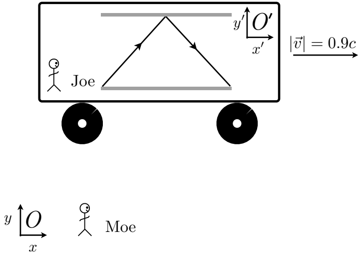

- Moe's view: also horizontal motion, but must preserve causality.

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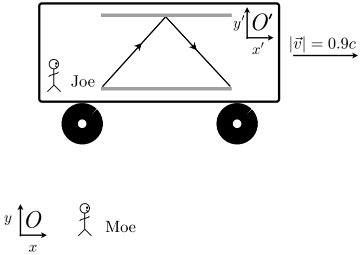

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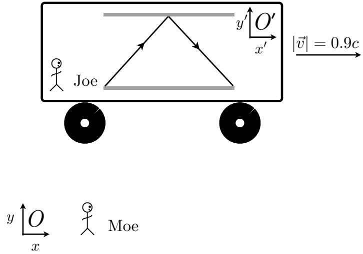

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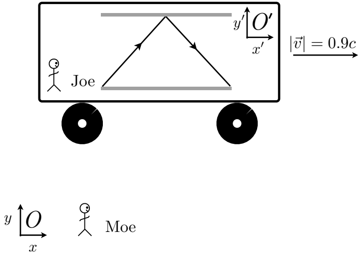

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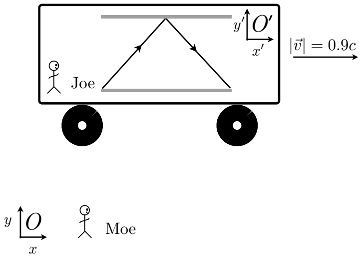

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- Moe is the *moving observer* since he moves with respect to clock.

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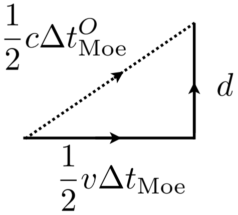

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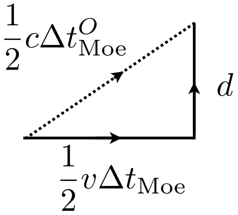
Time dilation



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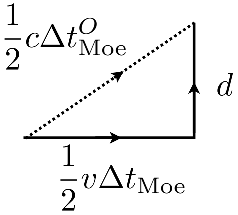

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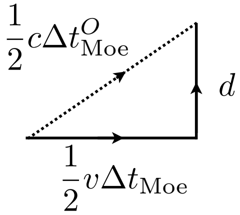

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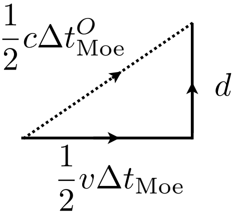

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(light beam distance observed by Moe)$^2 = d^2 + (\frac{1}{2}v\Delta t_{\text{Moe}})^2$

But we know from Joe that $d = \frac{1}{2}c\Delta t'_{\text{Joe}}$.

Time dilation

Put it together:

$$\left(\frac{1}{2}c\Delta t_{\text{Moe}}\right)^2 = d^2 + \left(\frac{1}{2}v\Delta t_{\text{Moe}}\right)^2 = \left(\frac{1}{2}c\Delta t'_{\text{Joe}}\right)^2 + \left(\frac{1}{2}v\Delta t_{\text{Moe}}\right)^2$$

Solve for Δt_{Moe}, we can relate the two times:

Time dilation

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$$\left(\frac{1}{2}c\Delta t_{\text{Moe}}\right)^2 = d^2 + \left(\frac{1}{2}v\Delta t_{\text{Moe}}\right)^2 = \left(\frac{1}{2}c\Delta t'_{\text{Joe}}\right)^2 + \left(\frac{1}{2}v\Delta t_{\text{Moe}}\right)^2$$

Solve for Δt_{Moe}, we can relate the two times:

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Time is *dilated* (stretched out) for moving observers!
Only agree if $v = 0$.

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Time dilation:

Two events take place at the same location. The time interval Δt between the events as measured by an observer moving with respect to the events is always *larger* than that measured by an observer who is stationary with respect to the events. The 'proper' time Δt_p is that measured by the stationary observer.

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The times agree *only* if $v = 0$, i.e., no relative motion of observers.

Time dilation

Caveat for time dilation

The analysis above used to derive the time dilation formula relies on both observers measuring the same events taking place at the same physical location at the same time, such as two observers measuring the same light pulses. When timing between spatially separated events or dealing with questions of simultaneity, we must follow the formulas developed

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General caveat

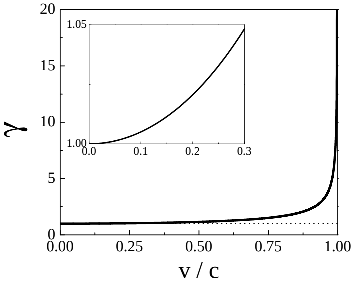
The principles of special relativity we have been discussing are only valid in *inertial* or non-accelerating reference frames. When accelerated motion occurs, a more complex analysis must be used (general relativity).

Lorentz factor γ

Degree of time dilation - strong function of speed:

$\Delta t'_{\text{moving}} = \gamma \Delta t_{\text{stationary}}$ with $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \geq 1.$

For $v \ll c$ (all everyday stuff), $\gamma \approx 1$ and the difference is negligible, but it is *measurable*. Key for GPS!



v [m/s]	v/c	γ	$1/\gamma$
0	0	0	∞
3×10^6	0.01	1.00005	0.99995
3×10^7	0.1	1.005	0.995
6×10^7	0.2	1.02	0.980
1.5×10^8	0.5	1.16	0.866
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2.85×10^8	0.95	3.20	0.312
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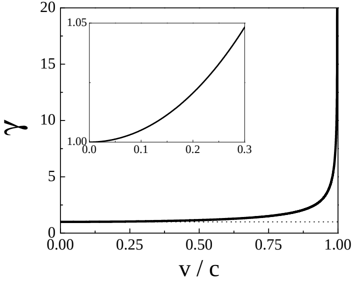

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(Note 1 m/s ~ 2 mph for the SI-impaired. Even $v = 0.01c$ is stupid fast.)

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- We measure distance using velocity and time . . .
- Longer time for moving observer $\Longrightarrow$ shorter distance!

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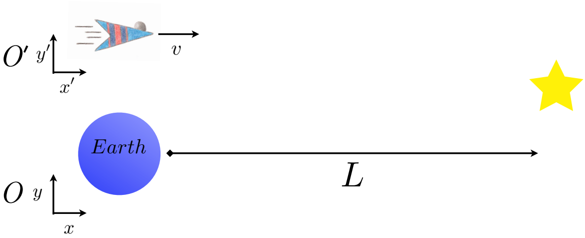

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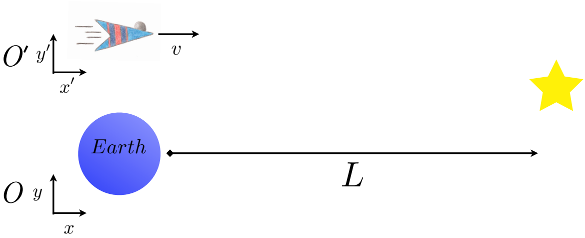

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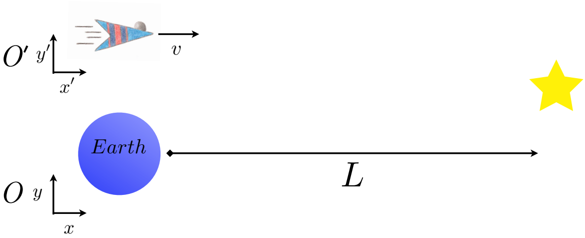

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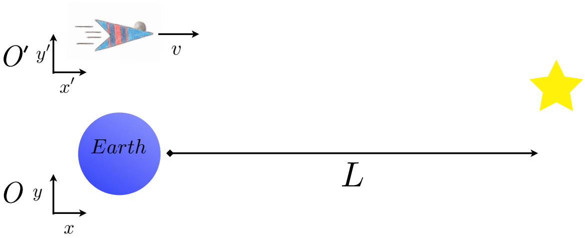

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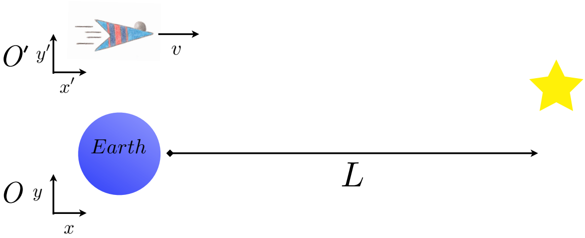

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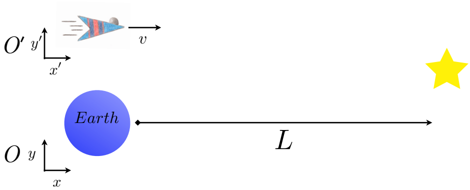
By now the answer is probably clear: it depends on who you ask!

Length Contraction: according to an earth-bound observer



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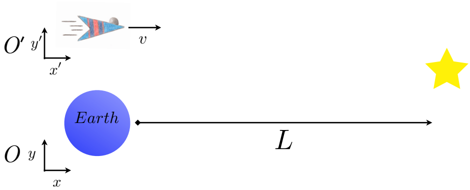

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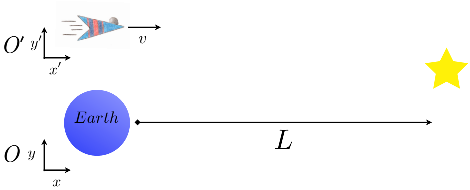

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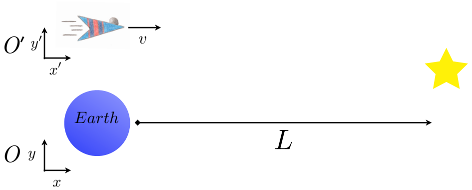

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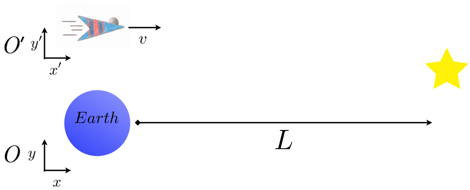

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- This is how long the journey takes according to earth-bound observers
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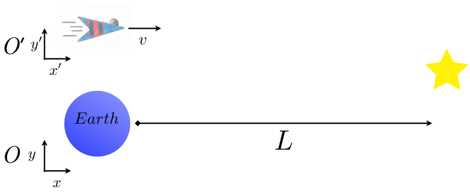
- But, earth-bound observers are in motion relative to the ship!

Length Contraction: according a ship observer



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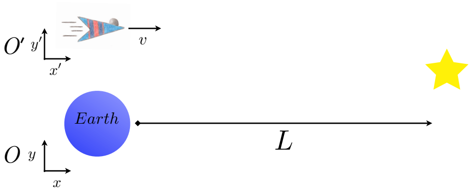

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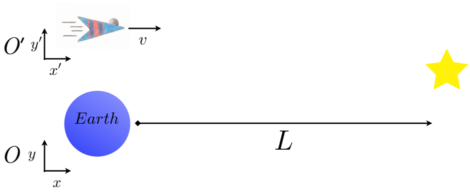

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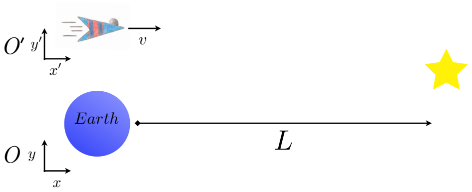

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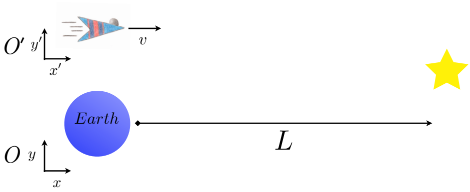

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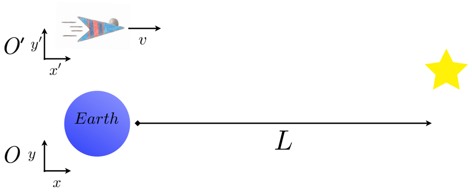

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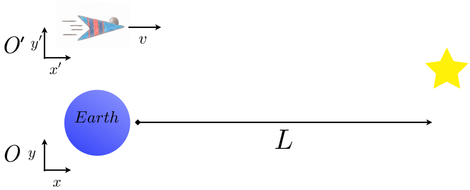
- Thus, time passes more slowly on earth

Length Contraction: how far away is the star?



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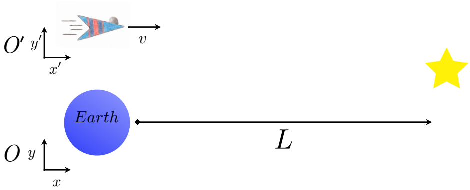

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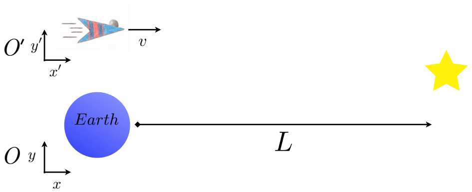

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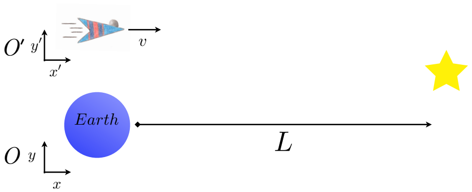

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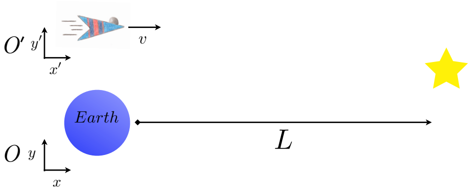

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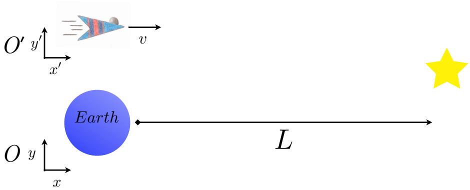

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Again: who is the moving observer? The one in motion relative to events of interest.

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The length of an object (or the distance to an object) as measured by an observer in motion is *shorter* than that measured by an observer at rest by a factor $1/\gamma$. The proper length, L_p, is measured at rest with respect to the object.

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That is, objects and distances appear shorter by $1/\gamma$ if you are moving relative to them.

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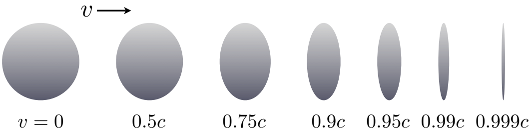
Do I divide or multiply by γ?

Note $\gamma \geq 1$. Think qualitatively about which quantity should be larger or smaller. In this example, we know the spaceship's time interval should be smaller than that measured on earth, so we know we have to *divide* the earth's time interval by γ.

Length Contraction



- Not just distances, but spatial dimensions as well!

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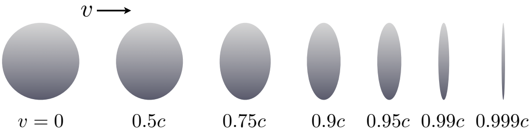

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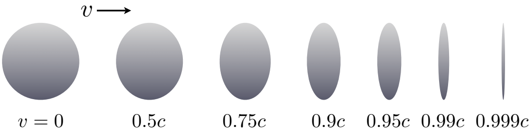

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- Sphere moving toward you flattens along the direction of motion

Outline

Problem

The period of a pendulum is measured to be $3.00\,\text{s}$ in its own reference frame. What is the period as measured by an observer moving at a speed of $0.950c$ with respect to the pendulum?

Problem

9.61 **sec.** The proper time is that measured by in the reference frame of the pendulum itself, $\Delta t_p = 3.00\,\text{sec}$.

The moving observer has to observe a *longer* period for the pendulum, since from the observer's point of view, the pendulum is moving relative to it.

Observers always perceive clocks moving relative to them as running slow. The factor between the two times is just γ:

$$\Delta t'_{\text{moving}} = \gamma \Delta t_p = \frac{3.0\,\text{sec}}{\sqrt{1-\frac{0.95^2c^2}{c^2}}} = \frac{3.0\,\text{sec}}{\sqrt{1-0.95^2}} \approx 9.61\,\text{sec} \qquad (4)$$

Problem

If you are moving in a spaceship at high speed relative to the earth, would you notice a difference in your pulse rate? In the pulse rate of the people back on earth? Explain, briefly.

Problem

no; yes. There is no relative speed between you and your own pulse, since you are in the same reference frame, so there is no difference in your pulse rate (possible space-travel-related anxieties aside).

There is a relative velocity between you and the people back on earth, however, so you would find their pulse rate *slower* than normal. Similarly, they would find *your* pulse rate slower than normal, since you are moving relative to them.

Relativistic effects are always attributed to the other party – you are always at rest in your own reference frame.

Problem

A stick of length $L = 1\,\text{m}$ is at rest on one system and is oriented with its length along the x axis. What is the apparent length of this stick as viewed by an observer moving at a speed v with respect to the first system?

Problem

Along the direction of motion, the moving observer will see contracted lengths. If the relative motion is along the x axis, then the meter stick appears shorter by a factor γ for the moving observer:

$$L'_{obs} = \frac{1\,\text{m}}{\gamma} = 1\,\text{m}\sqrt{1 - \frac{v^2}{c^2}}$$

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- The speed of light is the same for *all* observers, we rely on it.

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Next time: we'll learn how to connect different observers' measurements of time, distance, and velocity in a more general sense. Also: do an illustrative problem or two.