PH253

Lecture 3: relativity (continued)

Time dilation

Two events take place at the same location. The time interval Δt between the events as measured by an observer moving with respect to the events is always *larger* than that measured by an observer who is stationary with respect to the events. The 'proper' time Δt_p is that measured by the stationary observer.

$$\Delta t'_{moving} = \gamma \Delta t_{stationary} = \gamma \Delta t_p \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1.12}$$

In other words, time is stretched out for a moving observer compared to one at rest.

GPS

Uses geosynchronous satellites with atomic clocks on board
ask at least 4 satellites what time it is, with known positions compare times

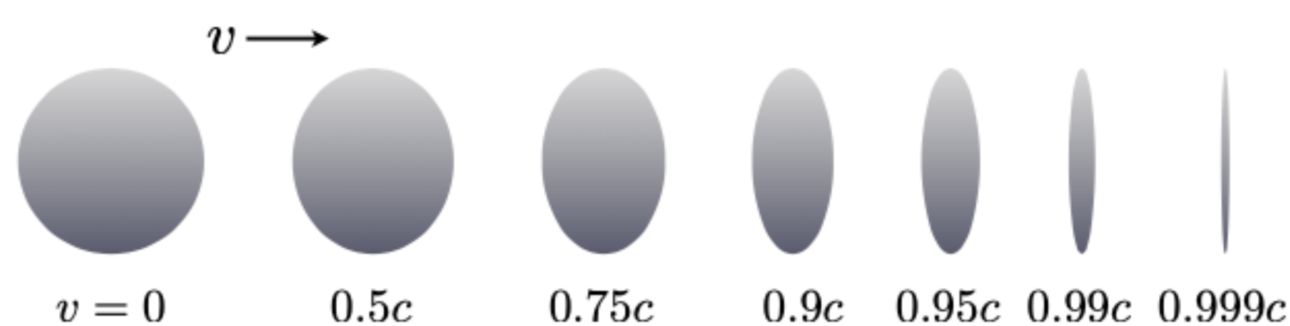
$$\Delta t'_{\mathrm{GPS}} = \gamma \Delta t_{\mathrm{Earth}}$$

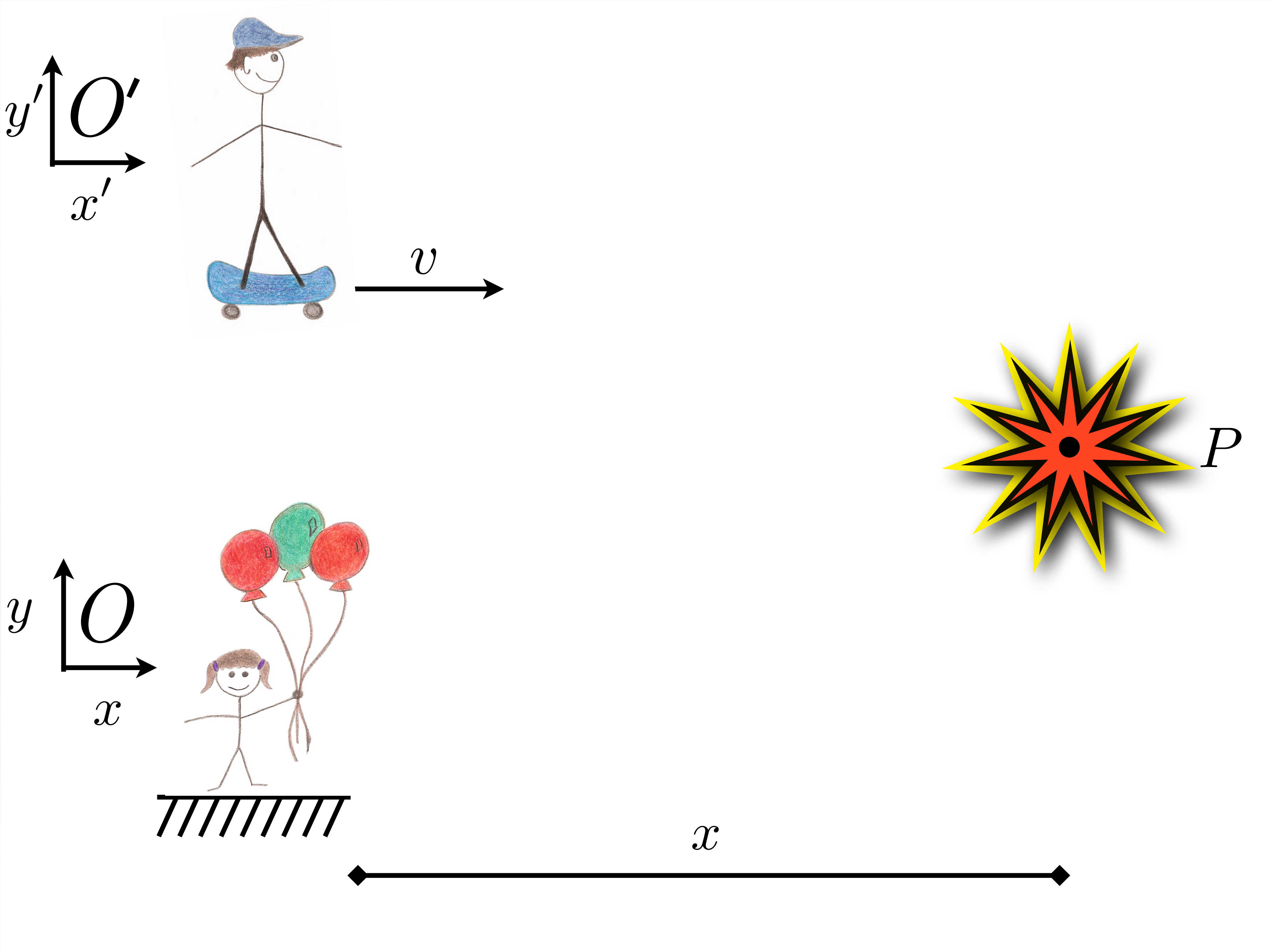
$$\begin{aligned}
\text{time difference} &= \Delta t_{\mathrm{Earth}} - \Delta t_{\mathrm{GPS}} = \Delta t_{\mathrm{Earth}} - \gamma \Delta t_{\mathrm{Earth}} && (1.15)\\
&= \Delta t_{\mathrm{Earth}}\,(1-\gamma) = \Delta t_{\mathrm{Earth}} \left[1 - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right] && (1.16)\\
&= \left[24\,\frac{\mathrm{h}}{\mathrm{day}} \cdot 60\,\frac{\mathrm{min}}{\mathrm{h}} \cdot 60\,\frac{\mathrm{s}}{\mathrm{min}}\right] \left[1 - \frac{1}{\sqrt{1-(1.3\times 10^{-5})^2}}\right] && (1.17)\\
&\approx [86400\,\mathrm{s/day}]\left[-8.32\times 10^{-11}\right] \approx -7.2\times 10^{-6}\,\mathrm{s/day} = -7.2\,\mu\mathrm{s/day}
\end{aligned}$$

Length Contraction The length of an object (or the distance to an object) as measured by an observer in motion is *shorter* than that measured by an observer at rest by a factor $1/\gamma$. The proper length, L_p, is measured at rest with respect to the object.

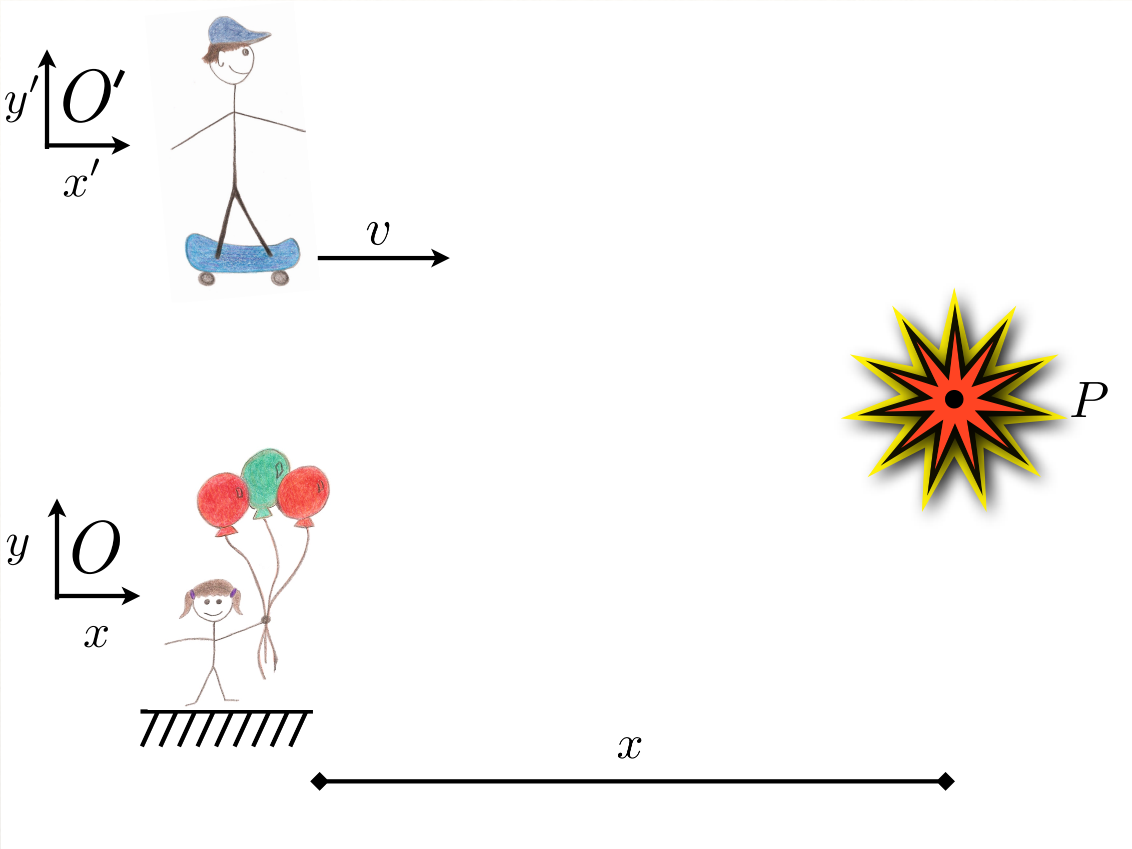
$$L'_{\text{moving}} = \frac{L_{\text{stationary}}}{\gamma} = \frac{L_p}{\gamma} \tag{1.28}$$

That is, objects and distances appear shorter by $1/\gamma$ if you are moving relative to them.



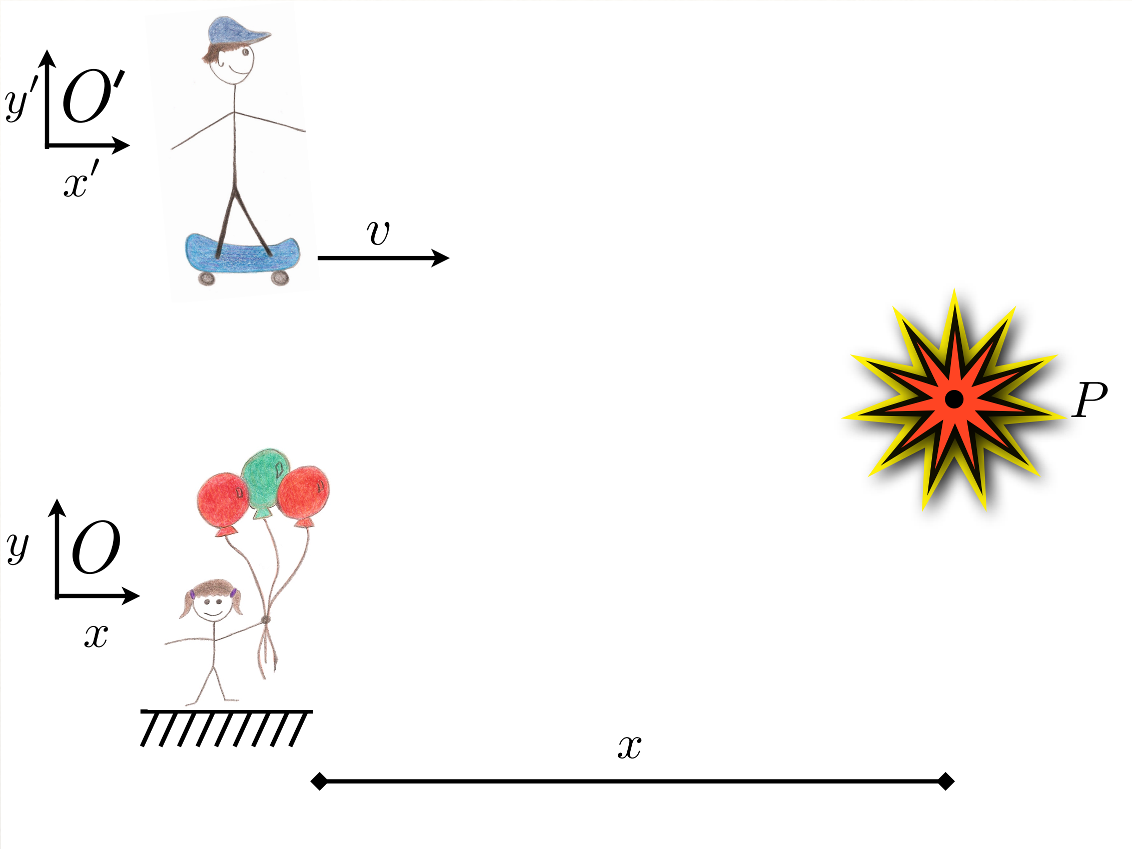


y'
O'
x'
v
P
y
O
x
x



According to girl, light arrives after a time

$$t_{arrival} = x/c$$



Boy in O′? He moves relative to star

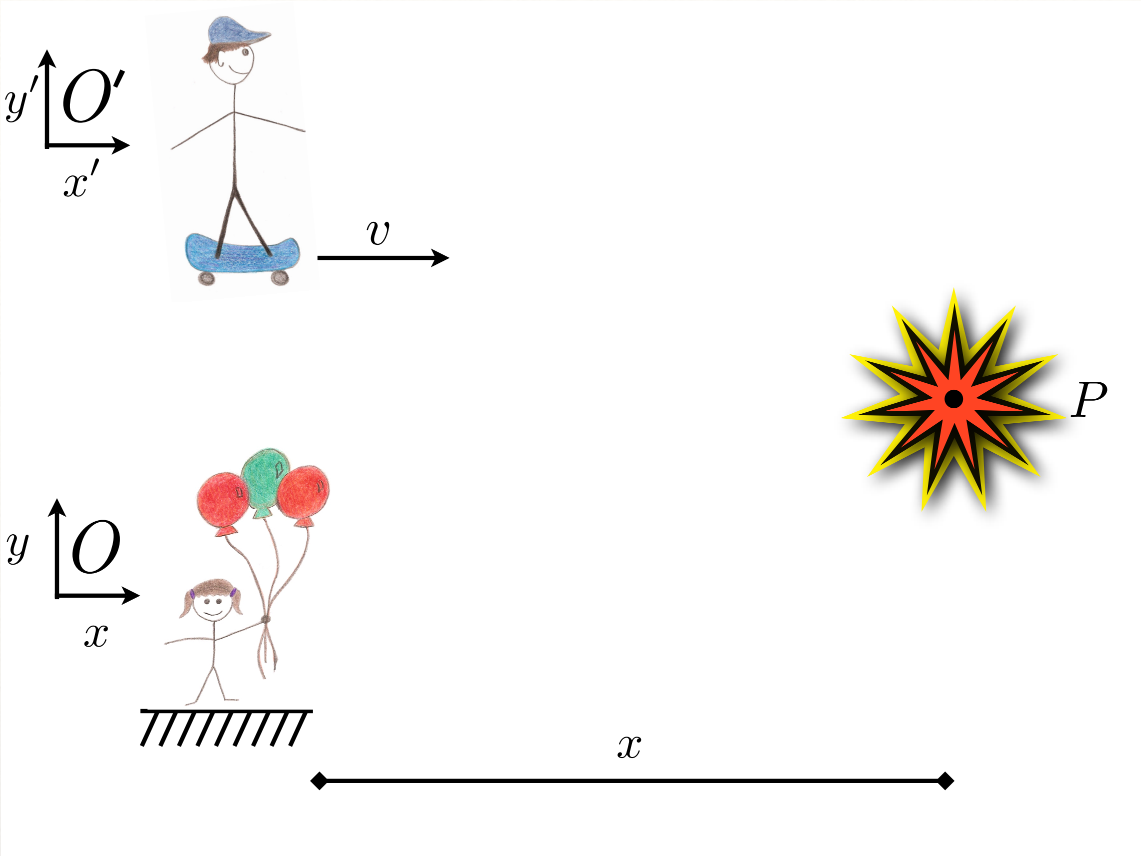
distance appears length contracted - shorter by factor γ

total distance is (him to star) - (rate he closes gap)

(him to star) = x/γ

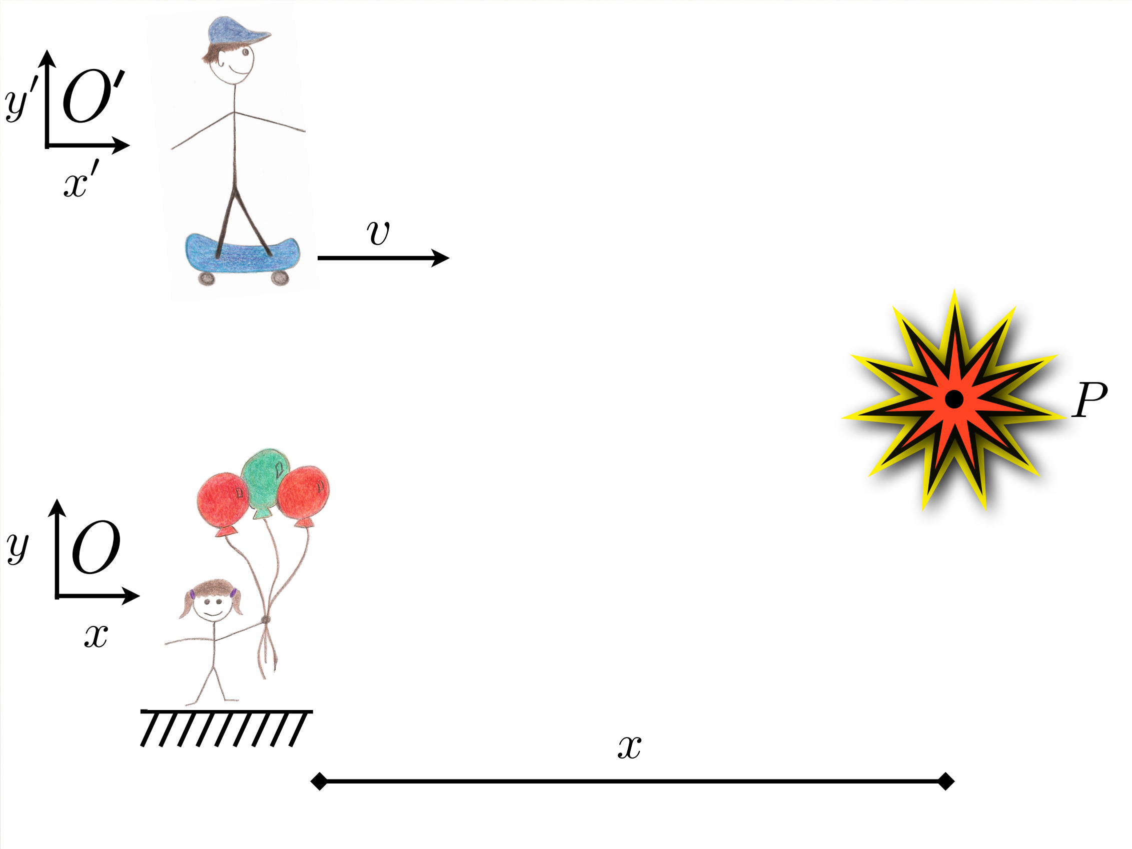
(rate he closes gap) = vt' *time in his frame*

$$x' = \frac{x}{\gamma} - vt'$$



Girl again? distance is (her to boy) + (boy to nova)
(her to boy) = vt
(boy to nova) = contracted because he is in motion!

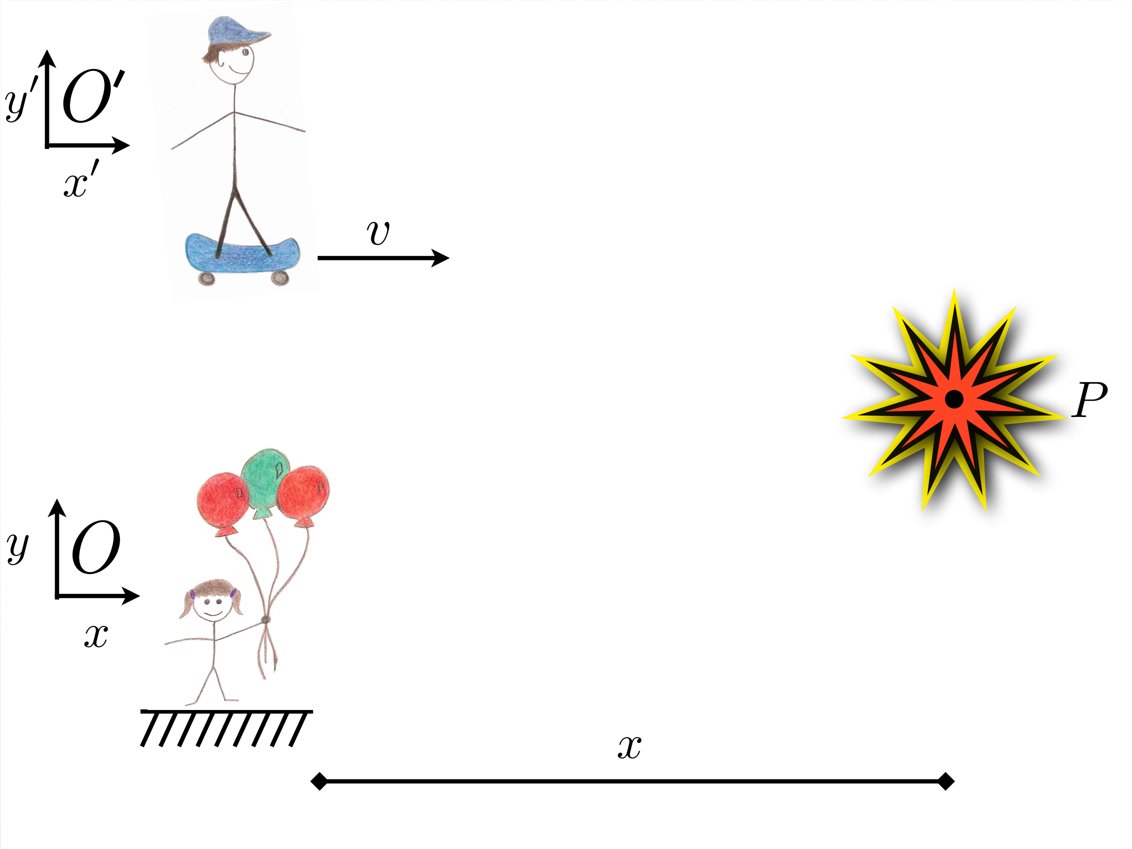
$$x = vt + \frac{x'}{\gamma}$$



Combine and algebra ensues:

$$x = vt + \frac{x'}{\gamma} \qquad x' = \frac{x}{\gamma} - vt'$$

$$\begin{aligned} x &= \gamma\left(x' + vt'\right) \\ &= \gamma\left(\gamma\left(x - vt\right) + vt'\right) \\ &= \gamma^2 x - \gamma^2 vt + \gamma vt' \end{aligned}$$



Combine and algebra ensues:

$$\gamma v t' = \left(1-\gamma^2\right) x + \gamma^2 v t$$

$$\implies \quad t' = \gamma t + \frac{\left(1-\gamma^2\right) x}{\gamma v} = \gamma \left[t + \frac{1-\gamma^2}{\gamma^2} \left(\frac{x}{v} \right) \right] = \gamma \left[t - \frac{vx}{c^2} \right]$$

can relate their times!

Time measurements in different non-accelerating reference frames:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

Two parts:
first term is ordinary time dilation
second is our *failure of simultaneity at a distance*

Elapsed times between events in non-accelerating reference frames:

$$\Delta t' = t'_1 - t'_2 = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) \tag{1.41}$$

If observer in O stationary relative to the events (x_1, t_1) and (x_2, t_2) measures a time difference between them of $\Delta t = t_1 - t_2$ and a spatial separation $\Delta x = x_1 - x_2$, an observer in O' measures a time interval for the same events $\Delta t'$. Events simultaneous in one frame ($\Delta t = 0$) are only simultaneous in the other ($\Delta t' = 0$) when there is no spatial separation between the two events ($\Delta x = 0$).

Go through this again to compare positions

Lorentz transformations between coordinate systems:

$$x' = \gamma (x - vt) \quad \text{or} \quad x = \gamma (x' + vt') \tag{1.42}$$

$$y' = y \tag{1.43}$$

$$z' = z \tag{1.44}$$

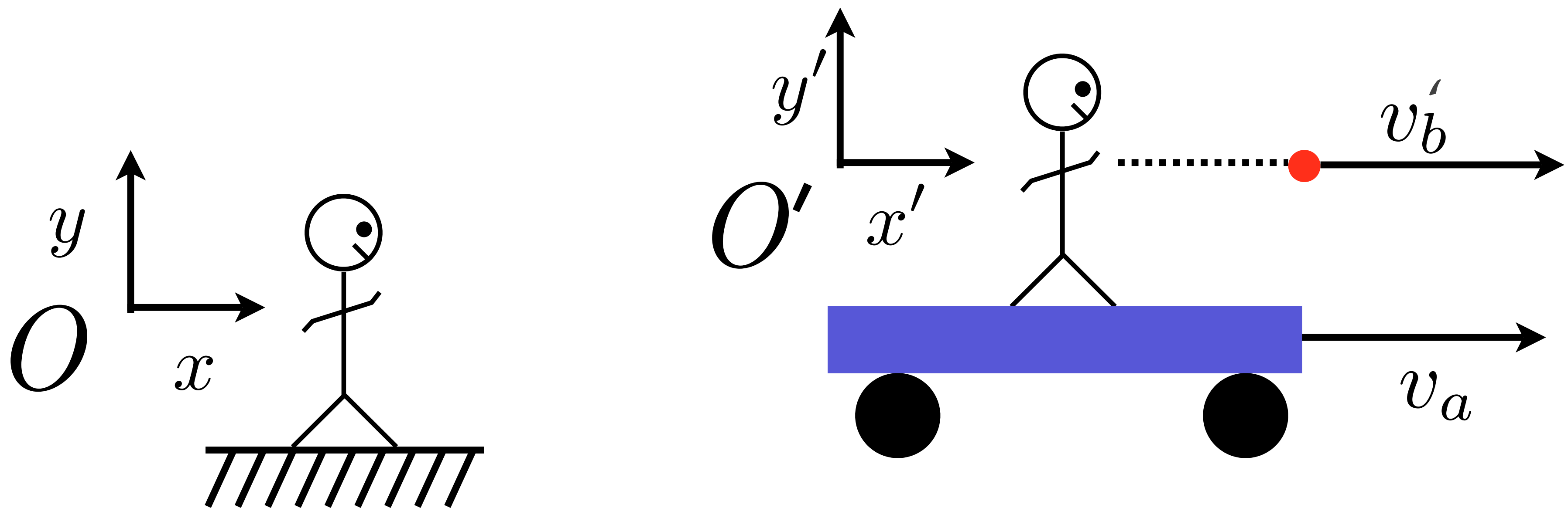
$$t' = \gamma \left(t - \frac{vx}{c^2} \right) \quad \text{or} \quad t = \gamma \left(t' + \frac{vx'}{c^2} \right) \tag{1.45}$$

Here (x, y, z, t) is the position and time of an event as measured by an observer in O stationary to it. A second observer in O', moving at velocity v along the x axis, measures the same event to be at position and time (x', y', z', t').

Relativity for observers in relative motion at constant velocity:

1. Moving observers see lengths contracted along the direction of motion.
2. Moving observers' clocks 'run slow', less time passes for them.
3. Events simultaneous in one frame are not simultaneous in another unless they occur at the same position
4. All observers measure the same speed of light c

Velocity addition: how fast is the ball going?

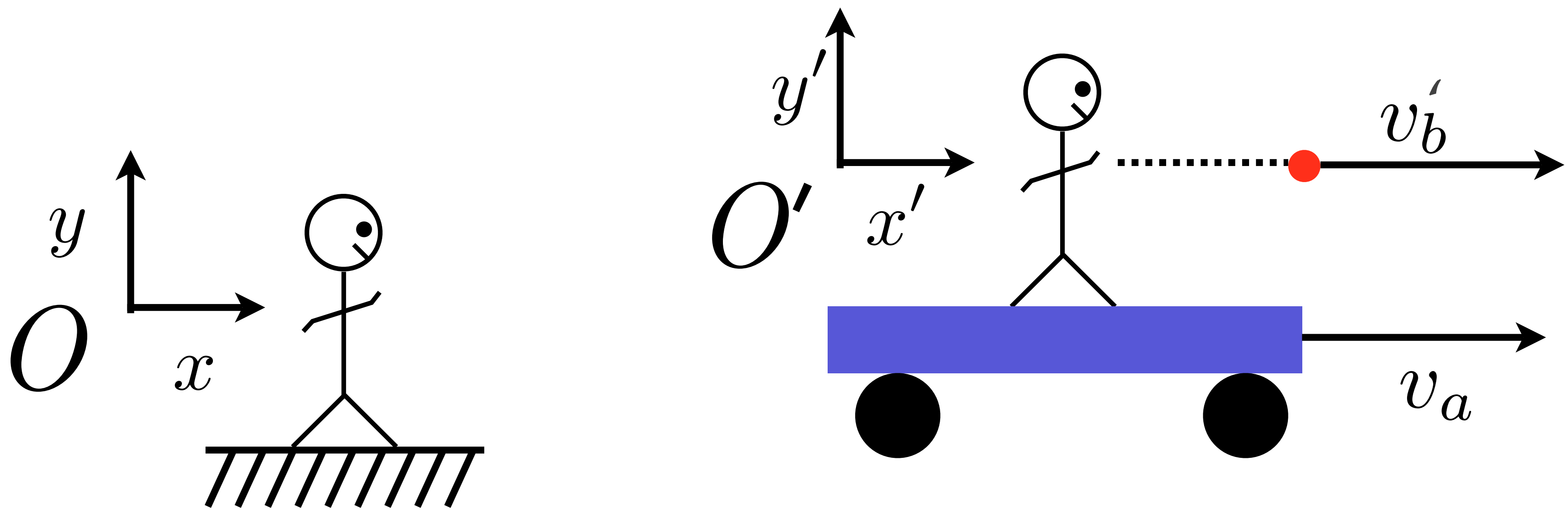


from person on car, what is position?

$x_b' = v_b' t'$

make sure all quantities are from the right frame!

Velocity addition: how fast is the ball going?



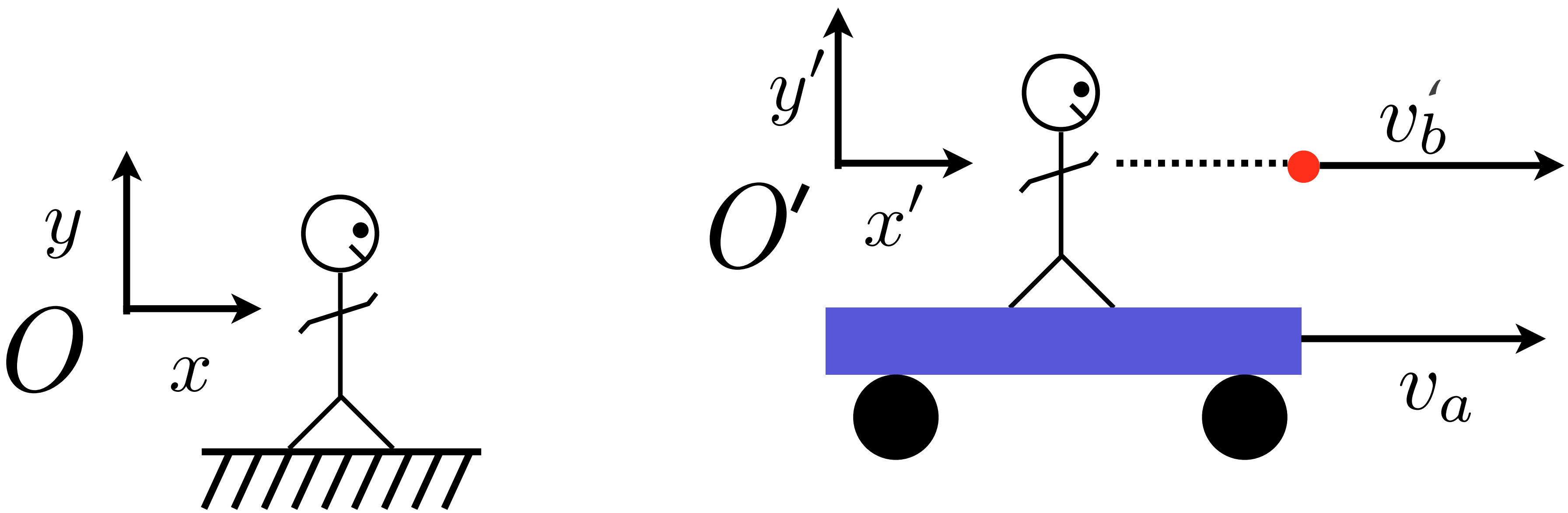
from person on ground, what is position?

use Lorentz transformation - both length contraction & time dilation

$$x_b = \gamma(x_b' + v_a t') = \gamma(v_b' t + v_a t')$$

make sure all quantities are from the right frame!

Velocity addition: how fast is the ball going?



divide position by time to get velocity *all from same frame*

first transform time too!

$$t = \gamma\left(t' + \frac{v_a x'}{c^2}\right) = \gamma\left(t' + \frac{v_a v_b' t'}{c^2}\right)$$

Velocity addition: how fast is the ball going?

Combine:

$$v_b = \frac{x}{t}$$

$$v_b = \frac{\gamma\left(v_b' t' + v_a t'\right)}{\gamma\left(t' + \frac{v_a v_b' t'}{c^2}\right)}$$

$$v_b = \frac{v_b' + v_a'}{1 + \frac{v_a v_b}{c^2}}$$

velocity of ball observed from the ground $= v_b = \dfrac{v_a + v_b'}{1 + \frac{v_a v_b'}{c^2}}$

Velocity addition: how fast is the ball going?

Relativistic velocity addition:
We have an observer in a frame O, and a second observer in another frame O′ who are moving relative to each other at a velocity v. Both observers measure the velocity of another object in their own frames (v_{obj} and v'_{obj}). We can relate the velocities measured in the different frames as follows:

$$v_{obj} = \frac{v + v'_{obj}}{1 + \frac{v v'_{obj}}{c^2}} \qquad v'_{obj} = \frac{v_{obj} - v}{1 - \frac{v v_{obj}}{c^2}} \tag{1.53}$$

Again, v_{obj} is the object's velocity as measured from the O reference frame, and v_{obj} is its velocity as measured from the O′ reference frame.

how to know which one to use?
do what you would normally do (add or subtract), then fix denominator

Velocity addition: how fast is the ball going?

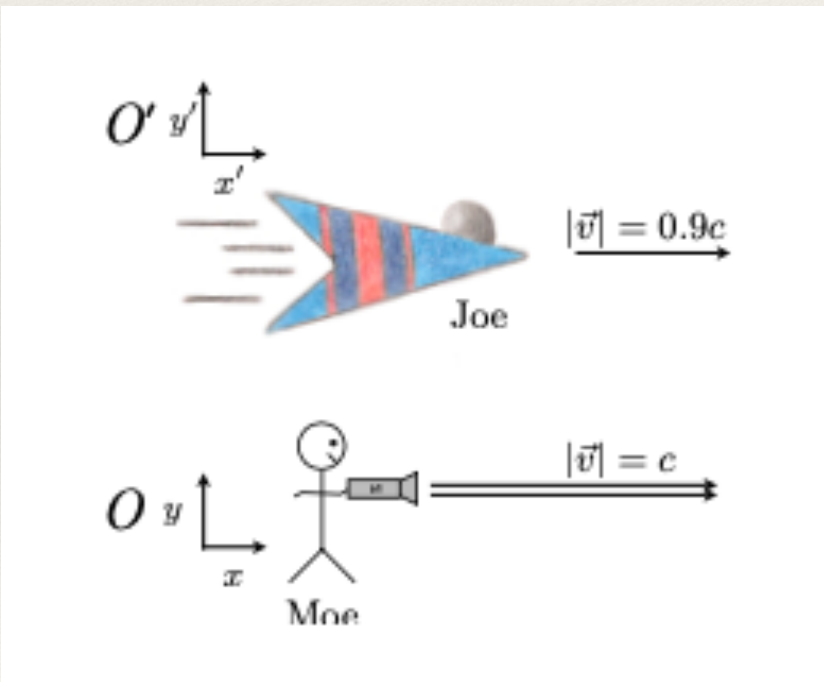
Throw a ball out a car window at v=0.5c, car at 0.75c?

Ordinarily, $v_{\text{ball relative to ground}} = 0.5c+0.75c=1.25c$ — NO
Using velocity addition? Would normally add them …

$$\begin{aligned} v_{\text{ball}} &= \frac{v_{\text{car}} + v'_{\text{ball}}}{1 + \frac{v_{\text{car}} v'_{\text{ball}}}{c^2}} \\ &= \frac{\frac{3}{4}c + \frac{1}{2}c}{1 + \frac{\left(\frac{3}{4}c\right)\left(\frac{1}{2}c\right)}{c^2}} \\ &= \frac{\frac{5}{4}c}{1 + \frac{3}{8}c} = \frac{10}{11}c \approx 0.91c \end{aligned}$$

Velocity addition: flashlight out the window?

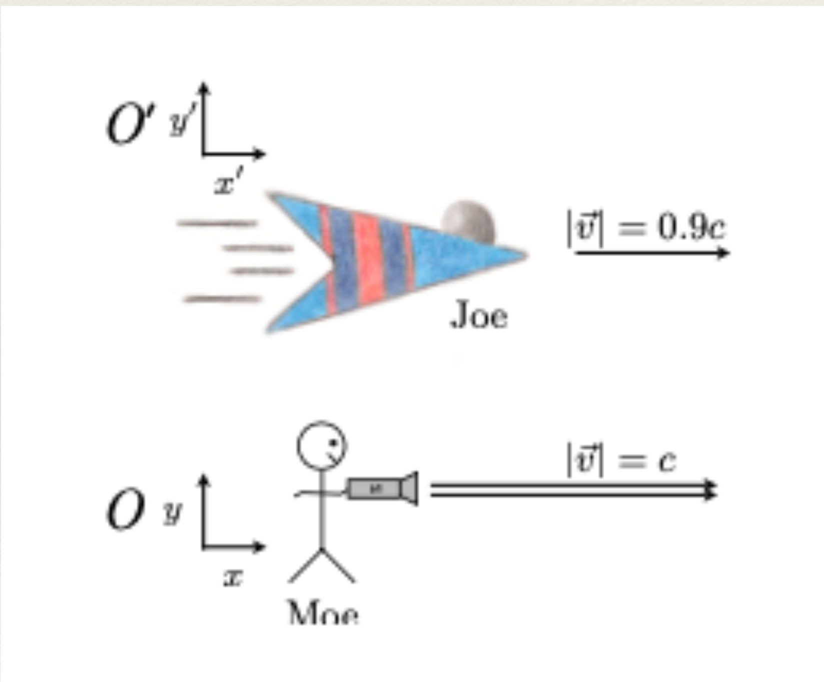
How fast does Joe in the rocket see the light beam go?
Would normally *subtract* light and rocket velocities.



$$v'_{\text{light}} = \frac{v_{\text{light}} - v_{\text{rocket}}}{1 - \frac{v_{\text{rocket}} v_{\text{light}}}{c^2}} = \frac{c - 0.99c}{1 - \frac{(0.99c)(c)}{c^2}} = \frac{0.01c}{1 - 0.99} = c$$

Velocity addition: flashlight out the window?

What if Joe has the flashlight in the rocket?
Add rocket + light speed?!?



$$
\begin{aligned}
v_{\text{light}} &= \frac{v_{\text{rocket}} + v'_{\text{light}}}{1 + \frac{v_{\text{rocket}} v'_{\text{light}}}{c^2}} \\
&= \frac{0.99c + c}{1 + \frac{(0.99c)(c)}{c^2}} \\
&= \frac{1.99c}{1 + 0.99} = c
\end{aligned}
$$