PH253 Lecture 9: Photons

Photoelectric effect, Compton scattering

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Outline

1. Remaining problems
2. Atoms have discrete energy levels
3. Photoelectric effect
4. Compton Scattering

Outline

Thermal spectra now well explained

- Quantization/discreteness of energy was critical
- Oscillating charges only emit "packets" with $E = hf$
- Results in peaked, continuous spectrum of emitted light
- Reduces to classical result in limit discreteness is not noticeable
- Problem: neon sign.

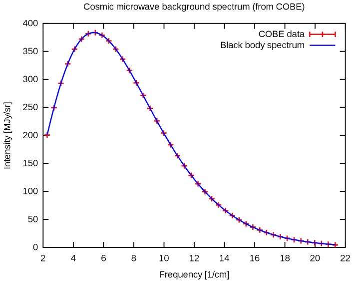


Figure: Source: `https://en.wikipedia.org/wiki/Cosmic_Background_Explorer` h/t: `https://xkcd.com/54/` you should now be able to identify the science get the bonus points [hover].

Absorption & Emission are discrete

Neon signs emit only specific colors of light (mostly red).

Same for absorption or emission of light by gases.
2 ways to view light-matter interaction

1. Sparse gas. Send in white light, see what gets through
   - only specific discrete wavelengths absorbed
   - even with quantized oscillators, got continuous spectrum
2. Excite a sparse gas with high voltage
   - only specific discrete wavelengths emitted
   - *same* wavelengths as for absorption!
   - implies the process is the same for both!

Not a thermal process, and not due to accelerated charges (also continuous).

Also recall pesky problem of atoms not being stable.

Absorption & Emission are discrete

Key implication: somehow atoms have discrete energies too, not just emitted energy

1. What if emission is a vibration of sorts?
2. If so, vibrating string has discrete modes ...
3. Somehow we misunderstand the structure of the atom
4. Start with an appeal to experiment

Absorption & Emission are discrete

Hydrogen spectrum

1. Discrete absorption and emission.
2. Characteristic series that repeats for IR, visible, UV, etc.
3. Write down all the frequencies of emission for each range (1,2,3) as f_{11}, f_{12}, f_{12}, etc.
4. Curious: sum of any 2 f's gives a 3$^{\text{rd}}$ observed f!

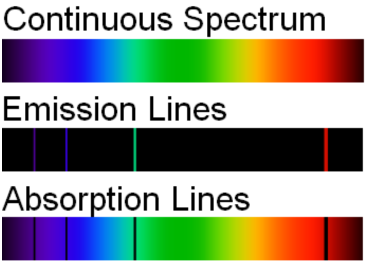


Figure: Source: https://upload.wikimedia.org/wikipedia/commons/6/6a/Spectral_lines_en.PNG?uselang=en-gb

Absorption & Emission are discrete

Sum of 2 observed frequencies is always a 3$^{\text{rd}}$ observed frequency.

1. Strongly implies stable energy states of atoms are discrete
2. *Not* just the radiation that has allowed energy "bundles"
3. Why doesn't the discharge/absorption expression "flicker" if emission is discrete?
4. There are *many* atoms. Think pixels - only notice if you zoom in enough.
5. Bohr had an explanation

Outline

Bohr' idea

1. Take Planck further: atoms only exist in certain discrete states
2. Planck: emit discrete, absorb continuously.
3. Bohr: absorption also discrete
4. Light emitted as quanta when atom transitions to new state

Discrete energy levels

1. Atom emits/absorbs light when transitioning between energy states/levels
2. Upward transition when absorbing (atom gains hf of light energy)
3. Downward transition when emitting (atom emits light energy hf)
4. Between two levels, light energy is $\Delta E = E_i - E_j = hf$
5. Given discrete set of levels, only certain energies possible

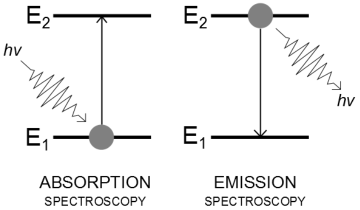


Figure: Source: https://commons.wikimedia.org/wiki/File:Absorption_or_emission_spectroscopy.png

Light emission/absorption

1. This explains the sum rule
2. Going from level 4 to 2 includes 4-3, 2-3 intermediate steps as possibilities
3. $hf_{42} = E_4 - E_2 = (E_4 - E_3) + (E_3 - E_2) = hf_{43} + hf_{32}$
4. Recover sum rule!
5. Boltzmann factor $P(E) = e^{-E/k_BT}$ explains why this doesn't happen constantly …
6. …and why there are often many more emission lines than absorption lines.

Occupancy considerations

1. Can't absorb radiation to go from 3 to 4 unless you're already in 3!
2. I.e., need some atoms excited already, transition amongst lowest more occupied levels easier
3. Probability of being in state 3 compared to state 1 (lowest)?
4. $N_3/N_1 = e^{-E_3/k_BT}/e^{-E_1/k_BT} = e^{-(E_3-E_1)/k_BT}$
5. Energy difference of order 2 eV, room temp is $k_BT \sim 0.025$ eV. $N_3/N_1 \sim 10^{-80}$.
6. At room temp, nearly all atoms are in the lowest state! Transitions from higher levels up very rare.

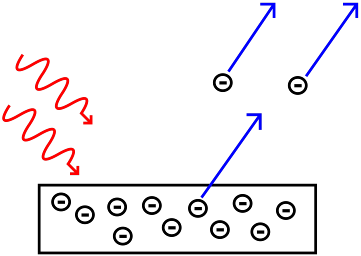
Emission vs absorption

1. At room temp, all emissions possible if gas is excited electrically
2. Only absorptions from 1 to higher levels are likely
3. Very high temp: start to see other absorptions (e.g., 2, 3 to higher)
4. Making radiation emission discrete has consequences
5. Namely: energy states of atoms themselves must also be discrete
6. What other empirical evidence can we draw on?

Outline

Photoelectric effect

1. Basically: shine light on a clean metal surface
2. Electrons are ejected. One at a time, always same charge.
3. Light has momentum, makes some sense
4. Consistent with wave behavior on its face
5. Problems: in the details
6. If light = continuous wave, how can it come in discrete energy bundles?
7. If absorption is continuous, why are only discrete charges emitted?



Photoelectric effect

1. Planck's energy quanta were a mathematical formality, not literal.
2. Einstein: take Planck literally! If energy is discrete, so is light itself.
3. Why? Experiment.
4. Light ejects electron
5. Absorbing light increases electron's KE
6. NEG metal plate: if KE $< e\Delta V$, no current
7. Measures energy distribution of electrons vs. light freq/intensity

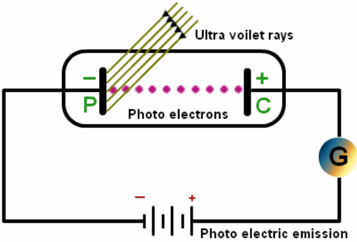


Figure: https://edignite.com/courses/hertz-photo-electric-effect/

Photoelectric effect

1. $K = \frac{1}{2}mv^2 = hf$ – light bundle gives KE to electron.
2. (assume $v \ll c$ so non-relativistic)
3. If $\frac{1}{2}mv^2 \geq e\Delta V$, e$-$ collected, I measured
4. ΔV required to stop e^- measures KE!
5. Vary ΔV for a given light frequency, find ΔV_{stop}
6. ΔV_{stop} is min ΔV where no e^- collected ($I = 0$)
7. "Stopping potential"

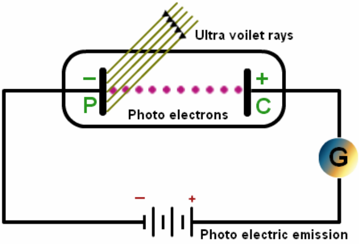


Figure: https://edignite.com/courses/hertz-photo-electric-effect/

Photoelectric effect

1. Stopping potential is *independent of intensity of light*
2. Higher intensity does *not* mean higher energy!
3. Light intensity and energy are not related
4. If they were, expect stopping $\Delta V_{\text{stop}} \sim$ intensity
5. Classical: e^- absorb EM waves continuously …
6. …so higher intensity = higher KE = larger ΔV_{stop}
7. Experiment? KE independent of intensity

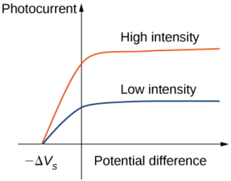


Figure: Source: `https://opentextbc.ca/universityphysicsv3openstax/chapter/photoelectric-effect/`

Photoelectric effect

1. Classical: expect time delay - wait for e^- to absorb enough energy
2. Experiment: no delay ($\ll 10^{-9}$ s)
3. Classical: any frequency f ejects e^- if intensity is high enough
4. Experiment: e^- only come out if f is above a critical value
5. Experiment: critical f is material-dependent
6. Classical: KE and f are unrelated
7. Experiment: KE $\uparrow$ as $f \uparrow$

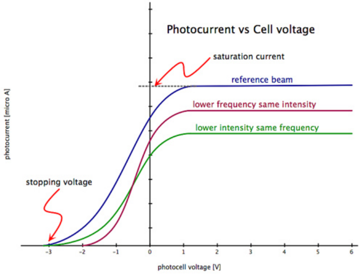


Figure: Source: `https://i.stack.imgur.com/nr93Y.jpg`

Einstein's model

1. Perhaps the incident light *is* the "quanta"
2. Light exists as little bundles of energy hf
3. Light emitted/absorbed as quanta and travels as such
4. Massless particles, energy hf – photons!
5. Still expect $1/r^2$ intensity drop
6. Relativity: if $m = 0$, travel at $v = c$ in straight lines
7. Energy $\neq$ intensity, intensity = *more photons*
8. At given frequency, all particles have same energy

Einstein's model

1. Now it is a mechanics problem
2. Photon-electron collision
3. If photon energy > binding energy of e^-, ejected
4. Photon energy too small, e^- excited but remains bound
5. Binding energy material-dependent
6. Remember photons have momentum too
7. e^- absorbs photon energy & momentum
8. If $hf > E_{\text{binding}}$, ejected

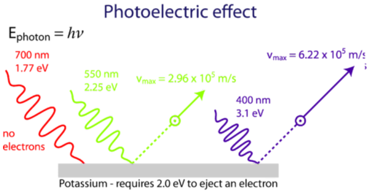


Figure: Source:
https://www.ck12.org/physics/photoelectric-effect-in-physics/lesson/Photoelectric-Effect-CHEM/

Einstein's model

1. Electrons bound to crystal with energy E_{binding}
2. Mostly e^- near surface, mostly ejected with $\vec{v} \perp$ surface
3. Energy balance:

$$E_{\text{photon}} = KE_{e^-} + E_{\text{binding}} = KE + W \tag{1}$$

$W = E_{\text{binding}} = \varphi$ aka "work function." Simplify:

$$KE_{e^-} = hf - W = hf - \varphi = hf - E_{\text{binding}} \tag{2}$$

Stopping potential is then $e\Delta V_{\text{stop}} = hf$, critical point where $KE_{e^-} = 0$. Any higher f ejects e^-

Einstein's model

$$KE_{e^-} = hf - W = e\Delta V_{\text{stop}} \tag{3}$$

1. This explains it all!
2. Need $hf > W$ for any ejection ($KE_{e^-} > 0$)
3. Need photon energy to exceed binding energy of e^-
4. KE linear in f above that
5. Happens instantly: each photon gives all its energy to one e^-
6. Intensity irrelevant, nothing to do with energy.
7. $E_{\text{light}} = (\text{num. photons})(\text{energy per photon}) = Nhf$
8. Intensity $\sim N/\Delta t$

Einstein's model

$$KE_{e^-} = hf - W = e\Delta V_{\text{stop}} \tag{4}$$

1. Weak beam does the same as intense one - just less photons/sec
2. But … since W is material-dependent, so is critical f
3. Why film is less sensitive to red light - less E per photon
4. Why UV is more damaging than visible light
5. What more does this imply?
6. If light is particles, how can we have ray diagrams and wave interference?
7. Depends on the *scale* of the probe

Wave vs. particle

1. Relevant length scale is λ
2. Probe scale $\lesssim \lambda$ - wave like. Close enough to see waveiness
3. Probe scale $\gtrsim \lambda$ - too far away, see "beam" of particles.
4. Everyday stuff: visible light is $\lambda \sim 400 - 700\,\text{nm}$
5. So we mainly see stream of particles - light rays
6. Tiny structures: can observe wave-like nature
7. Really: photon is *neither*, failure of imagination
8. More on this to come …

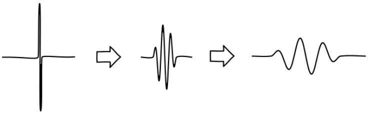


Figure: Source: https://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/quantum_theory_waves/index.html

Implications from relativity

Planck-Einstein:

$$E_{\text{photon}} = hf = \hbar\omega \qquad \hbar = \frac{h}{2\pi} \tag{5}$$

Relativity:

$$E^2 = m^2c^4 + p^2c^2 \qquad v_{\text{light}} = c \tag{6}$$

Only consistent if $m = 0$ for photons! Implies

$$E^2 = c^2p^2 \qquad \text{or} \qquad |p| = E/c \tag{7}$$

Same result we had from classical E&M
$|\vec{p}| = hf/c = h/\lambda$ - larger λ, lower p, "gentler" collision

Implications

1. Photons have momentum by virtue of energy, but no mass
2. Emission of a photon means body loses energy
3. Which decreases its mass by E^2/c
4. Absorb photon: mass-energy equivalence $\implies$ mass increase
5. Tiny though . . . green photon $\sim 2\,\text{eV}$, $m_{e^-}c^2 = 511,000\,\text{eV}$
6. Radiation pressure from classical E&M now more natural - collisions
7. How to prove photons have momentum? Collide them with stuff.
8. Compton effect!

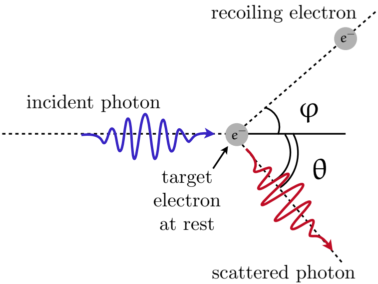
Outline

Compton scattering

An incident photon of frequency f_i, energy $E_i = hf_i$, and momentum $p_i = h/\lambda_i$ strikes an electron (mass m) at rest.

The photon is scattered through an angle θ, and the scattered photon has frequency f_f, energy $E_f = hf_f$, and momentum $p_f = h/\lambda_f$.

Electron recoils at angle φ relative to the incident photon, acquires kinetic energy KE_e and momentum p_e.



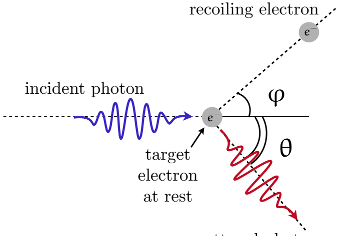
Compton scattering

Conserve energy and momentum:

$$hf_i = hf_f + KE_e = hf_f + \sqrt{m^2c^4 + p_e^2c^2} - mc^2 \tag{8}$$

Noted KE_{e^-} is total energy minus its rest energy mc^2.
Conservation of p in both directions:

$$p_i = p_e \cos\varphi + p_f \cos\theta \qquad p_e \sin\varphi = p_f \sin\theta \tag{9}$$



Compton scattering

Math ensues.

$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{mc}\left(1 - \cos\theta\right) \tag{10}$$

This is the *Compton equation.*

h/mc has units of length - the *Compton wavelength*

$\lambda_c = h/mc \approx 2.42 \times 10^{-12}\,\mathrm{m}.$

Scale at which quantum effects dominate

Compton scattering

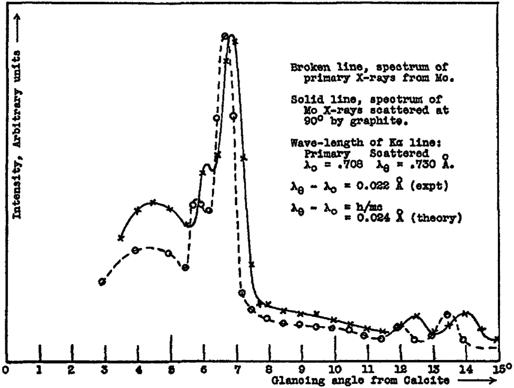


Fig. 4. Spectrum of molybdenum X-rays scattered by graphite, compared with the spectrum of the primary X-rays, showing an increase in wave-length on scattering.

Figure: Original data. Physical Review **21**, 483-502 (1923)

Compton scattering

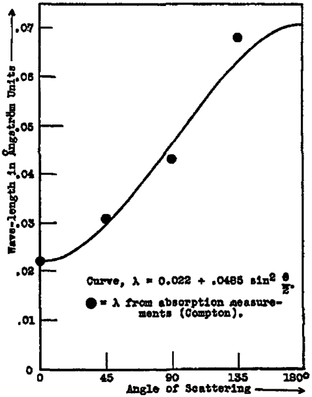


Figure: It works! Physical Review **21**, 483-502 (1923)

Compton scattering

Compton wavelength sets fundamental limitation on measuring the position of a particle.

- Depends on the mass m of the particle.
- Can measure the position of a particle by bouncing light off it
- But! Need short wavelength for accuracy.
- That means higher p and E for the photon!
- (Which disturbs position . . . uncertainty)
- If photon energy $> mc^2$, when it hits particle being measured there is enough energy to create a new particle of the same type!
- Meaning you still don’t know where the original one is.

Compton scattering

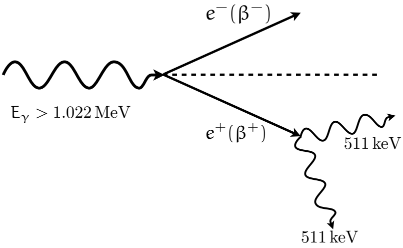


Figure: Pair production

A photon can “decay” into an electron and a positron (electron antiparticle). Try to measure electron with high energy photon? Now you have 3 particles.