

PH255: Modern Physics Laboratory

Patrick R. LeClair, Jerry Busenitz, Andreas Piepke, Dawn R. Williams

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The Care & Feeding of Laboratory Notebooks¹

On the following page, we reproduce an excellent guide from MIT's OpenCourseWare on maintaining a laboratory notebook. (This guide is available under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States license. See <http://ocw.mit.edu/terms/#cc> for terms and conditions.)

Requirements for Experimental Notebooks

MIT Department of Physics
(Dated: February 14, 2008)

Learning to maintain a laboratory notebook is one of the most important skills you will develop in Junior Lab. A good laboratory notebook is essential when you begin to write papers or to develop oral presentations summarizing your experimental efforts. A clear well-written narrative that includes experimental schematics, plots of raw data, and details of your analysis methods will enable you to receive quick feedback and assistance during lab sessions from peers, TA's and section instructors.

A poorly maintained notebook will prove immensely frustrating to you and your instructor. It is very difficult to answer questions like 'why didn't the experiment work' or 'why was my result off by an order of magnitude?' without being able to clearly and easily trace your efforts using your notebook. Don't count on being able to recall the gain of your amplifier even one day after a lab session!!! The following is a list of specific guidelines to follow when performing laboratory work.

1. Create a *descriptive* table of contents and make an entry every time you add new material:

Date ——— **Contents** ——— **Page** Don't use generic entries like "Day 1" or "Analysis". Instead, produce records of significant milestones: e.g. "Calibration of NaI Scintillation Counter Using Ba-133 and Na-22 Check Sources", "Plot of monochromator linearity over the visible spectrum", "Montecarlo Simulation of Mean Slant Path Distance in Muon TOF Experiment".

2. Sign and date each page demonstrating authenticity.
3. Don't ever erase, use white-out, or tear out pages of a lab notebook. Indicate "mistakes" by simply drawing a single, neat line through the item. These may prove to not so incorrect as initially thought and will very often be useful as a guide to how the experiment was done and provide clues on how to better execute the experiment next time.
4. Loose-leaf pages are never acceptable within a lab notebook. Graphics or tables generated by computer must be neatly taped into the notebook. Remember to annotate these types of graphics with as much information about how they were created as possible.
5. Preparatory questions and solutions should be written in your lab notebooks.
6. Following the preparatory questions, list the objectives of the experiment. Restate the essential physics of the experiment in your own words!!!

7. After listing the objectives, identify the things you will have to do, the data you must obtain and identify the required calibrations.
8. On the first day of a new experiment you should sketch a block diagram of the experimental apparatus..
9. Identify the location of the data files or long analysis programs if they are too big to directly enter or tape into your notebook. Analysis scripts, functional forms for non-linear fits, etc. should always be present in your notebooks.
10. Note typical "readings" and instrumental settings so to be able to quickly setup an experiment on subsequent days.
11. Sketch waveforms at various places within the signal chain. This will help ensure your understanding of each component and permit you to rapidly identify equipment failure.
12. When tabulating data, make neat columns with headings and always note correct units.
13. Don't wait until after the session has ended to visually examine the quality of your data. Make preliminary, hand drawn plots of data, **AS THEY ARE ACQUIRED**. These initial plots will very often save you time and frustration in making sure that your data are reasonable and suggestive of the behavior you expect. **The importance of making preliminary plots and analyses in real-time cannot be overstated.**
14. Your notebook should contain diagrams, narratives, tables of raw data, formulas, computations, reduced data, error analysis and conclusions in a neat compact, orderly arrangement.

Bring your notebook to every lab session and to all oral exams. Failure to do so will result in penalties to your grade!

Laboratory Safety

It is required that you read this entire section before doing any PH255 lab work. After reading the safety information below, please print and sign the lab safety rules acknowledgment form on page 10.

UA safety information and procedures can be found at <http://bama.ua.edu/~ehs/>.

2.1 General rules

Two primary rules for lab safety are that *you never work alone in the lab*, and *you never operate equipment without proper training*. At least one other person must be present in the lab for you to work on any experiment or exercise. The instructor will usually be happy to assist you if you need any extra time in the laboratory beyond normal class hours. Proper training includes reading the laboratory procedure for a given experiment and asking the instructor for a quick demonstration of the operation of the equipment involved. A short summary of some of the basic rules:

- If you don't know what you are doing WITH CERTAINTY, ask for help.
- Don't assume that you can just 'figure it out,' ask for proper instructions.
- Make sure you are trained on all equipment and procedures before beginning.
- Never work alone.
- If you must work alone, see the preceding rule.
- If others urge you to follow unsafe practices, refuse.
- Taking shortcuts to speed up an experiment is not only potentially dangerous, it usually has the opposite effect in the end. A hastily done experiment usually needs to be repeated.
- If an injury occurs, remain calm, dial 911, and only provide first aid if you are absolutely certain of what to do.

Other general rules are to *always put things back in their proper place when you are finished*, and *leave the lab in a better condition than you found it*. This is more than just a common courtesy to others working in the lab, safety and organization go hand in hand.

2.2 High Voltage Safety & Electronic Equipment Considerations

Some of the exercises and experiments in this course involve the use of high voltage. To ensure your own safety and to prevent damage to equipment in the lab, certain precautions must be followed. While OSHA defines "high voltage" to be anything in excess of 600 V, for the purposes of this

course, we will treat *all* electrical devices as if they had high voltage present.

- Before connecting or disconnecting any cable, make sure that the power supply is in the STANDBY or OFF mode. NEVER connect or disconnect a cable with power supplied.
- Never, ever work alone. If an electrical accident does occur, having a lab partner present may be the difference between life and death. We have first-hand experience in these matters.
- Some supplies have capacitors inside which may not be discharged for a considerable amount of time after powering off the supply. Do not open any electrical equipment unless instructed to do so and supervised by an instructor.
- Always keep one hand in your pocket or behind your back when testing any circuit in which there may be high voltage present. This will help prevent an electrical shock from sourcing current through your heart.
- Remember that it is current that kills. A good (e.g., sweaty) connection of 6 V across your body can kill as well as a poor connection of 600 or 6000 V.
- Unless specifically instructed to do so, do not change the polarity of an applied voltage. Under no circumstances change the polarity with the high voltage on, as this will damage the power supply.
- Gas discharge tubes (e.g., for atomic spectra) use a 5000 V dc supply. Never insert or remove a tube while the power is on, and never touch the metal connectors on the ends of the tube while inserting or removing a tube. Only handle the central glass portion of the tubes. The tubes may also become hot after operation, let the tubes cool down before attempting to remove them.
- When you have finished your exercise or experiment, power down all electronics (except computers, they stay on).
- Keep all cable ends and connectors off the floor. If a cable ends or connector get stepped on, it will be damaged, and the mating connector will also be damaged when you attempt to put them together. Connectors and cables cost anywhere from \$10 to \$100 each – the damage bill adds up quickly!

Electrical equipment represents the large majority of potential hazards in the lab, particularly in terms of risk indexed against opportunity. There are many sources of moderate or high voltage, and many occasions to operate electronic equipment. However, if you follow proper safety procedures, the risk to you is negligible.

Electricity is potentially dangerous because your body operates via electrical nerve impulses. Even a small amount of external current can disturb or interrupt such transmissions. The amount of current is what matters, as shown in the table below.ⁱ

ⁱSource data from http://www.allaboutcircuits.com/vol_1/chpt_3/4.html. This page has an excellent discussion of electrical safety.

Bodily effect	dc	60 Hz ac	10 kHz ac
Slight sensation felt at hand(s)	Men = 1.0 mA	0.4 mA	7 mA
	Women = 0.6 mA	0.3 mA	5 mA
Threshold of perception	Men = 5.2 mA	1.1 mA	12 mA
	Women = 3.5 mA	0.7 mA	8 mA
Painful, but voluntary muscle control maintained	Men = 62 mA	9 mA	55 mA
	Women = 41 mA	6 mA	37 mA
Painful, unable to let go of wires	Men = 76 mA	16 mA	75 mA
	Women = 51 mA	10.5 mA	50 mA
Severe pain, difficulty breathing	Men = 90 mA	23 mA	94 mA
	Women = 60 mA	15 mA	63 mA
Possible heart fibrillation after 3 seconds	Men = 500 mA	100 mA	
	Women = 500 mA	100 mA	

Keep in mind that these figures are only approximate, and will vary from individual to individual. However, note that with more than just a few milliamperes, you **can't let go** of the electrical source. With good contact, your arm-to-arm resistance is about $1\text{ k}\Omega$. A normal 110 V ac wall outlet thus gives 110 mA, more than enough to kill you if improperly handled!

If someone in the lab does receive an electrical shock, first turn off the power to the offending instrument or pry the victim off with insulated materials (gloves, yardstick, plastic). Apply CPR if you know how (you have learned CPR, right?), and have someone immediately dial 911.

Some other important suggestions if an accident occurs:

- If you don't know what you are doing, GET HELP.
- Double check that the power is off, and tape its power switch off so no one else can turn it on.
- Use a plastic-handled screwdriver to short out the device before touching wires or electrodes.
- If you need to move the offending wire(s), use a long insulating item, such as a wooden meter stick.
- If you must touch the offending wire(s), keep one hand in your pocket, touch the wire with the **back** of your hand first (higher resistance, no chance of grabbing and not being able to let go).
- Never, EVER work alone.
- Be sure all electrical equipment is grounded in at least one place, and do not defeat the third pin on electrical plugs.

2.3 General Precautions When Using Lasers

The helium-neon and diode lasers used in the laboratory have a total beam power of < 1 mW and should not be capable of damaging your eyes. However, we will not test this assumption. NEVER allow a laser beam to enter your eye directly. Take care not to look at the laser beam reflected from a specular (mirror-like) surface. When the beam has been broadened by a lens or diffusely reflected (e.g., by paper or a matte-finished surface), it is generally quite safe, since the beam energy density has been strongly reduced. See <http://bama.ua.edu/~ehs/New%20Web/laseprog.htm> for further laser safety guidelines.

2.4 Radiation Safety

The radioactive sources used in this lab are expressly intended for classroom use. They are sealed sources of very low activity, and according to federal safety standards, do not require special precautions (for example, it is not necessary to have a film badge to monitor your exposure). However, we will treat all radiation sources with proper respect, and assume that they are all potentially dangerous. Some common sense rules to follow when handling or using radioactive sources in the lab:

- The radioactive sources used in this laboratory are stored in a labeled red box inside a shielded metal box. WHEN YOU HAVE FINISHED USING A SOURCE, PUT IT BACK. Do not leave any source lying around the lab unattended.
- Do not handle the sources any more than necessary. Before doing an exercise or experiment, think about how to configure the equipment with the goal in mind of keeping the maximum practicable distance between yourself and the source. This will help reduce the amount of time you spend in the proximity of the source.
- If you suspect that there is radioactivity where you aren't expecting it, there is a survey meter (Geiger counter) in the lab for checking out this kind of situation.
- If you wish to use them, lead bricks are available to provide shielding around the source. This is primarily to reduce the influence of background radiation on your experiment, they are not required for safety. To avoid getting any lead on your hands, use the gloves provided in the lab while handling the bricks, or any other lead objects in the lab. Wash your hands after handling lead, even if you wore gloves. Use only painted lead bricks; unpainted and oxidized bricks will have a white powdery coating, these should not be used.
- Never remove a source from the laboratory, even temporarily. The staff are legally responsible for the sources and must periodically account for their presence and condition.
- Keep sources away from your body.
- Never bring a radioactive source near your eyes because they are particularly sensitive to radiation.
- Be aware of the sources being used in neighboring experiments.

Meticulous care must be taken by all students and staff to insure that every source removed from the repository be returned immediately after its use. For further information on UA radiation safety, see <http://bama.ua.edu/~ehs/New%20Web/radprog.htm>

2.4.1 Exposure and Dosage

An excellent guide on radioactivity and radiation protection can be found at <http://pdg.lbl.gov/1999/radiorpp.pdf>. This section is adapted from the MIT OpenCourseWare for course 8.13-14, “Experimental Physics I & II.” The safety guide at <http://ocw.mit.edu/courses/physics/> under course 8.13-14 is available under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States license. See <http://ocw.mit.edu/terms/#cc> for terms and conditions.

Ionizing radiation damages tissue; any exposure should therefore be minimized. The unit of radiation exposure is the rem (roentgen equivalent man). A new unit, called the Sievert (= 100 rem) is recommended by the International Commission on Radiation Units and Measurements (ICRU). Your inescapable dosage from cosmic rays and other background sources is 360 mrem/yr, which works out to 4.2×10^{-2} mrem/hr. The recommended limit to exposure for a member of the general public is 100 mrem/yr, averaged over any consecutive five years. If you follow the lab guidelines, your exposure will be only a small fraction of the dose you receive from the natural background. A meter is available for you to check the radiation levels yourself.

Radioactive sources emit three types of radiation: high energy helium nuclei (alpha rays), electrons (beta rays), or photons (gamma rays). Most of the sources in the lab emit only gamma radiation. Of the sources which do emit alpha or beta particles, most are enclosed in plastic or metals, which prevent particulate radiation from escaping. Any exceptions will be obviously noted. Any such specially marked sources should never be handled. Handling of open alpha- or beta-emitters can result in dangerous dosages to the skin.

The strength of a radioactive source is measured in curies (Ci). A one-curie source has an activity of 3.7×10^{10} disintegrations/s. The “absorbed dose” is a quantity that measures the total energy absorbed per unit mass; it is measured in rads, where 1 rad = 100 erg/g. The “equivalent dose” is measured in the units discussed above, the rem. The equivalent dose is derived from the absorbed dose by multiplying by a “radiation weighting factor” which is a measure of how damaging a particular type of radiation is to biological tissue. For photons (gamma rays) and electrons and positrons (beta particles), the radiation weighting factor is unity; for helium nuclei (alpha particles), it is 20; for protons with energy greater than 2 MeV it is 5; and for neutrons it ranges from 5 to 20, depending on the energy. When you use the meter in the lab, the readings are in rads, and you must consider the type of particle when you work out the equivalent dose.

For gamma rays with energy greater than 1 MeV, a useful approximation is that the equivalent dose due to a source with an activity of C microcuries is $5.2 \times 10^{-4} C E_{\gamma} R^{-2}$ mrem/hr, where R is the distance from the source in meters and E_{γ} is the energy of the gamma ray in MeV. For gamma rays with energy less than 1 MeV, this formula is still approximately true for a full body dose. However, low-energy gamma rays deposit their energy in a smaller mass of tissue than high-energy gamma rays and can cause high local doses. For example, the local dose to the hands from handling a 10 KeV source can be up to 25 times the value given by the above formula; hands, however, have a higher tolerance to radiation than inner organs or eyes.

The protective value of shielding varies drastically with the energy of the photons. The intensity of a soft X-ray beam of “soft” (i.e. < 1 KeV) can be reduced by many orders of magnitude with a millimeter of aluminum while 1.2 MeV gamma rays from ^{60}Co are attenuated by only a factor of 2 by a lead sheet one-half of an inch thick. The best way to keep your dosage down is to put distance between you and the source. If you stay a meter away from most sources in the lab, you will be receiving, even without any lead shielding, a dose which is much less than your allowable background dose. If, however, you sit reading the write-up with a box of sources a few inches away, you may momentarily be receiving ten to a hundred times the background level.

2.5 Cryogenics

Liquid nitrogen is chemically inert, but it can cause severe frostbite. Wear gloves and protective glasses or a face shield when transferring or transporting liquid nitrogen. Do not put liquid nitrogen in closed containers – the boiloff of liquid nitrogen will cause pressure to build up until something explodes.

2.6 What to do if an Injury Incident Occurs

1. Don't panic. The accident scene may be very scary. Resist the urge to rush in without thinking, or run away.
2. Take charge of yourself and others. Take a few seconds to think and develop a plan. Dial 911. Give firm, clear instructions to bystanders.
3. Approach the victim safely. Turn off any electricity, move the victim away from conductors with insulating tools (yardstick, gloves). Stabilize falling objects.
4. Perform emergency first aid **if you know what to do**. You may do more harm than good if you do not know the correct procedures. Perform CPR, arrest bleeding if necessary. Do not move victim if possible.
5. Stay with the victim until help arrives, or delegate a bystander if you must deal with equipment.

A summary of the preceding safety rules:

- If you don't know what you are doing FOR SURE, seek help.
- Get proper training for whatever equipment and procedures you encounter, and
- DON'T assume you are so smart you can derive it all on the spot.
- Never work alone if at all avoidable.
- If you must work alone, make sure someone knows where you are, what you are doing, and when you should be done.
- If others urge you to follow unsafe practices, refuse loudly and firmly.
- It is your body, and you need it for a long time.
- If an injury occurs, remain calm, dial 911, and only provide first aid if you know what to do FOR SURE.

LAB SAFETY RULES

You must have a signed copy of this form on file to work in the PH255 laboratory.

The safety policies and procedures listed below must be followed in the laboratory. Failure to comply with these rules may result in dismissal or suspension from the laboratory.

- Be aware of the locations of fire extinguishers and first aid kits.
- The emergency telephone is in the hallway (dial 911 for emergencies).
- For non-emergency issues dial 8-5454 to contact campus police.
- All accidents or injuries, however slight, must immediately be called to the attention of the laboratory instructor who will advise the student on the course of action to be taken.
- Unauthorized experiments are prohibited. Follow the laboratory directions carefully.
- Food and beverages are prohibited in the laboratory.
- Do not remove any laboratory equipment or supplies from the laboratory.
- Never work in the laboratory without supervision.
- Never look directly into a laser beam or point a laser beam at someone else.
- Use care when handling electrical equipment in order to avoid electrical shock.
- Notify the instructor in case of damaged power cords or other problems with electrical equipment.
- Careless acts are prohibited.

I have read and understand these rules and agree follow them to the best of my ability.

Student Name:

Student Signature:

CWID:

Course Number:

Date:

Electron Diffraction

3.1 Introduction

The purpose of this lab is to observe the wave properties of electrons, verify the de Broglie relation, and determine lattice spacings in graphite carbon.

3.2 Equipment Description and Setup

The electron diffraction tube comprises a “gun,” which emits a narrow converging beam of electrons within an evacuated clear glass bulb on the surface of which is deposited a luminescent screen. Across the exit aperture of the “gun” lies a micro-mesh nickel grid onto which has been vaporized a thin layer of graphitized carbon; the beam penetrates through this carbon “target” to become diffracted into two rings corresponding to separations of the carbon atoms of 0.123 and 0.213 nanometers. The source of the beam of electrons is an indirectly-heated oxide-coated filament inside the cathode structure, the heater of which is connected to 4mm sockets at the rear of the tube, labeled “+ FIL” and “- FIL”. The electrons are accelerated by a potential difference between the cathode and anode of the gun. The ground of the accelerating voltage is connected to the “Cathode” socket at the rear of the tube and the positive voltage is connected to the socket at the top of the tube. Figure 3.1 shows annotated photos of the diffraction tube.

The electron diffraction tube is connected to a high voltage supply, a bias voltage supply, and a low voltage supply (Figure 3.2). To avoid burning out the graphite grating or the filament, an interface is provided at the rear of the tube for making the required connections. Do not turn on any power supply until the connections have been checked by the instructor.

1. Connect the low voltage power supply with the positive terminal connected to the socket labeled +FIL and the negative terminal connected to the socket labeled -FIL.
2. Connect the positive terminal of the external bias power supply to the +FIL socket and connect the negative terminal of the bias power supply to the socket labeled Cathode.
3. The positive potential (anode potential) of the high voltage tube should **already be connected** to the terminal on top of the tube (if not, **see the instructor**) and there should be a banana cable connection branching out from this cable; attach this banana cable to the socket labeled Cathode.
4. Connect the socket labeled Cathode to earth ground, which should be available on the bias power supply.

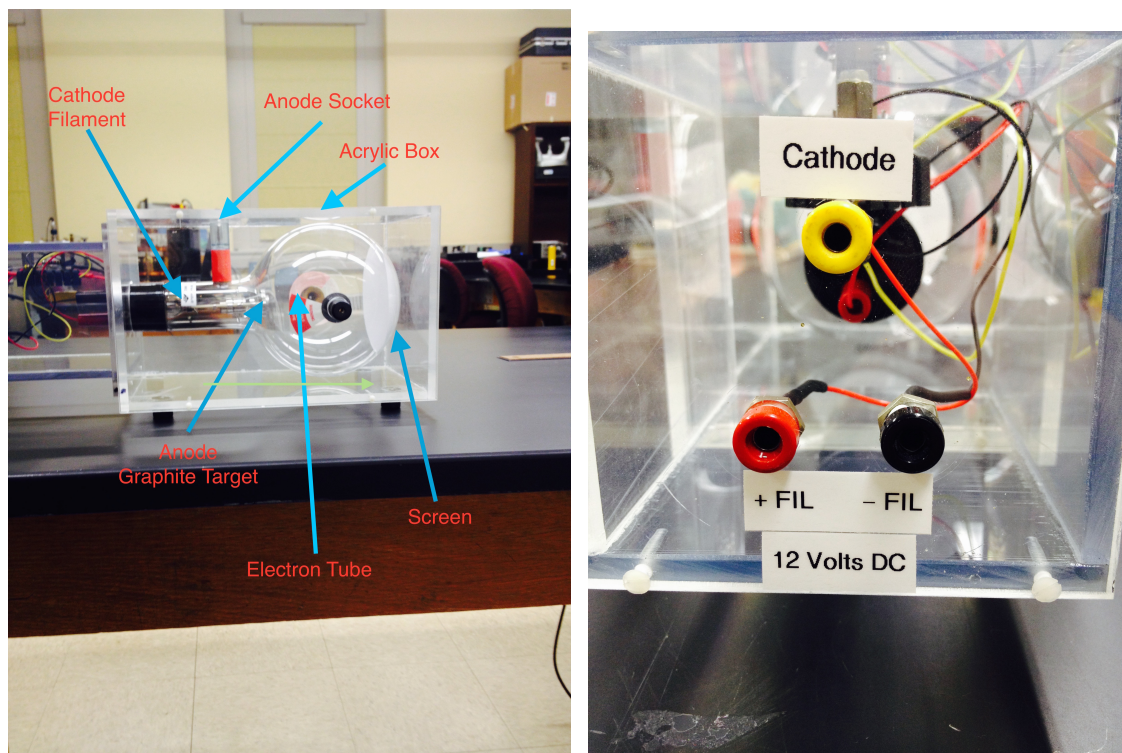


Figure 3.1: Side view (left) and rear view (right) of the electron diffraction tube

External Bias: This bias is used to focus the rings which are observed on the end of electron diffraction tube. This bias should be initially set at approximately 40 volts and then reduced as needed. You will observe that decreasing the external bias will increase the anode current. (See comment below regarding maximum anode current.) The paragraph below on target protection explains how the external bias works.

Anode Voltage: This voltage accelerates the electrons. During the experiment it will be varied between 2500 and 5000 volts. **AS A SAFETY PRECAUTION, THE LEAD CONNECTING THE HIGH VOLTAGE POWER SUPPLY TO THE ELECTRON DIFFRACTION TUBE SHOULD NEVER BE DISCONNECTED.** After turning on the high voltage power supply, raise the voltage gradually to 5000 volts, noting the anode. The instructor will show you how to precisely monitor the anode current and voltage; see also the notes below on the Bertan power supply, which is used as the high voltage power supply. **THE ANODE CURRENT SHOULD NEVER EXCEED 200 μA .** As mentioned above, you can use the external bias to reduce the anode current.

Protection of the Carbon Target:

The graphitized carbon through which the electron beam is confined to pass is only a few molecular layers in thickness and can be punctured by current overload. The external bias voltage is used to control the electron beam current as follows. The cathode is essentially a can which surrounds the filament and which has a small opening (exit aperture) on the anode side. An internal resistor connects one lead of the filament to the cathode. Recall that you connected the power supplies

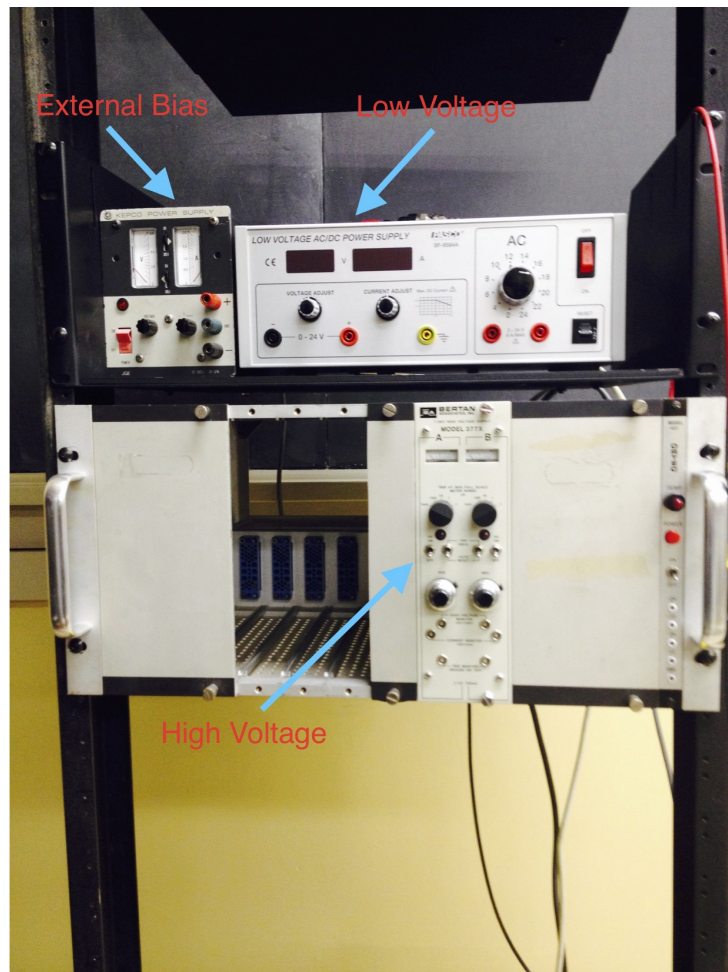


Figure 3.2: Power supplies used for the electron diffraction experiment

so that the cathode and negative terminal of the bias are tied to earth ground, which means that they are held at an absolute potential of 0 V. The low voltage power supply maintains a constant potential difference between the two filament leads. The positive terminal of the external bias sets its end of the filament to an absolute potential equal to its output potential. For example, when the external bias is set at 40 V, the voltage of the filament is about 40 V, which is positive compared to the 0 V potential of the cathode. Therefore, when an electron is thermally ejected from the filament, it immediately experiences a retarding force acting upon it towards the filament. At a potential difference of 40 V, practically all thermally ejected electrons are stopped before they can reach the cathode and therefore no electrons escape the cathode through the hole to be accelerated toward the anode. The voltage applied through the external bias is sometimes called a “negative bias” or “reverse bias.” In order to produce an observable current of electrons being accelerated between the cathode and anode, one must lower the external bias far enough so that some electrons are able to reach the cathode exit aperture before being stopped.

Notes on Bertran HV supply.

1. The NIM bin must be on for the supply to work.
2. Dual channel. You need only one of them, A or B. The output connectors are in the back.
3. Voltage is set with dial, 1000 V per full turn.
4. Set voltage to zero before enabling output using toggle switch.
5. There are two LEMO outputs, one for reading the voltage setting and the other for the current being drawn. Suggestion: set voltage coarsely using dial while monitoring current. Then connect voltage readout to DVM for fine voltage setting. Always monitor current while making major changes to the HV level or to the external bias.

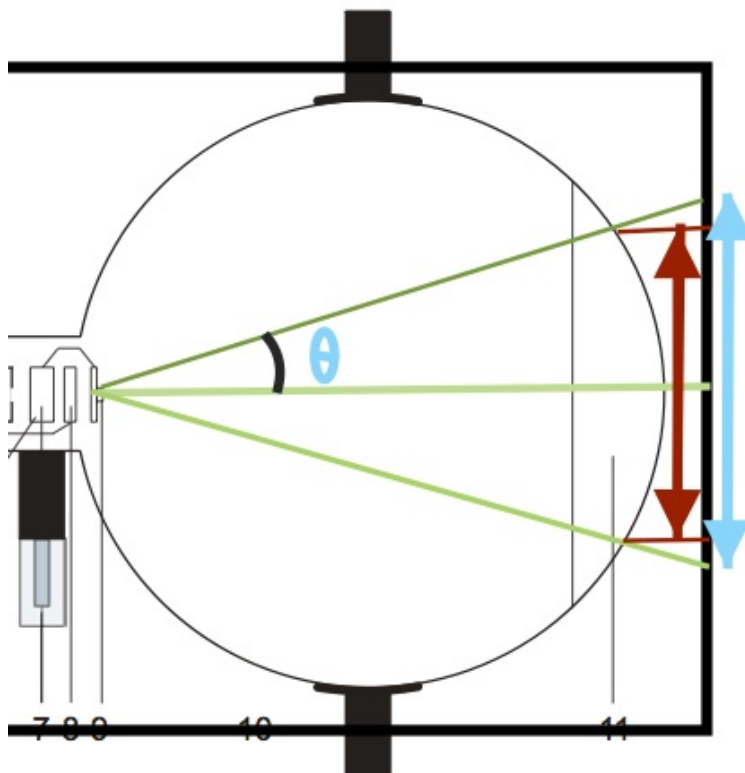
3.3 Procedure

Connect the tube electron tube into the power supplies as described above. **After the instructor has checked the wiring**, first turn on the low voltage supply, set it to 12 V, and wait a few minutes for the filament (which should begin to glow) to stabilize. Second, turn on the external bias power supply and set it to 40 V. Third, turn on the high voltage power supply and turn up the anode potential to 5000 V, monitoring the anode current as you do so. Finally, decrease the external bias potential (perhaps to about 20 V) until the image of two wide rings and a central dot is clearly visible on the screen.

First, measure the ring diameters while the anode voltage is at 5000 volts. Then reduce the anode voltage to 4500 volts, refocus using the external bias, and measure the ring diameters for that voltage. As the rings are fuzzy, you should measure the inside diameter and outside diameter of **each ring** - 4 diameters in total. Continue in this fashion, reducing the anode voltage in steps of 500 volts until 2500 volts is reached. You may find it useful to turn off the room lights or use

a black cloth as a hood in order to improve your measurements. If you do decide to turn off the lights, first check with other teams working in the room.

The distance from the graphite target to the front of the acrylic box is approximately 125 mm and the radius of curvature of the fluorescent screen is approximately 50 mm. Knowing this, you can calculate the extrapolated diameter (blue double arrow) from the measured diameter (red double arrow). You will calculate the extrapolated diameter in your analysis and then use it to calculate the electron diffraction angle θ .



3.4 The de Broglie Hypothesis of Electron Diffraction

Louis de Broglie hypothesized that particles of momentum p would exhibit the properties of waves with wavelength λ given by

$$\lambda = \frac{h}{p}$$

where h is Planck's constant. In our experiment, the electrons of charge e are accelerated through a potential different V . Approximating the initial momentum of the electrons as zero and applying the law of conservation of mechanical energy,

$$p = \sqrt{2meV}$$

where m is the mass of the electron. The observed diffraction rings correspond to first-order interference maxima satisfying the condition for interference maximum:

$$d \sin \theta = \lambda$$

where d is the lattice spacing corresponding to the maximum. In the small angle approximation

$$\sin \theta = \tan \theta$$

In terms of the distance L from the graphite target to the outside surface of the acrylic box and the ring diameter D extrapolated to the outside surface of the acrylic box,

$$\tan \theta = \frac{D}{2L}$$

Equating the two expressions for λ ,

$$\frac{h}{\sqrt{2meV}} = \frac{dD}{2L}$$

In carrying out the experiment, d and L are set in the construction of the apparatus, you choose and set an accelerating voltage V , and then you measure D . We can solve the above relationship for D in terms of d , L , and V , yielding a prediction for D in terms of experimental parameters.

$$D = \frac{\sqrt{2}Lh}{d\sqrt{me}} \frac{1}{\sqrt{V}} \equiv \alpha \frac{1}{\sqrt{V}}$$

where

$$\alpha \equiv \frac{\sqrt{2}Lh}{d\sqrt{me}}$$

In other words, according to the de Broglie hypothesis applied to this experiment, D should vary linearly with $1/\sqrt{V}$ with slope α .

3.5 Analysis and Discussion

1. For each ring (inner, outer) and accelerating voltage, average your measurements of D . Tabulate the results as a function of the accelerating voltage.
2. Calculate the extrapolated ring diameter from the measured average ring diameter. Explain your method.
3. What are the significant sources of uncertainty in the determination of the ring diameters. Make a quantitative estimate of the uncertainty.
4. For each ring, plot the extrapolated diameter versus the inverse square root of the accelerating voltage.

5. Fit a straight line to each plot. Compare the quality of the fit to the prediction that the experimental data should lie along a straight line. If the prediction is consistent with the data, further compare the fitted slope of the straight line with the predicted value for the slope.

Speed of Light

Choose one of the two experiments to measure the speed of light.

4.1 Experiment 1

For the Foucault method Pasco apparatus, we will use the Pasco manual, found online at:

https://www.pasco.com/file_downloads/Downloads_Manuals/Basic-Speed-of-Light-Apparatus-Manual-0S-9262.pdf

4.2 Experiment 2

4.2.1 Introduction

This experiment represents a direct measurement of the speed of light by dividing the distance traveled by the light by the time required for the trip. A diagram of the apparatus is shown in Figure 4.1. The beam from a laser amplitude-modulated at 1 MHz is directed to a mirror or a series of mirrors so the beam will travel over a distance D before it is incident on a photodiode. The output of the photodiode is connected to one input of a dual beam oscilloscope so the 1 MHz waveform can be observed. The beam from the laser is directed through a beam splitter so a reference beam can be directed over a very short distance d to a second photodiode. The second diode output is connected to the second input of the dual channel oscilloscope so two 1 MHz waveforms can be observed simultaneously. There should be a shift in time Δt between the two 1 MHz signals, and this time difference represents the difference in time required for the laser beam to traverse the long and short light paths. Thus the speed of light can be calculated from the equation

$$c = \frac{D - d}{\Delta t}$$

4.2.2 Experimental Setup

The experimenters have some choice in how to set up the equipment. They can choose the long path D by using one or two mirrors and they can choose the short path length d by position of the beam splitter. Because the beam will diverge over a long distance, a lens should be used to focus the beam on the photodiode. Some time will be required to adjust the mirrors so the two laser beams will fall on the two photodiodes. The light path adjustments should be continued until strong signals (around 0.5 V - 1 V peak to peak), are observed on the oscilloscope.

Use the ring stands and lab jacks to set different parts of the apparatus to the similar heights. Handle the mirrors only on the edges. It may be necessary to clean the mirrors, which should be done carefully, as the mirrors are coated on their front surfaces.

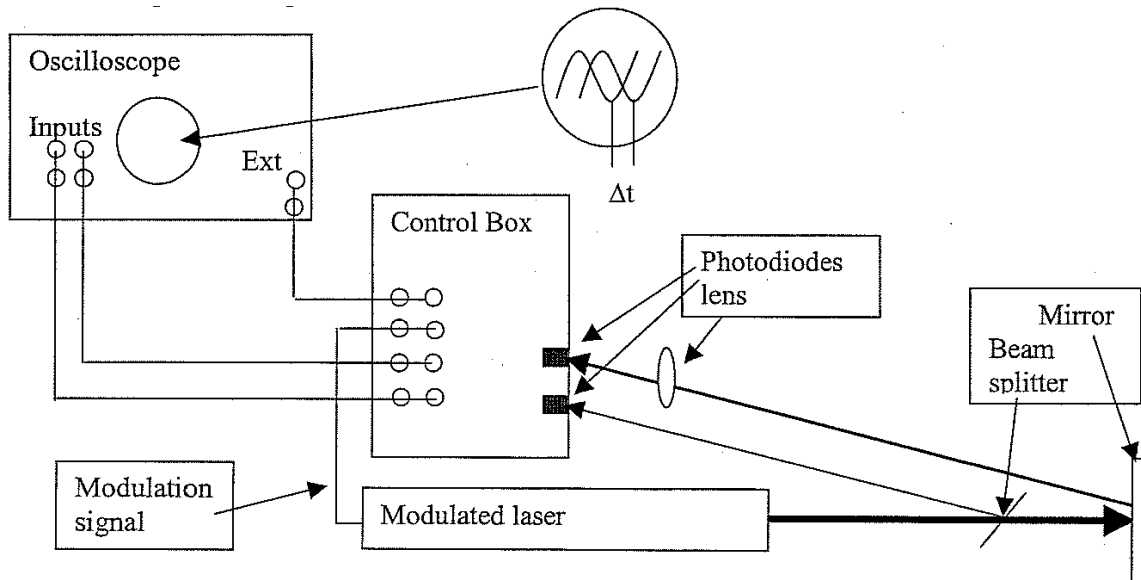


Figure 4.1: Schematic of apparatus

The photodetector is powered by a 9 V battery on a circuit board, which is attached to the top panel of the photodetector. If the battery needs replacing, remove the corner screws to lift the top panel out.

Make sure the switch next to the amplifier on the photodetector is set to OFF.

Figure 4.2 and figure 4.3 show photos from previous setups of the experiment.

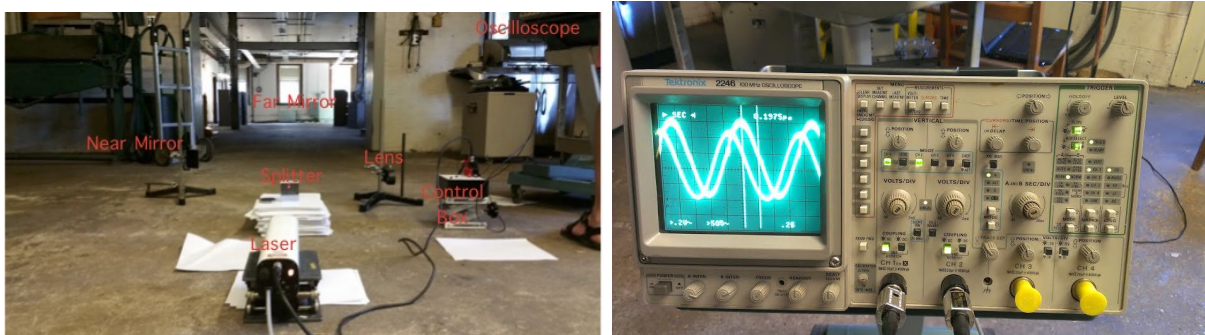


Figure 4.2: Left: Overview photo of setup. Right: Oscilloscope traces for a particular pair of beam paths

Note that the connection between the control box and the external trigger on the oscilloscope should not be made; rather trigger the oscilloscope on the photodiode output for the path. The instructor will show you how to use the oscilloscope.

4.2.3 Experimental Procedure

After the equipment setup is complete, the time difference can be measured directly on the oscilloscope. Different oscilloscope sweep times should be used and the Δt measurement should be made

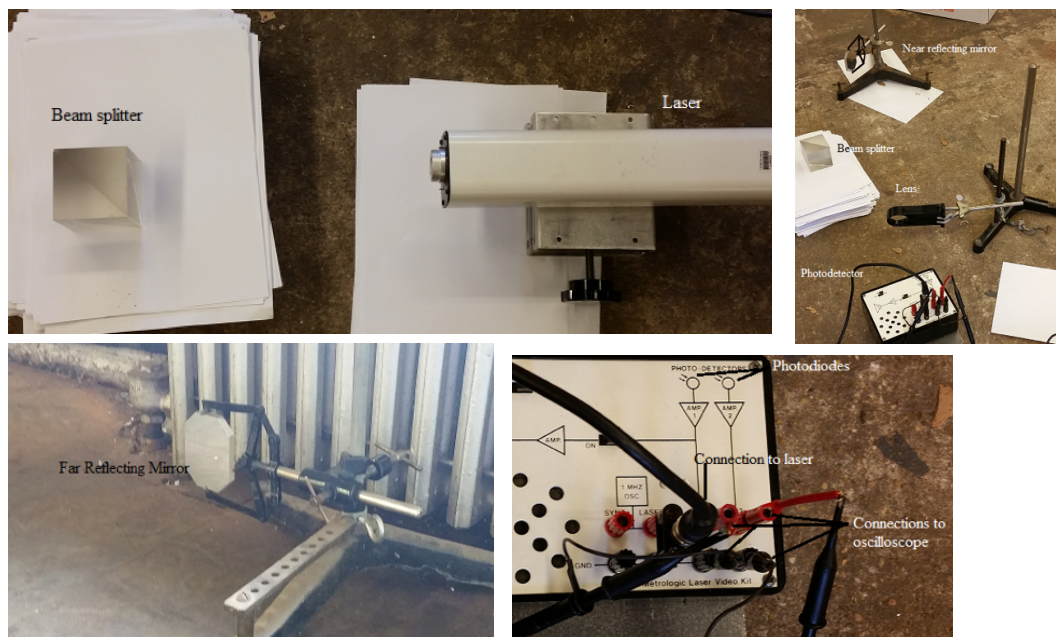


Figure 4.3: More photos showing the setup of the apparatus. Clockwise from top left: laser and beam splitter; beamsplitter, lens and photodetector; closeup of photodetector showing connections to oscilloscope; far reflecting mirror.

several times for each sweep time. After measurements are made at one distance D , this distance should be changed and the experiment repeated.

4.2.4 Analysis and Discussion

1. Average your measurements of Δt for each sweep time and value of D . Tabulate the results.
2. Estimate the uncertainties in the measurements of d , D , and Δt . In the case of Δt , discuss the uncertainty of the individual measurements as well as the uncertainty after averaging. Explain how you got your estimates.
3. Plot $D - d$ versus Δt (averaged) and fit a straight line to it, requiring that the intercept be zero. Compare the slope with the accepted value for the speed of light. Discuss any difference between the measured and accepted values in terms of the experimental uncertainties.

Counting Statistics

5.1 Hypothesis

Gamma ray emission from a radioactive source occurs as random, independent events. As such, the distribution of emission times should follow Poisson statistics. For a large number of decays and a small decay probability, the statistics should therefore be approximately Gaussian.

5.2 Introduction

No matter how many measurements of a quantity we make, we will never know its true value. If we make a large number of measurements under nominally identical conditions, then the average of this collection of measurements gives us an estimate of the true value. We might logically expect that the more measurements we make, the better our estimate of the true value. In some cases, the underlying statistics of the randomness in the measurements allows us to determine how far our estimate is from the true value. This is the case for radioactive decay, where the probability for decay λ is the same for all identical atoms in the sample of interest. This makes radioactive decay an example of a sequence of independent random events, in which the occurrence of any event has no effect on the occurrence of any other. Repeated measurements of independent, random events occurs often in physics, and the goal of this laboratory is to learn how to analyze such experiments.

Since the decay of an unstable nucleus is entirely random, we cannot predict when a particular atom will decay. Because it is equally likely to decay at any moment, however, we can predict the mean number of atoms which have decayed after a prescribed amount of time. If the probability per unit time of a given atom decaying is λ , then the mean number of original atoms remaining after a time t out of an original population of $N(0)$ atoms is governed by an exponential decay:

$$N(t) = N(0)e^{-\lambda t} \quad (5.1)$$

Closely related to the decay rate above is the *half life* of the isotope $t_{1/2}$, the amount of time on average it takes for half of a given sample of radioactive material to decay. From the exponential decay, it is clear that

$$t_{1/2} = \frac{\ln 2}{\lambda} \quad (5.2)$$

If a particular isotope has a decay probability of $\lambda = 1 \times 10^{-5} \text{ s}^{-1}$, meaning any given atom has a probability of 1 in 10^5 of decaying during any given second, this implies a half life of $t_{1/2} = 6.93 \times 10^4 \text{ s}$ (19.25 hr). The exponential decay is only an approximate solution for real radioactive decay for two primary reasons. First, the exponential function is continuous, while the number of atoms remain-

ing can only take on integer values. Second, since the decay process is random, the exponential decay is only statistically true. We only expect the exponential behavior to hold for an extremely large population of atoms. Since we are normally dealing with very large numbers of atoms, of the order of Avogadro's number, the approximation is usually quite good.

The analysis above applies to a particular sort of experiment, one in which we simply measure the number of non-decayed atoms as a function of time. The present experiment is somewhat different, in that we will make repeated measurements of the number of decay events occurring within a certain fixed amount of time. If the measurement time is small compared to the half life, then the number of decays observed will be small compared to the total number of atoms present, so we may assume that the number of non-decayed atoms in the sample is not changing significantly over the course of our measurement. On the other hand, the decay *rate* should be constant in this case, with the number of decays in a given fixed amount of time distributed about a characteristic mean value. What we wish to establish is that their distribution is governed by the statistics of independent random events.

How can one judge whether a certain process really has a steady rate on time scales of the experiment? There is only one way: make repeated measurements of the number of counts x_i in a time interval t_i , and see if there is a trend in the successive values of x_i/t_i . These ratios are sure to fluctuate, since we are inherently dealing with a random variable, and the question is whether the observed fluctuations are consistent with a fixed steady rate. To answer this question, we will need the expected distributions for a truly random process.

5.3 Binomial, Poisson, and Gaussian Statistics

From the decay probability per unit time λ and the fact that individual decays are independent events, we can calculate the probability that out of a sample of N atoms, we measure x decays per unit time. We do this in the same way we would calculate the odds when throwing dice or tossing coins, using the binomial distribution. If the probability per unit time of decay is λ , the probability per unit time of no decay is $1 - \lambda$. Putting these two facts together, the probability per unit time of x decays from N atoms isⁱ

$$P_\lambda(x|N) = \binom{N}{x} \lambda^x (1 - \lambda)^{N-x} = \frac{N!}{x! (N-x)!} \lambda^x (1 - \lambda)^{N-x} \quad (5.3)$$

Though this an exact expression, it is unwieldy and computationally useless given the appearance of numerous factorials. For a very large collection of atoms and a small decay probabilityⁱⁱ, we can take the limit of the binomial distribution[1] as $N \rightarrow \infty$ and obtain the somewhat friendlier Poisson distribution:

ⁱRecall that $\binom{n}{k} = n!/k!(n-k)!$.

ⁱⁱSay, of order $\lambda N \lesssim 10$.

$$P_{\mu}(x) = \frac{\mu^x}{x!} e^{-\mu} \quad \text{Poisson distribution} \quad (5.4)$$

Here μ is the expected number of decays per unit time (the population mean), x the number of decays per unit time actually observed, giving $P_{\mu}(x)$ as the probability for measuring x decays per unit time if μ are expected. The population mean μ is related to the decay rate and number of atoms by $\mu = N\lambda$. The use of the Poisson distribution is justified when the number of events per unit time is steady over the course of the measurement. On the other hand, if our experimental data is found to obey Poissonian statistics, we can be reasonably sure that we are observing a process governed by a steady decay rate.

Though an improvement, the Poisson distribution remains unwieldy and restricted to sets of integers, owing to the appearance of $x!$. If the true number of decays per unit time μ is largeⁱⁱⁱ, the Poisson (or Binomial) distribution can be approximated by the normal (Gaussian) distribution:

$$P(x) = \frac{1}{\sqrt{2\pi\mu}} e^{-(x-\mu)^2/2\mu} \quad \text{Gaussian distribution} \quad (5.5)$$

This is part of the Central Limit Theorem, one of the most important results in all of statistics. Crucially, the x in the Gaussian distribution can be treated as a continuous variable if necessary, while the x in the Poisson distribution must be an integer.

The usual way of writing the normal (Gaussian) distribution is it in terms of the mean μ which defines the center of the distribution, and the standard deviation σ which defines the spread of the data. The relationship between μ and σ for a random process is simply $\sigma = \sqrt{\mu}$, giving

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad \text{Gaussian distribution} \quad (5.6)$$

In the normal distribution, it can be shown that approximately 68% of the observations lie in the interval $[\mu - \sigma, \mu + \sigma]$, 95.5% in the interval $[\mu - 2\sigma, \mu + 2\sigma]$, and 99.7% in the interval $[\mu - 3\sigma, \mu + 3\sigma]$.

Although a true normal distribution is defined in terms of the true mean μ and the standard deviation σ , we cannot determine either of these quantities exactly with a finite number of measurements, we can only measure better and better approximations. In practice, we replace μ by the observed mean \bar{x} , defined as:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad (5.7)$$

where N is the total number of observations and x_i are the individual observations. We replace σ by the sample standard deviation s , defined as:

ⁱⁱⁱSay, $\mu > 100$.

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2} \quad (5.8)$$

When only a single observation x is made, the sample standard deviation may be determined by $s = \sqrt{x}$. Thus, the mean error for *any single result or data point* x is \sqrt{x} and the fractional error is given by $1/\sqrt{x}$. This is a very important result for analyzing experimental data. As a practical point, now you know how to put error bars on a frequency distribution.

The standard deviation is a measure of the random statistical uncertainty in the measurement, and it becomes part of the experimental error associated with physical measurement. If you have measured an average count to be 695 with $s=26$, and another experimenter has measured a count of 680, then their count agrees with your count, within the experimental error. Nothing is made of the difference between the values 695 and 680 because, in all probability, the two results are the same. A related quantity is the *probable error* P is defined to be that deviation which is expected to be exceeded in half of the observations. For a normal distribution, P is given by

$$P = 0.6745\sigma \quad (5.9)$$

We can also estimate the relative uncertainty of the measured *mean* of a number of measurements, a quantity known as the *standard deviation of the mean* $\sigma_{\bar{x}}$. This quantity tells you that if you measure x repeatedly, the sample mean \bar{x} has an uncertainty $\sigma_{\bar{x}}$ compared to the true mean μ given by:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} \quad (5.10)$$

For an estimate, σ can be approximated by s , $\sigma_{\bar{x}} \approx s/\sqrt{N}$. Based on your observations, you can expect 68% of any measurements of x made in the same way will have a mean of $\bar{x} \pm \sigma_{\bar{x}}$. Put another way, if you are primarily interested in the average value of x , then $\sigma_{\bar{x}}$ tells you the uncertainty in that average with respect to repeated measurements^{iv}. The standard deviation of the mean is the typical manner of reporting average quantities with statistical uncertainty:

$$(\text{best value of } x) = \bar{x} \pm \sigma_{\bar{x}} \quad (68\% \text{ confidence}) \quad (5.11)$$

Putting this all together, if we take N measurements for our sample, the frequency with which we would expect to measure x decays would be, using sample mean and standard deviation:

$$\text{frequency}(x) = \frac{N}{s\sqrt{2\pi}} e^{-(x-\bar{x})^2/2s^2} \quad (5.12)$$

^{iv}Remember that the standard deviation σ tells you that 68% of subsequent measurements will fall within $\pm\sigma$ of the mean \bar{x} , whereas the standard deviation of the mean tells you that a *collection* of measurements has a 68% chance of yielding a mean of \bar{x} .

“Frequency” here is the number of times that x is the result of your measurement, if you take N measurements with a mean value of \bar{x} . The data in Fig. 5.1 below are compiled from 5120 consecutive measurements of the number of 662 keV gamma rays emitted in 40 ms measurement intervals from a ^{137}Cs source. The frequency of the measured rates is well-described by a normal distribution, which can be determined from only the sample average and standard deviation. The data here have been sorted in to ‘bins’ of width 2 counts.[1] The ‘binning’ process just means that rather than tallying the frequency of exact numbers of counts, we would tally the frequency of observations falling within fixed-width ranges, known as ‘bins.’ In this case, we counted the frequency of observations falling in the range $[60, 62]$, $[62, 64]$, etc. Another way of viewing the binning process is as a sort of ‘rounding’ – for a bin width of 2, we round downward to the nearest multiple of 2. A bin width of 5 would mean rounding down to the nearest multiple of 5, and a bin width of 1 just means rounding downward to the nearest integer.^v

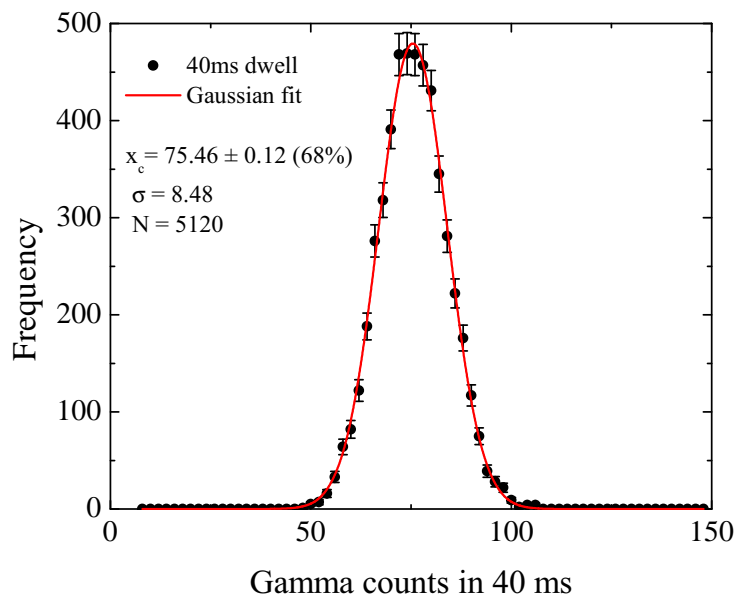


Figure 5.1: Gaussian fit to data with an average of $\bar{x}=75.5$ counts in 40 ms. The sample standard deviation is $s=8.48$ counts, in good agreement with $\sqrt{\bar{x}}=8.7$. The sample standard deviation of the mean is $s_{\bar{x}}=0.119$, so our average count rate is reported as $\bar{x}=75.5\pm 0.12$. (The counts are sorted in to ‘bins’ of width 2 for this analysis.)

5.4 Objective

In this experiment you will explore the statistics of random, independent events in physical measurements. The random events used in this study will be pulses from a scintillation detector exposed to gamma rays from a radioactive ^{137}Cs source. The ^{137}Cs parent nuclei undergo beta decay to create

^vOf course, this ‘rounding’ process is only for constructing the frequency distribution plot, we do not actually round the data – that would be throwing away data! We merely sort it and construct a frequency table based on intervals determined by the bin width. If we actually rounded it, we could never choose a smaller bin size after that...

excited ^{137}Ba daughter nuclei, which relax to their ground state by emitting 662 keV gamma rays. The gamma ray emission from the excited state of ^{137}Ba occurs randomly, as does the decay of its parent isotope, ^{137}Cs . Since the 30 year half-life of ^{137}Cs is continually replenishing the short-lived ^{137}Ba , this isotope's activity will be nearly constant for far longer than a lab period, and will suffice for a few hours of measurement.

The scintillation detector software control will allow a measure the source activity 1,024 times for a specified duration known as the “dwell time.” From these measurements, you can construct a frequency histogram and compare with Gaussian or Poisson distributions. Dwell times (i.e., the measurement time interval) can be changed to change the average number of counts with the dwell period for further comparison with theoretical predictions. The overall goal is to become familiar with the statistical analysis of random uncertainties.

5.5 Preparatory Questions

Include your responses to these questions in your report.

1. Describe how a scintillation counter works, starting from the entrance of an energetic charged particle or photon, and ending with an electrical pulse at the output of the photomultiplier. Why are subsequent signals independent?
2. Suppose the mean counting rate of a certain detector of random events is 2.1 counts per second. What is the probability of obtaining zero counts in a one-second counting interval? What is the most likely interval between successive pulses?

5.6 Relevant Reading

Taylor[1], Ch. 4, 5, 11

Pfeffer & Nir[2], Ch. 5.3.1-3, 5.5.1

5.7 Supplies

1. Spectech UCS30 spectrometer
2. NaI(Tl) detector
3. Multi-channel analyzer (MCA) / control electronics
4. Pb cylinder for shielding
5. ^{137}Cs source (check out from locked cabinet)
6. detector holder and shelf housing
7. lab notebook and USB drive for saving spectra

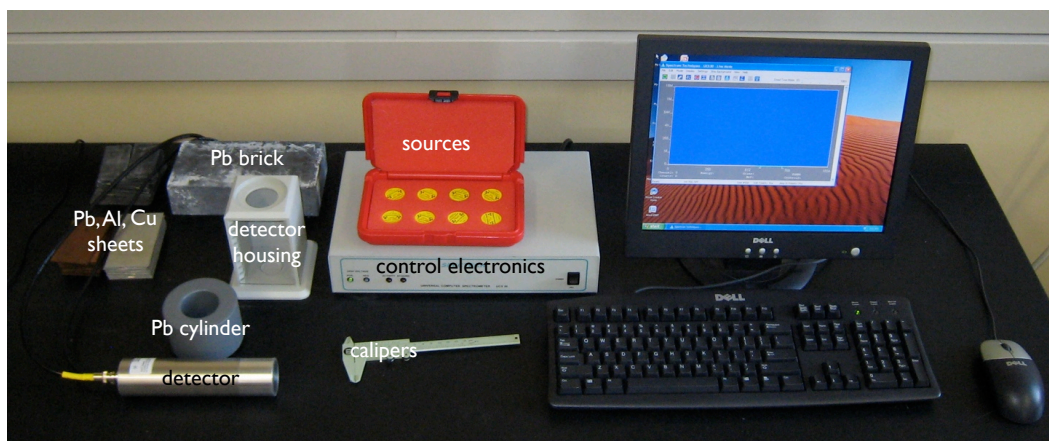


Figure 5.2: The Spectech UCS30 spectrometer and accessories

5.8 Suggested procedure

5.8.1 Startup

1. Turn on the power to the Spectech UCS30
2. Run the program “UCS USB” from the shortcut on the desktop
3. Place the ^{137}Cs source on top of the table, and place the Pb cylinder around it
4. Place the cylindrical NaI detector inside the Pb cylinder
5. Run the energy auto-calibration from “Settings/Energy Calibration/Auto Calibrate” - this will take several minutes. The final spectrum should look like the one in Fig. 5.3.
6. Once the calibration is finished, click on the “X” to clear the spectrum, and you are ready to set up the experiment and begin taking data.

5.8.2 Setting up Pulse Height Analysis (PHA) mode

1. Under the “Mode” menu, select “PHA (preamp in).” This should be the default value.
2. Open the detector settings (Settings, High Voltage/Amp/ADC).
3. Adjust the Discriminator Units of Channels settings to select only the 662 keV gamma peak. The values are in channels where the maximum channel is 1024; about 590 and 730 for the Low and High values respectively should work well (the peak should be at about channel 660).
4. Run a spectrum using the green diamond “go” button and verify that you are only scanning the 662 keV peak. Once verified, stop counting with the red octagon’ “stop” button and clear the spectrum with the “X”.
5. Under the “Mode” menu, select ”MCS (Internal).”
6. Under Settings/MCS/Dwell, choose a dwell time. This is the time interval over which the counts within the detection limits are measured.

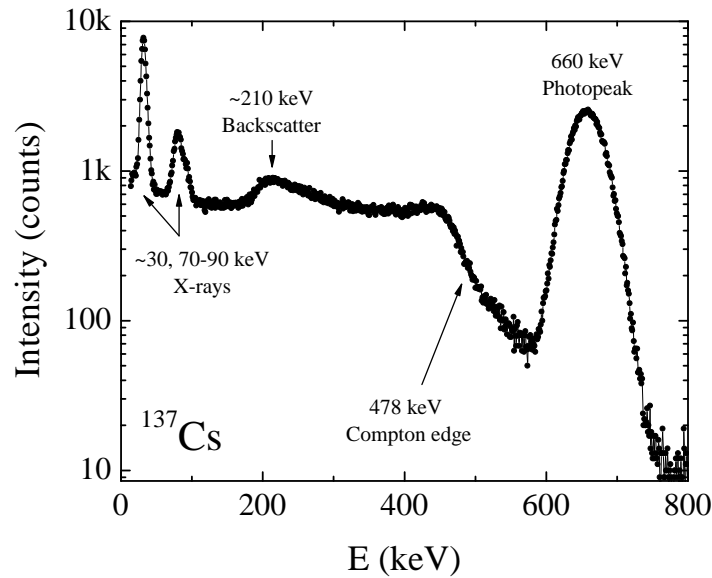


Figure 5.3: Gamma spectrum of ^{137}Cs . Aside from the main photopeak at $\approx 660\text{ keV}$, clear Compton edge and backscatter features are visible, as well as X-ray absorption edges. Note the logarithmic scale on the vertical axis.[3]

5.8.3 Running the experiment

Once a dwell time is selected, running a spectra (with the “go” button) will make 1024 measurements of the total number of counts within the detection region, with each measurement as long as the specified dwell time. Do this, and save the resulting data (name it sensibly!). **Be sure to save the file in comma-separated value (CSV) format so you can read it in, e.g., Excel later!**

In this experiment, you will want to vary the dwell time such that you have a total number of counts varying by a factor of 5-10. The goal is to accumulate data at three different dwell times, corresponding to three different mean counts per dwell time, *such that the total amount of acquisition time at any dwell time is the same*. Thus, if your first spectra is at a dwell time of 200 ms, for 1024 points your acquisition time is 204.8 s. Your second data set at, e.g., 100 ms would require two separate spectra to have the same amount of total acquisition time, since 1024 points at 100 ms corresponds to 102.4 s, or half the measurement time as your first spectra. Though you are free to choose your own parameters, a suggested plan would be

- 1 spectrum at 200 ms dwell time
- 2 spectra at 100 ms dwell time
- 5 spectra at 40 ms dwell time

During your data analysis, you will combine the 5 spectra at 40 ms dwell time to make one data set which can be compared with the single spectra acquired at 200 ms dwell time. It is important

that you run the spectra for each dwell time in order and in quick succession. The total acquisition time, once the experiment is set up, should take one hour or less, which should leave time to begin analysis during the laboratory period. *The data analysis for this experiment is longer than average.* However, the data acquisition itself is more straightforward than usual.

5.9 Data analysis

Though it is somewhat cumbersome for our purposes, we suggest that you use MS Excel for your analysis, though you are free to use any statistical/analysis software package you are comfortable with (e.g., Originlab, Sigmaplot, etc.). See your instructor for assistance.

For each of your runs with different dwell times, import all of the data (perhaps with a new worksheet or file for each dwell time to avoid confusion). If you have 5 measurements at a dwell time of 40 ms, for instance, import all 5 data sets and paste them sequentially into a single set of columns to make one large data set.^{vi} If you have followed the procedure correctly, the product of dwell time and the number of points should be a constant for each cumulative data set (e.g., 5 sets giving 5120 points at 40 ms, 2 sets giving 2048 points at 100 ms, and one set giving 1024 points at 200 ms, all giving 204.8 s acquisition time). A cursory qualitative analysis can be performed by simply plotting the number of counts versus the acquisition time or measurement number. Does the data seem to be evenly distributed about a mean, or is a trend visible? Indicate the mean in your graph as a horizontal line.

As a first level of quantitative analysis, you should do the following:

1. Compute the sample mean \bar{x} and sample standard deviation s . Both are built-in functions in Excel.
2. Compare s to σ calculated by $\sigma = \sqrt{\mu}$ with $\mu \approx \bar{x}$. Calculate and comment on the numerical difference between s and σ , e.g., which is larger, and why.
3. Compute P , Eq. 5.9.
4. Count the number of times that the deviation $|x - \bar{x}|$ is greater than σ and show that this occurs for approximately 1/3 (strictly, 31.7%) of the observations. (A column of $|x - \bar{x}|$ and the function COUNTIF will be useful in Excel.)
5. Show that the deviation $|x - \bar{x}|$ is greater than P approximately 50% of the time.

Next, you should determine whether the count rate (or equivalently, the decay probability) is constant *quantitatively*. For each dwell time make a new column to find the cumulative average, defined as

^{vi}When pasting in the sets, be sure to add to the time column so it increases.

$$r_c(j) = \frac{\sum_{i=1}^{i=j} x_i}{\sum_{i=1}^{i=j} t_i} \quad (5.13)$$

where x_i is the number of counts detected in time t_i . Note that the sum in the denominator is just the time at which data point j was recorded after the start of the measurement, which is j times the dwell time (or the first column in your data file), and the numerator is the sum of the data from the first point through point j . For a truly random process with a steady mean rate μ , $r_c(j)$ should converge to μ in the asymptotic limit. Plot $r_c(j)$ and demonstrate that the cumulative average converges, and thus that the decay rate is constant over the course of the measurement.^{vii} Include error bars to demonstrate convergence.^{viii}

If you are satisfied that the decay probability is constant, construct a histogram of the number of times that a given x was observed versus x (similar to Fig. 5.1. It may be convenient to distribute the data into fixed-width bins as described above, and as demonstrated in the Excel template. Fit a Gaussian distribution to your data. If you use \bar{x} as an estimate of μ and s as an estimate of σ , a calculated Gaussian curve with the proper normalization condition should fit your data reasonably well (the normalization for your distributions can be found from the total number of readings). Comment on the accuracy of the fit.

5.10 Discussion and topics for your report

Are the decay *rates* consistent with the dwell times chosen? That is, do the rates scale with the decay time properly? Are the standard deviations of the rates for different dwell times related as expected?

For each dwell time, what are the 68% confidence limits for the number of counts?

Should you use the standard deviation σ_N or the standard deviation of the mean $\sigma_{\bar{N}}$ in comparing different data sets?

For the observed distribution with the lowest mean rate, take the highest deviation from that mean and test whether you would be justified in concluding that the detector was malfunctioning at that time. Do not forget to account for how many chances there were for such a deviation to occur . . .

^{vii}A way to double-check that the distribution is random about the mean is to calculate the reduced deviation, or z score: $z(x_i) = (x_i - \bar{x}) / \sigma$. Calculate $z(x_i)$ in a new column. If the deviations are random, then the sum of this column should be very close to zero.

^{viii}Recall that the error bar for a given point is just the square root of the number of counts, if the decay process is truly random.

5.11 Format of report

You may choose one of the following formats for your report:

1. two-page memo (excluding required plots)
2. group oral presentation
3. formal written lab report
4. research proposal

Further details and templates are provided for each of these formats. Keep in mind that over the course of all of the one-week experiments you must do each type of report.

A Quick Review of Standard Deviation

Suppose a series of measurements is made of the value of some unknown quantity. Usually these measured values will not all be the same. A statistical analysis of the measured values estimates the quantity and its variability. Analysis of the uncertainty determines the probability that the “true” value lies within a certain range. Of course, the percent difference between two measured values gives some idea of the range of measured values to be expected, but this is not a very reliable indicator. Whenever a measurement is reported, a determination of the reliability of a measurement is equally important to report. The mathematical methods used to determine statistical uncertainty are commonly referred to as error analysis.

A mathematically complete treatment of error analysis is beyond the scope of this document, and is a part of your required reading[1]. It is necessary to understand the basic methods of error analysis to properly report the results of the experiments. Some basic assumptions are necessary to estimate the uncertainties encountered in the measurements and analysis of data. First, it is assumed that differences in measurements are due to small random fluctuations that are just as likely to make the measurement higher as it is to make it lower. In some cases there is a systematic error which always makes a measurement smaller or larger than the “true” value. Examples of systematic errors include parallax in reading a meter stick, friction in balance or meter bearings, tightening of a micrometer screw too much, failure to account for air resistance, etc.

In well-designed experiments, systematic errors are accounted for, noted and measured. Under these conditions, a very large number of measurements of the same quantity should distribute themselves symmetrically about the simple arithmetic mean or average, which is the “best” value of the quantity. The expected variations of the measurements can be described by a quantity called the “standard deviation”

The standard deviation is computed in a straightforward manner. Suppose the quantity x is measured n times. The measured values are labelled x_1, x_2, \dots, x_n . First, we calculate the *mean*, or average of all the values, denoted \bar{x} . This is just as you would expect:

$$\bar{x} = \frac{1}{n} \sum_i^n x_i \quad (5.14)$$

Next, calculate the deviation of *each* measurement from the mean, $x_i - \bar{x}$ and square the result: $(x_i - \bar{x})^2$. We square the result so that deviations above and below the mean don't cancel each other out, but add to the overall deviation. Finally, add the squared deviations together, divide by the number of measurements n and take the square root of the result:

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (5.15)$$

A sneaky, and somewhat less tedious formula is given by

$$\sigma = \frac{1}{n} \sqrt{n \left(\sum_{i=1}^n x_i^2 \right) - \left(\sum_{i=1}^n x_i \right)^2} \quad (5.16)$$

A large standard deviation indicates that the data points are spread far from the mean and a small standard deviation indicates that they are clustered closely around the mean. The reported standard deviation of a group of repeated measurements should give the precision of those measurements. When deciding whether measurements agree with a theoretical prediction, the standard deviation of those measurements is of crucial importance: if the mean of the measurements is too far away from the prediction (with the distance measured in standard deviations), then the theory being tested probably needs to be revised. This makes sense since they fall outside the range of values that could reasonably be expected to occur if the prediction were correct and the standard deviation appropriately quantified.

According to sampling theory, there is a 68% probability that any additional measurement made of the quantity x will lie within $\pm\sigma$ of the mean and a 95% probability that it will lie within $\pm 2\sigma$ of the mean. In most of the experiments of this course, repeated measurements are performed about five or ten times. Using the above analysis for less than five independent measurements of a quantity is generally not considered to be statistically reliable.

The statistical treatment assumes that all errors are random and ignores any systematic error, such as improper meter calibration, wind resistance, etc. which might be present. In any laboratory situation, it is the responsibility of the experimenter to determine whether or not these systematic errors are significant and to include them in the estimate of accuracy if they are. On occasion, it will not be feasible to repeat an experiment several times and find a mean and standard deviation. It may be possible, however, to determine or estimate the uncertainties in the measurements of

the quantities used in computing the final result. For example, a manual measurement of a time interval is accurate to ± 0.1 second, due to reaction times in starting and stopping the timer.

^{137}Cs Decay and Gamma Emission

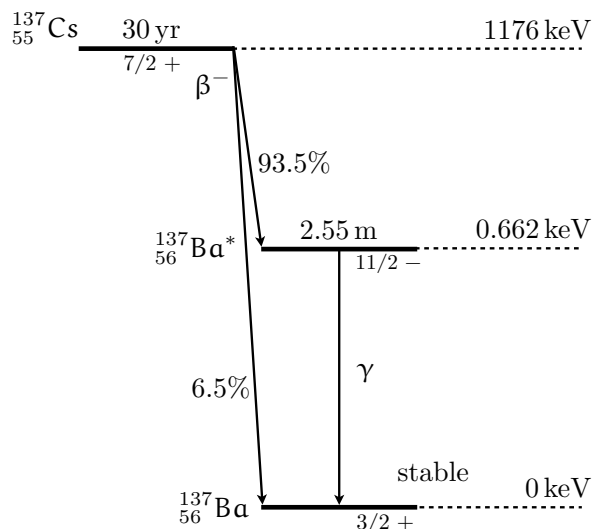


Figure 5.4: Energy level diagram showing the decay of ^{137}Cs to ^{137}Ba . The ^{137}Cs decays via two paths, 93.5% follow a two-step process resulting in the emission of a 662 keV gamma ray. The decay of the Ba nucleus from its excited state is an internal transition, and contributes to the decay percentages but not the gamma decay factor.[4]

A ^{137}Cs nucleus can decay via two routes to the ^{137}Ba ground state, shown schematically in Fig. 5.4. First, a single beta emission of 1.176 MeV can occur with no subsequent gamma emission, bringing the ^{137}Cs directly to the ground state. However, this single-step process occurs for only about 6.5% of all nuclei. The other 93.5% decay via two-step process. First, a 514 keV beta is emitted, resulting in metastable and short-lived $^{137}\text{Ba}^*$. The metastable $^{137}\text{Ba}^*$ subsequently (2.55 min half life) decays to the ^{137}Ba ground state by emission of a 662 keV gamma.

Though ^{137}Cs has only a single gamma emission^{ix}, there are x-ray emissions that can be observed in the energy range under study. We list a few of the higher energy emissions in Table 5.1.

Table 5.1: X-rays from ^{137}Cs

E (keV)	I (%)	Assignment	E (keV)	I (%)	Assignment
31.45	0.00026	Ba $K_{\alpha 3}$	36.38	0.68	Ba $K_{\beta 1}$
31.72	2.04	Ba $K_{\alpha 2}$	36.65	0.0079	Ba $K_{\beta 5}$
32.19	3.76	Ba $K_{\alpha 1}$	37.26	0.215	Ba $K_{\beta 2}$
36.30	0.35	Ba $K_{\beta 3}$	37.35	0.0481	Ba $K_{\beta 4}$

^{ix}Strictly speaking, there is a 283.53 keV gamma, but its intensity is about 10^5 times smaller than the 662 keV peak.

Gamma ray attenuation

6.1 Introduction

The term “gamma ray” (or γ -ray) is an approximate classification for photons of energies $E \sim 100 \text{ keV} - 10 \text{ MeV}$ ($f \sim 10^{19} - 10^{24} \text{ Hz}$; see Fig. 6.7 in the Appendix). When gamma rays pass through matter, their intensity may be attenuated by interactions with the matter just as with any other photons. Gamma rays have three primary mechanisms of interaction: photoelectric effect, Compton scattering, and pair production. If gamma rays are emitted in a narrow beam toward a block of intervening matter, these three interactions cause a loss of intensity along the beam direction, either by deflection along a different direction or absorption. These processes are shown schematically in Fig. 6.1 below.

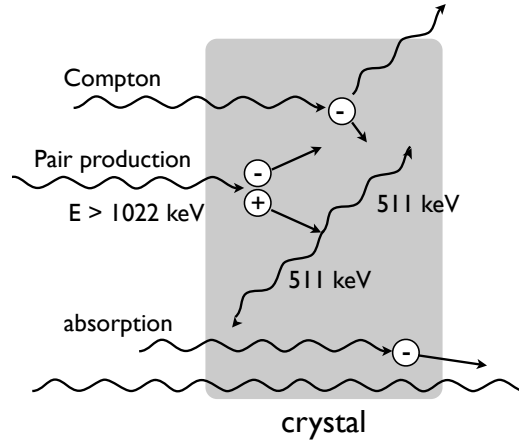


Figure 6.1: Gamma rays can interact with crystals in a number of ways: Compton scattering, pair production, absorption, and photoeffect (not shown). The combination of these effects is a dispersion in energy, as well as a deflection of many gamma rays from the original beam direction.

Thus, the interaction of high-energy X-rays and γ rays involves three different mechanisms, which we describe briefly below:

1. **Photoelectric effect:** the photons in the incident radiation are of high enough energy to release electrons from atoms and molecules in the absorbing or detecting material. Photons in the beam are thus absorbed, leading to a corresponding reduction in intensity. This photoeffect will lead to a peak in the γ energy spectrum, known as the photopeak or full absorption peak. This corresponds to gamma rays depositing all of their energy in the detector material. The energy of the gamma rays can be determined from the photopeak energy and compared to an energy level diagram for the parent nucleus and its decay products.
2. **Compton effect:** when a beam of high-frequency electromagnetic radiation passes through a material containing free electrons, an interaction takes place between the incident photons and

the free electrons. In this interaction, photon-electron scattering, energy and momentum are transferred from the photons in the incident beam to the electrons. X-ray and γ -ray energies are rather large compared to the binding energies of the outermost electrons of atoms, so the outermost electrons can be treated as essentially free. As a result of energy transfer to the electrons in the absorbing material, both the intensity and the energy of the beam is reduced.

3. **Pair production:** When a positron and electron collide, they annihilate each other and produce photons. Energy conservation dictates that the total energy of the emitted photons equals that of the annihilated particles. Given that the rest energy of an electron or positron is 511 keV, the total energy of the emitted photons must be at least 1.022 MeV. When high-energy photons (> 1.022 MeV) interact with matter, the reverse process can also occur: the energy of the photon is converted to an electron-positron pair. This process conserves charge as well as linear and angular momentum, and is usually referred to as *pair production*. As with the photoelectric effect, the annihilation of photons results in a reduction of gamma ray intensity.

Two other mechanisms are worth noting, though they do not change the essential physics of the problem.ⁱ A fourth mechanism, Rayleigh scattering, corresponds to the elastic (photon-energy-conserving) scattering of light, which simply changes the direction of incident photons. Though no energy is lost, this process will reduce the measured intensity of radiation simply because some photons will be deflected out of the detection region. Essentially, at a given energy, this may be regarded as a geometrical factor.

Finally, in principle we must also consider direct absorption of incident photons by the nucleus itself, a process called photonuclear absorption. This process usually results in the ejection of one or more neutrons and/or protons. This interaction can contribute $\sim 5-10\%$ to the total photon interaction, though within a fairly narrow energy region usually occurring somewhere between 5 MeV and 40 MeV. At the energies of the present experiment (< 2 MeV), this effect is negligible.

6.1.1 Gamma Absorption

In the simplest model, we do not distinguish between the different types of interactions discussed above but instead simply presume that, at a given incident γ energy, the number of gamma rays ΔN removed from the incident beam of original intensity N_o is proportional to the number of electrons and nuclei along the gamma ray's path through the material. That is, we assume a constant attenuation coefficient for the beam, which may be viewed as an energy-dependent, weighted sum of separate attenuations from all three mechanisms above.

The number of electrons and nuclei encountered is itself proportional to both the density of the material ρ and the path length traveled in the material x . The coefficient of proportionality is

ⁱOf course, there are still other mechanisms, such as nuclear-resonance scattering and Delbrück scattering, but they are negligible from our point of view.

known as the *mass attenuation coefficient* μ and has units of inverse distance. The loss of gamma ray intensity can then be expressed as

$$\Delta N = -N_o (\rho x) \left(\frac{\mu}{\rho} \right) \quad (6.1)$$

Typically, one deals with the density-normalized mass attenuation coefficient μ/ρ , which has units of cm^2/g , rather than the raw mass attenuation coefficient. Similarly, the quantity ρx is known as the “mass thickness,” with units of g/cm^2 . In any case, the solution to this first-order differential equation is well-known:

$$N(x) = N_o e^{-(\rho x)(\mu/\rho)} \quad (6.2)$$

$N(x)$ is the number of gamma rays remaining after passing through a material of thickness x . Two reduced quantities determine the gamma ray attenuation: the density-normalized attenuation coefficient μ/ρ , and the mass per unit thickness (often called “mass thickness”) ρx , which has units of g/cm^2 . An exponential law is not unexpected in this case; one obtains the same solution for any system in which the primary quantity grows or diminishes at a fixed rate.

6.1.2 Attenuation Coefficient

The simplistic model outlined above would certainly seem reasonable for a single energy-independent mechanism for beam attenuation. Considering all three interactions, however, we must at least consider the density-normalized attenuation coefficient to be a function of energy. Typically, the quoted coefficients are a sum of attenuation coefficients for the principle photon interactions:

$$\mu = \mu_{pe} + \mu_{coh} + \mu_{incoh} + \mu_{pp} \quad (6.3)$$

Here μ_{pe} is the coefficient for the photoelectric effect, μ_{coh} for coherent (Rayleigh) scattering, μ_{incoh} for incoherent (Compton) scattering, and μ_{pp} for pair production. Overall, when dealing with *monochromatic* radiation, one may simply consider an effectively constant μ at that particular energy, and not worry about the detailed contributions to the effective μ .

However, each process has a characteristic energy dependence, making μ a strongly varying function of energy. The photoelectric attenuation decreases rapidly with increasing energy, leading to an overall decreasing background for $\mu(E)$. The Compton effect has a roughly constant attenuation until $\sim 0.1 - 1$ MeV, after which it decreases rapidly with increasing energy. Pair production is possible only above 1.022 MeV and increases rapidly for higher energies. Taken together, $\mu(E)$ is a strongly decreasing function of energy until a few MeV, after which it increases gently, as shown in Fig. 6.2. At lower energies, one must also factor in characteristic x-ray absorptions.ⁱⁱ

ⁱⁱThe X-ray absorption energies correspond to incident photons matching the energy level spacing of the atoms in the detector material. This leads to strong absorption at these particular energies, and hence, strongly increased attenuation.

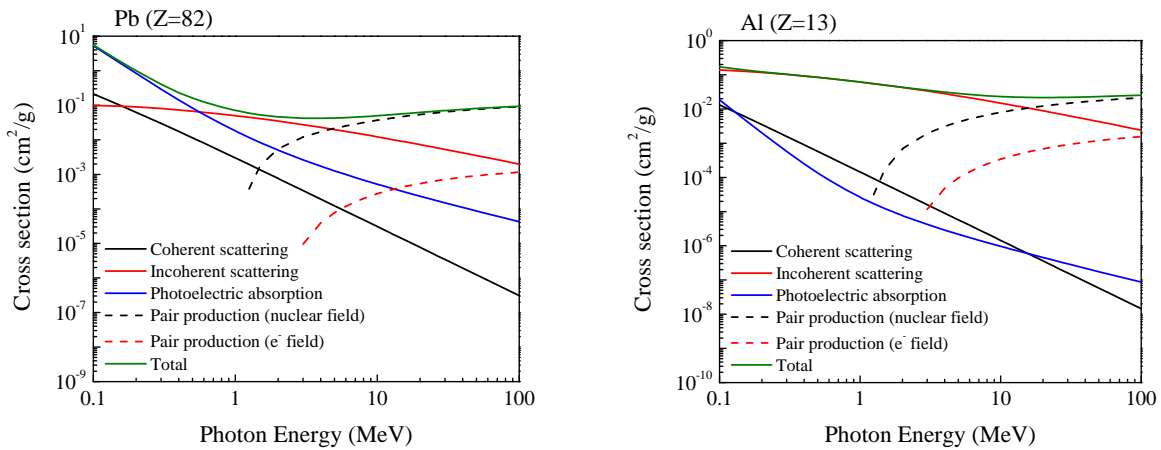


Figure 6.2: (left) The absorption cross-section of lead ($Z = 82$) for gamma rays showing the contributions from various processes. (right) The corresponding data for aluminum ($Z = 13$). Data from the NIST XCOM database <http://www.nist.gov/physlab/data/xcom/index.cfm>

Figure 6.3 shows the absorption cross-section for Pb as a function of photon energy. From this one can see the superimposed effects of Compton scattering, the photoelectric effect, and pair production as well as characteristic X-ray absorption energies. At the energies used in this experiment (^{57}Co and ^{137}Cs gammas), the value of μ should vary by approximately a factor of 20–30.

6.2 Objective

The primary purpose of this laboratory is to determine the mass absorption coefficients for lead, copper, and aluminum for a fixed gamma ray energy (662 keV) to ascertain their utility in shielding ionizing radiation. A secondary purpose will be to ascertain any energy dependence of the attenuation factor for lead by comparing the attenuation factors at two different energies (662 keV and 121 keV). Figure 6.3 shows the mass attenuation coefficient for lead for energies up to 20 MeV. The decaying background is a superposition of the effects of Compton scattering and photoelectric effects, while the sharp increases are due to the characteristic X-ray absorptions of Pb.

Hypothesis: The attenuation of gamma rays by a solid material can be described by a single parameter, the mass attenuation coefficient. This quantity should be energy-dependent and material-dependent.

6.3 Preparatory Questions

You should touch on these questions in your report.

1. Where are gamma rays on the electromagnetic spectrum?

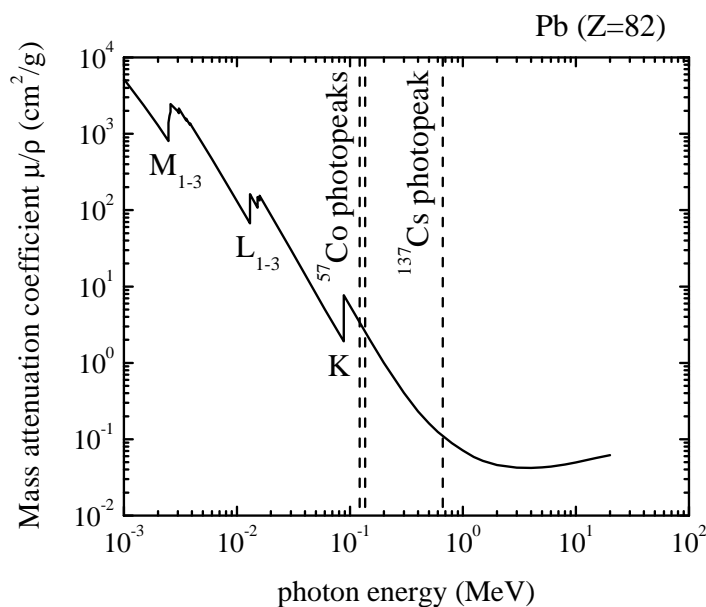


Figure 6.3: Density-normalized mass attenuation coefficient μ/ρ for Pb ($Z=82$) as a function of photon energy. Note the logarithmic scales. The energies of primary ^{57}Co and ^{137}Cs gamma ray photons are noted, as are the K, L, and M absorption energies for Pb.[5]

2. Can we be completely shielded from gamma rays? If not, then why bother?
3. Which material provides the best shielding against gamma radiation? Does the answer change as the gamma energy changes?
4. What are the possible ways in which photons with energies in the range from 1 to 2000 keV can interact with matter?

6.3.1 Relevant Reading

Taylor[1], Ch. 1-3

Pfeffer & Nir[2], Ch. 5.3.1-3, 5.5.1

http://en.wikipedia.org/wiki/Gamma_ray

See [5–7] for relevant nuclear data and [8] for example gamma spectra.

6.4 Supplies

1. Spectech UCS30 spectrometer
2. NaI(Tl) detector
3. Multi-channel analyzer (MCA) / control electronics
4. lead cylinder for shielding
5. sheets of Al, Cu, and Pb
6. radioactive sources

7. detector holder and shelf housing
8. calipers for measuring sheet thickness
9. lab notebook and USB drive for saving spectra

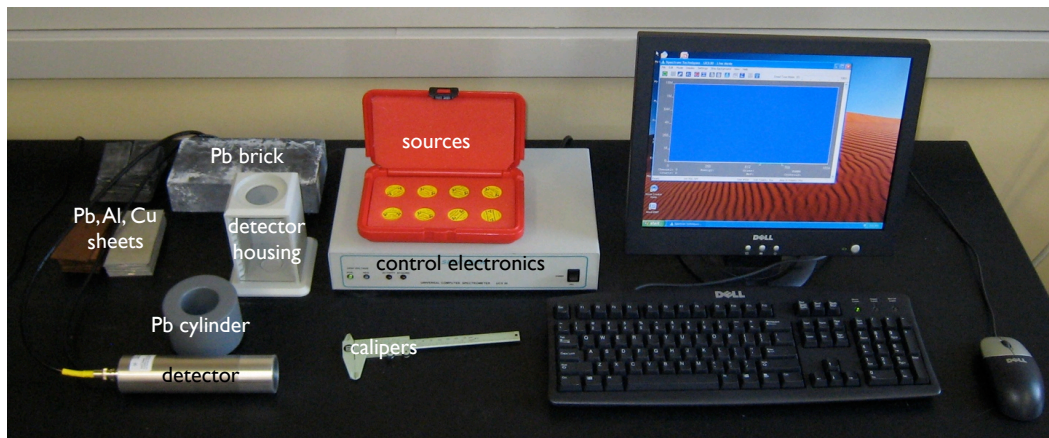


Figure 6.4: *The Spectech UCS30 spectrometer and accessories.*

6.5 Suggested procedure

6.5.1 Startup

1. Turn on the power to the Spectech UCS30
2. run the program “UCS30” from the shortcut on the desktop
3. under the Mode menu, select PHA (preamp in)
4. place the ^{137}Cs source on the table, and place the lead cylinder around it
5. place the cylindrical NaI detector inside the lead cylinder
6. Run the energy auto-calibration from “Settings/Energy Calibration/Auto Calibrate” - this will take several minutes. The final spectra should look like the one shown below in Fig. 6.5.

6.5.2 Running the experiment

1. Once the calibration is finished, place the NaI detector on top of the slotted stage of the detector housing with the lead cylinder around it
2. Place the ^{137}Cs source in the third slot from the top using the plastic holder and record the source-detector distance using calipers
3. Acquire a spectrum of the ^{137}Cs source alone for a preset amount of time. An example spectrum is shown in Fig. 6.5. In the gamma ray spectroscopy unit, you will learn more about the interpretation of these spectra.
 - From “Settings/Time” choose a “live time” of 300 seconds

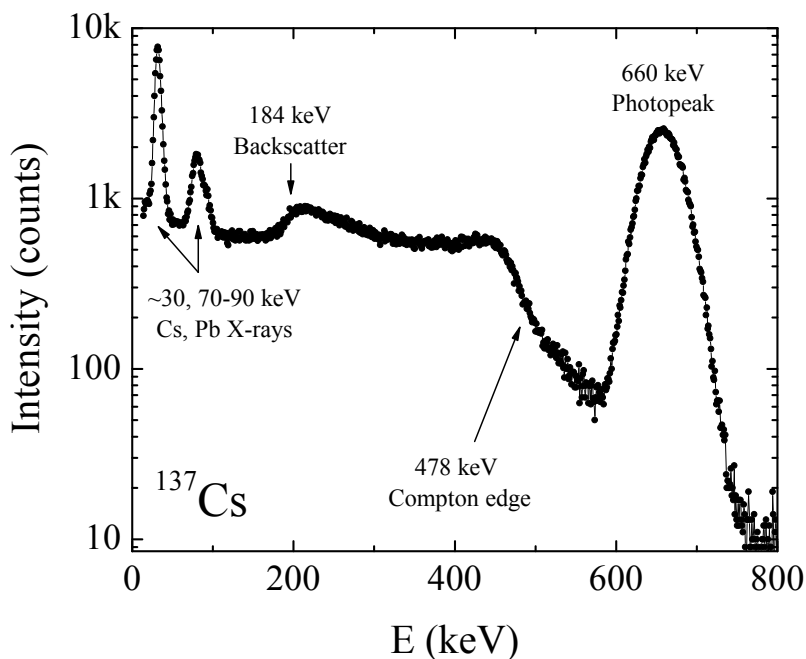


Figure 6.5: Gamma spectrum of ^{137}Cs . Aside from the main photopeak at ≈ 660 keV, clear Compton edge and backscatter features are visible, as well as X-ray absorption edges. Note the logarithmic scale on the vertical axis.[3]

- Click the “X” eraser icon on the toolbar to erase the existing spectrum
 - Click the green “Start Counts” button on the toolbar to begin recording your spectrum. When the allotted time has elapsed, the “Start Counts” button should be undimmed and the red “Stop Counts” button dimmed. A large peak around 660 keV should be visible.
 - On the main screen, highlight the region of interest (ROI) under the 660 keV peak and record the peak centroid, gross counts, and full-width at half-maximum (fwhm). From the main screen, record the number of counts at the centroid position by setting the channel to the centroid value.
 - If you do not have at least ~ 500 counts for the 660 keV photopeak, increase the “live time” as required.
 - Save the spectra using Save and choosing “Comma Separated File” under “Files of Type...”. Name your file sensibly!
4. Keeping your source at the same position, and keeping the same ROI, insert a sheet of Pb between the source and detector. Measure the thickness of this sheet with the calipers provided. **Use gloves when handling Pb, and wash your hands thoroughly when finished.**
 5. Acquire and save *the whole spectrum* using the steps above and record the data for the 660 keV peak.
 6. Repeat for increasing numbers of Pb sheets until you have a peak intensity (measured by gross counts) at least 3 times lower than you had without any Pb or you run out of sheets.

7. Record a spectrum without any Pb sheets to verify the unattenuated intensity.
8. Repeat the steps above for sheets of Cu or Al. You may need to add 2-4 sheets of Al at a time to achieve reasonable attenuation.

Once you have acquired all of your data for all three materials using the ^{137}Cs source, if time permits insert the ^{57}Co source. Using the 120 keV gamma ray peak, repeat the procedure above. An example spectra for ^{57}Co can be found in on page 6.8.

Note: Since you have recorded the entire ^{137}Cs spectrum each time, you can use several of the peaks present (not just the 660 keV peak) to plot an energy dependence of the attenuation coefficient.

6.6 Data analysis

As discussed briefly above, a narrow beam of monoenergetic photons with an incident intensity I_o ⁱⁱⁱ, penetrating a layer of material with thickness x and density ρ emerges with intensity I given by an exponential attenuation law

$$\frac{I}{I_o} = \exp \left[- \left(\frac{\mu}{\rho} \right) (\rho x) \right] \quad (6.4)$$

We can put this in a form more amenable to analysis by taking the natural logarithm of both sides:

$$\ln I = \ln I_o - \frac{\mu}{\rho} (\rho x) \quad (6.5)$$

Given a material of known density and thickness, it is clear that if we plot $\ln I$ (y -axis) versus ρx (x -axis), we will obtain a straight line of slope $-\mu/\rho$. Thus, by measuring the gamma ray intensity as a function of intervening material, we can extract the density-normalized mass attenuation coefficient.

For each attenuating material, make a table recording the thickness of the material and the corresponding counts at the peak position. Be sure to include data for zero thickness, i.e., the detector and source along. From this table, construct a plot with $\ln I$ on the y axis and the product of thickness and density (ρx) on the x axis. Perform a linear regression analysis to determine the slope of the plot, which should be $-\mu/\rho$. In order to compare with the NIST database, it is convenient to have your thickness in cm, and the material density in g/cm^3 (material densities can be found in on page 6.8). Be sure to include proper uncertainties in your determination of μ/ρ (e.g., arising from peak counts uncertainty and thickness uncertainty). An example plot is shown in Fig. 6.6

Determine μ/ρ for Cu, Pb, and Al using the 660 keV gamma ray of ^{137}Cs , and investigate the energy-dependence of μ/ρ for Pb using the 120 keV gamma ray peak of the ^{57}Co (and the 835 keV peak of ^{54}Mn if you have time).

ⁱⁱⁱIntensity is proportional to the number of photons detected, so this is not at odds with our previous discussion.

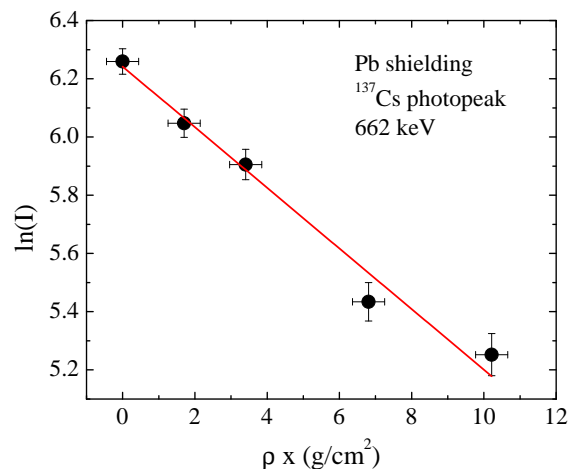


Figure 6.6: Natural logarithm of ^{137}Cs gamma ray (661 keV) intensity as a function of lead mass thickness ρx .^[3] The accepted value of μ/ρ for Pb at 661.6 keV is $0.105 \text{ cm}^2/\text{g}$.^[4, 5]

6.7 Format of report

You may choose one of the following formats for your report:

1. two-page memo (excluding required plots)
2. group oral presentation
3. formal written lab report
4. research proposal

Further details and templates are provided for each of these formats. Keep in mind that over the course of all of the one-week experiments you must do each type of report (and two formal lab reports).

6.8 Discussion and topics for your report

Find the accepted values of the density-normalized mass attenuation coefficients for Al, Pb, and Cu.^{iv} How do your values compare?

The aluminum cover on your detector is about 3.2 mm thick. What fraction of the 661.6 keV gamma rays are absorbed when passing through the aluminum cover? How about the 122 keV gamma rays?

Which material provides the best shielding against gamma radiation? Does your answer depend on the energy of the gamma rays? The online NIST database may be helpful.

^{iv}See, e.g., <http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html>.

Compare the range for gamma ray penetration to characteristic values for beta and alpha emission. How and why are they different?

Theoretical analysis of the photoeffect interaction for $E_\gamma \ll m_e c^2$ suggests that

$$\mu = K Z^n E_\gamma^3 \quad \text{or} \quad \ln \mu = \ln K + n \ln Z - 3 \ln E_\gamma \quad (6.6)$$

where K is a constant of proportionality, Z is the target atomic number, n is a coefficient between 4 and 5, and E_γ is the gamma energy in keV. Compton scattering effects are neglected here, they contribute very little to the absorption.

Plot the natural logarithm of your 662 keV mass absorption coefficients as a function of the natural logarithm of the atomic number of the absorbing material. Perform a linear regression analysis to find the slope and intercept of best fit. Based on the equation above, the slope should yield n . Is it between 4 and 5 as the model suggests?

For gamma energies near or larger than 511 keV the equation above is no longer valid. However it is tempting to determine the energy dependence anyway. If you measured μ/ρ for lead at more than two energies, graph the logarithm of μ/ρ as a function of the logarithm of the gamma energy. If your data resemble a straight line, is the slope close to three?

Compare the spectrum from the ^{137}Cs source with no absorber to the spectrum from the ^{137}Cs source with a thick lead absorber. What is the origin of the gammas between the photopeak and the Compton edge? What happened to the Ba K X-rays? The same questions arise with the ^{57}Co spectrum. What has happened?

Electromagnetic Spectrum

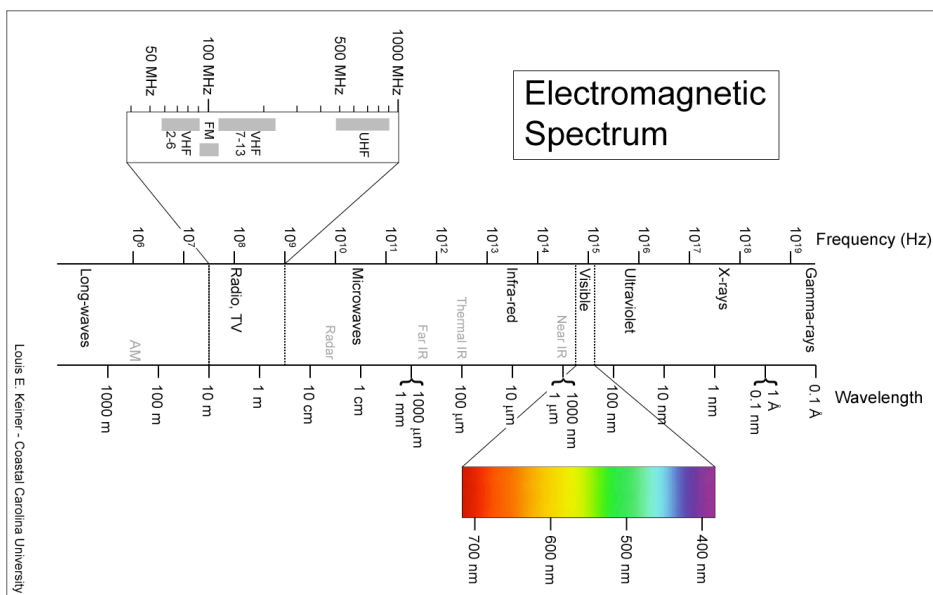


Figure 6.7: The electromagnetic spectrum. From http://en.wikipedia.org/wiki/Electromagnetic_spectrum.

Density of various materials

Table 6.1: Densities of absorber materials

Element	Density g/cm ²
Al	2.70
Cu	8.96
Pb	11.35

Gamma sources available

Table 6.2: Disk gamma source set

Isotope	Activity (μCi)	Half-life	Peaks (keV)
¹³³ Ba	1	10.8 yrs	81, 276, 303, 356, 384
¹⁰⁹ Cd	1	462 days	88
¹³⁷ Cs	1	30.2 yrs	662
⁵⁷ Co	1	272 days	122, 136
⁶⁰ Co	1	5.27 yrs	1173, 1333
⁵⁴ Mn	1	313 days	835
²² Na	1	2.6 yrs	511, 1275
unknown	1.5	?	?

Sources from Spectrum Techniques, Oak Ridge, TN (www.spectrumtechniques.com). USNRC and state license exempt quantities.

Additional data for ¹³⁷Cs and ⁵⁷Co

A ¹³⁷Cs nucleus can decay via two routes to the ¹³⁷Ba ground state, shown schematically in Fig. 6.8. First, a single beta emission of 1.176 MeV can occur with no subsequent gamma emission, bringing the ¹³⁷Cs directly to the ground state. However, this single-step process occurs for only about 6.5% of all nuclei. The other 93.5% decay via two-step process. First, a 514 keV beta is emitted, resulting in metastable and short-lived ¹³⁷Ba*. The metastable ¹³⁷Ba* subsequently (2.55 min half life) decays to the ¹³⁷Ba ground state by emission of a 662 keV gamma.

Though ¹³⁷Cs has only a single gamma emission^v, there are x-ray emissions that can be observed in the energy range under study. We list a few of the higher energy emissions below.

^vStrictly speaking, there is a 283.53 keV gamma, but its intensity is about 10⁵ times smaller than the 662 keV peak.

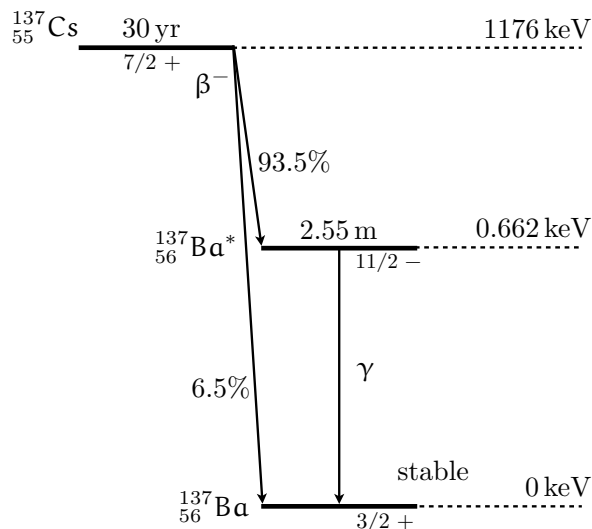


Figure 6.8: Energy level diagram showing the decay of ^{137}Cs to ^{137}Ba . The ^{137}Cs decays via two paths, 93.5% follow a two-step process resulting in the emission of a 662 keV gamma ray.[4] The decay of the Ba nucleus from its excited state is an internal transition, and contributes to the decay percentages but not the gamma decay factor.[4]

Table 6.3: X-rays from ^{137}Cs . From [6]

E (keV)	I (%)	Assignment	E (keV)	I (%)	Assignment
31.45	0.00026	Ba $K_{\alpha 3}$	36.38	0.68	Ba $K_{\beta 1}$
31.72	2.04	Ba $K_{\alpha 2}$	36.65	0.0079	Ba $K_{\beta 5}$
32.19	3.76	Ba $K_{\alpha 1}$	37.26	0.215	Ba $K_{\beta 2}$
36.30	0.35	Ba $K_{\beta 3}$	37.35	0.0481	Ba $K_{\beta 4}$

The ^{57}Co spectra and decay process is a bit more complicated, and we will leave the details as an exercise to the reader. Figure 6.9 shows an energy level diagram for the decay of ^{57}Co to ^{57}Fe . For all but $\sim 0.18\%$ of nuclei, the decay proceeds by electron capture to create metastable $^{57}\text{Fe}^*$ states, which decay to the ground state by gamma emission. This leads to the three characteristic gamma peaks at 136.5, 122.1, and 14.4 keV. Additional peaks appear in the spectrum due to x-ray emissions, sum peaks, *etc.* The feature of interest for this laboratory is the relatively intense 122.1 keV gamma.

Table 6.4: Gammas from ^{57}Co . From [6]

E (keV)	I_g (%)	E (keV)	I_g (%)
14.41	9.16	352.4	0.013
122.1	85.60	366.8	0.0013
136.5	10.68	569.9	0.017
230.3	0.0004	692.0	0.16
339.5	0.014	706.4	0.025

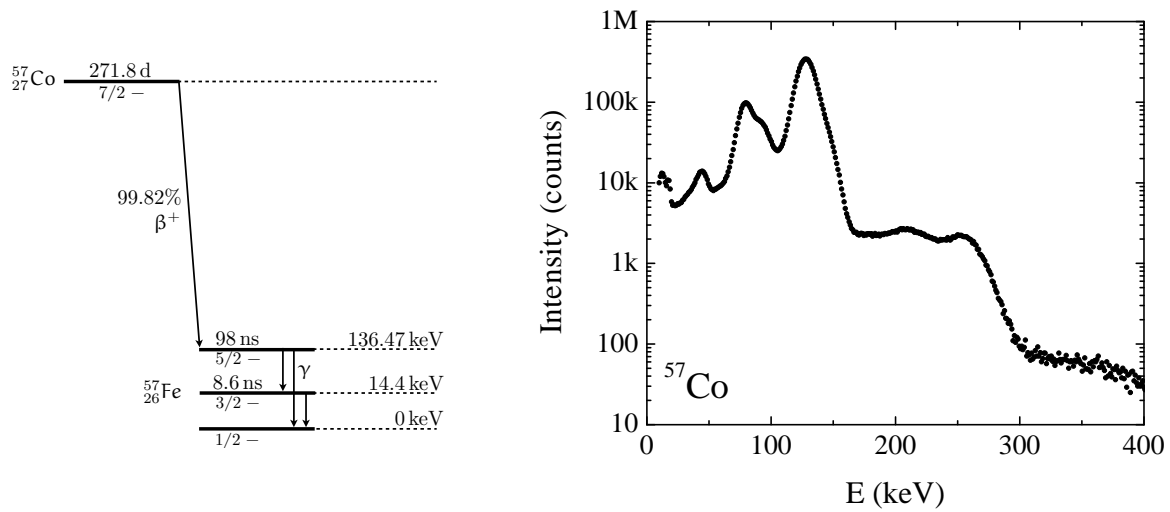


Figure 6.9: Left: Energy level diagram showing the decay of ^{57}Co to ^{57}Fe . Not shown is a more complicated path for 0.18% of nuclei, which take a decay path through higher ^{57}Fe excited states. Associated gamma rays, after the initial β^+ electron capture are 136.5, 122.1, and 14.4 keV.[4] Right: Gamma spectrum of ^{57}Co . Note the logarithmic scale on the vertical axis.[3]

Electron Charge to Mass Ratio e/m

7.1 Introduction

In this experiment, you will investigate the motion of an electron beam under the influence of a magnetic field, a simple method for measuring e/m , the charge to mass ratio of the electron.ⁱ The method is similar to that used by J.J. Thompson in 1897.

7.1.1 Motion of Electrons in a Perpendicular Magnetic Field

A beam of electrons is first created by thermionic emission from a heated metal filament, and then accelerated through a known potential V_{acc} . If the electrons are emitted from the filament with negligible kinetic energyⁱⁱ, conservation of energy allows us to determine the electrons' velocity:

$$\frac{1}{2}mv^2 = eV_{\text{acc}} \quad \implies \quad v = \sqrt{\frac{2eV_{\text{acc}}}{m}} \quad (7.1)$$

A pair of Helmholtz coils produces a uniform and measurable magnetic field B at right angles to the electron beam. Recall that in the presence of a perpendicular magnetic field, an electron of charge $-e$ will undergo uniform circular motion, as shown in Fig. 7.1, since the Lorentz force $\vec{F}_B = e\vec{v} \times \vec{B}$ is always at a right angle to the particle's velocity vector.

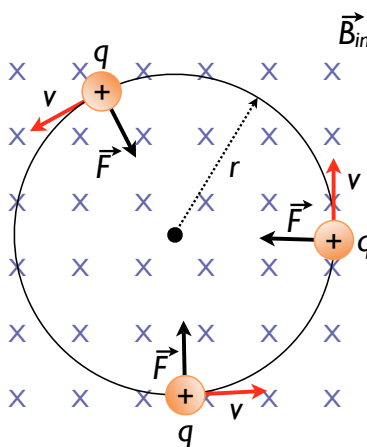


Figure 7.1: An electron moving at velocity v in a plane perpendicular to a magnetic field B undergoes circular motion.

ⁱMuch of this procedure is reproduced from Pasco document 012-03471D, “ e/m Apparatus.” This partial reproduction is only for use in PH255 at the University of Alabama, as dictated by the copyright notice in 012-04049J.

ⁱⁱA reasonable assumption, since even at a filament temperature of $T = 4000$ K the thermal energy is only $k_B T \approx 0.35$ eV, giving an electron velocity of $\sim 3 \times 10^5$ m/s. This is to be compared to the electron's velocity after acceleration, $\sim 10^7$ m/s.

If the electrons are in circular motion, then the magnetic force must provide the centripetal force F_C :

$$F_C = \frac{mv^2}{r} = F_B = evB \quad (7.2)$$

Solving for the charge to mass ratio e/m , we find:

$$\frac{e}{m} = \frac{v}{Br} \quad (7.3)$$

Using our previous relationship for the electrons' velocity,

$$\frac{e}{m} = \frac{1}{Br} \sqrt{\frac{2eV_{\text{acc}}}{m}} \implies \frac{e}{m} = \frac{2V_{\text{acc}}}{B^2 r^2} \quad (7.4)$$

Finally, if we know the radius of the Helmholtz coils and the current passing through themⁱⁱⁱ we can determine the magnetic field near the center of the coils:

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{N\mu_o I}{a} \quad (7.5)$$

Here N is the number of turns in each coil, $\mu_o = 4\pi \times 10^{-7} \text{ N/A}^2$, I is the current through the coils, and a the radius of the coils. Putting everything together, we can relate the e/m ratio to known experimental quantities:

$$\frac{e}{m} = \frac{v}{Br} = \frac{2V_{\text{acc}} \left(\frac{5}{4}\right)^3 a^2}{(N\mu_o I r)^2} \quad (7.6)$$

A knowledge of the accelerating voltage, the construction of the Helmholtz coils and their current, and a measurement of the radius of the electrons' circular path is sufficient to determine e/m .

7.1.2 The e/m Tube

Of course, electrons are quite small, and their velocities will be prodigious. How can one measure the radius of the circular path? An ingenious method is to perform the experiment inside a vacuum tube back-filled with a small pressure of a noble gas – in our case, 0.1 Pa of He. A partial vacuum is necessary so that most electrons can travel without colliding with He atoms. However, those few electrons in the beam that do collide with He atoms will excite them such that they will subsequently radiate visible light, allowing us to view the trail of the electron beam. This is a somewhat delicate balance: too much He and the mean distance between collisions for the electrons is too small to be of any use, too little He and the glowing beam will barely be visible. A schematic of the e/m tube is shown in Fig. 7.2

ⁱⁱⁱRecall that in the Helmholtz geometry, the coil spacing is equal to the coil radius.



Figure 7.2: Left: The e/m tube. Right: Electron gun.

The electron beam itself, as mentioned above, is created by thermionic emission from a heated metal filament, part of a device known as an “electron gun” also shown in Fig. 7.2. In the electron gun, the heater heats the cathode, which emits electrons. The electrons are accelerated by a potential applied between the cathode and anode. The grid, which helps to focus the electron beam, is held positive with respect to the cathode and negative with respect to the anode.

A rather unique feature of the Pasco e/m tube is that the socket rotates, allowing the electron beam to be oriented at any angle from 0 to 90° with respect to the Helmholtz coils. Thus, you can rotate the tube and examine the vector nature of the Lorentz force on moving electrons, changing the path from a purely circular to helical, for instance. You can also use small permanent magnets instead of the Helmholtz coils to further investigate the effects of magnetic fields on the electron beam.

Finally, the e/m tube also has two additional deflection plates, which can be used to demonstrate the effect of an *electric* field on the electron beam. This can be used to give a confirmation of the negative charge of the electron, and to demonstrate how an oscilloscope or cathode-ray tube works.

7.2 Objective

The objective of this experiment is to measure the charge-to-mass ratio of electrons e/m by applying a magnetic field perpendicular to an electron beam of known velocity. Introductory electromagnetism tells us that the electrons’ velocity is related to the accelerating potential, and the magnetic field is determined by the geometry and current of the Helmholtz coils.

Hypothesis: From the accelerating potential, coil current and geometry, and the beam radius, the ratio e/m can be determined.

7.3 Preparatory Questions

You should touch on these questions in your report.

1. Justify the formula given for the magnetic field near the center of a pair of Helmholtz coils.
2. Considering an ideal gas, to what pressure must the tube be evacuated for the mean free path of He atoms to be comparable to the beam radius of ~ 5 cm?
3. Compare thermal energy of the electrons assuming a filament temperature of 4000 K to the kinetic energy of electrons accelerated through a 200 V potential.
4. Given a 200 V accelerating potential, what magnetic field is required to produce a beam radius of 5 cm?

7.3.1 Relevant Reading

Taylor[1], Ch. 1-3

Review of introductory electromagnetism.

7.4 Supplies

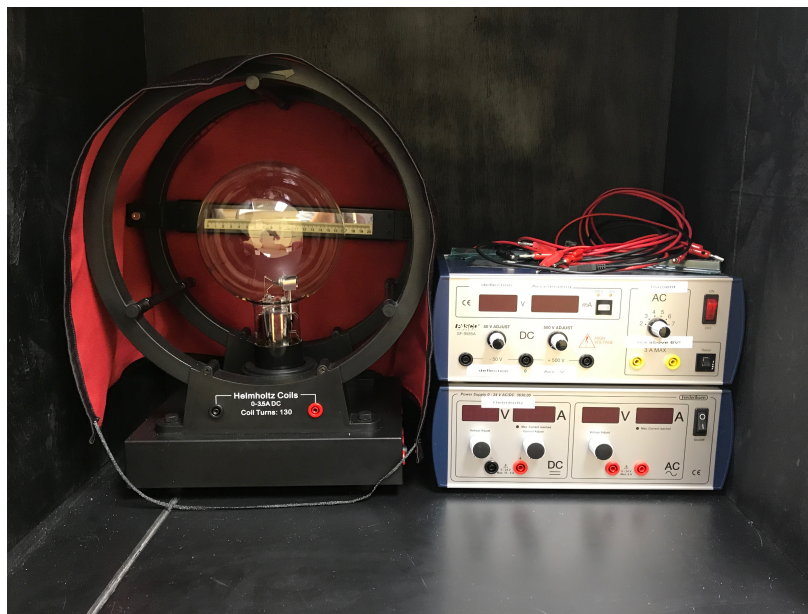


Figure 7.3: The e/m setup and associated electronics. The black shroud is for more easily viewing the electron beam, and can be removed.

1. Pasco e/m apparatus

- Helmholtz coils: radius and separation $a = 0.158$ m, $N = 130$, $B[\text{T}] = 7.40 \times 10^{-4} I [\text{A}]$
- Control panel: all connections are labeled.

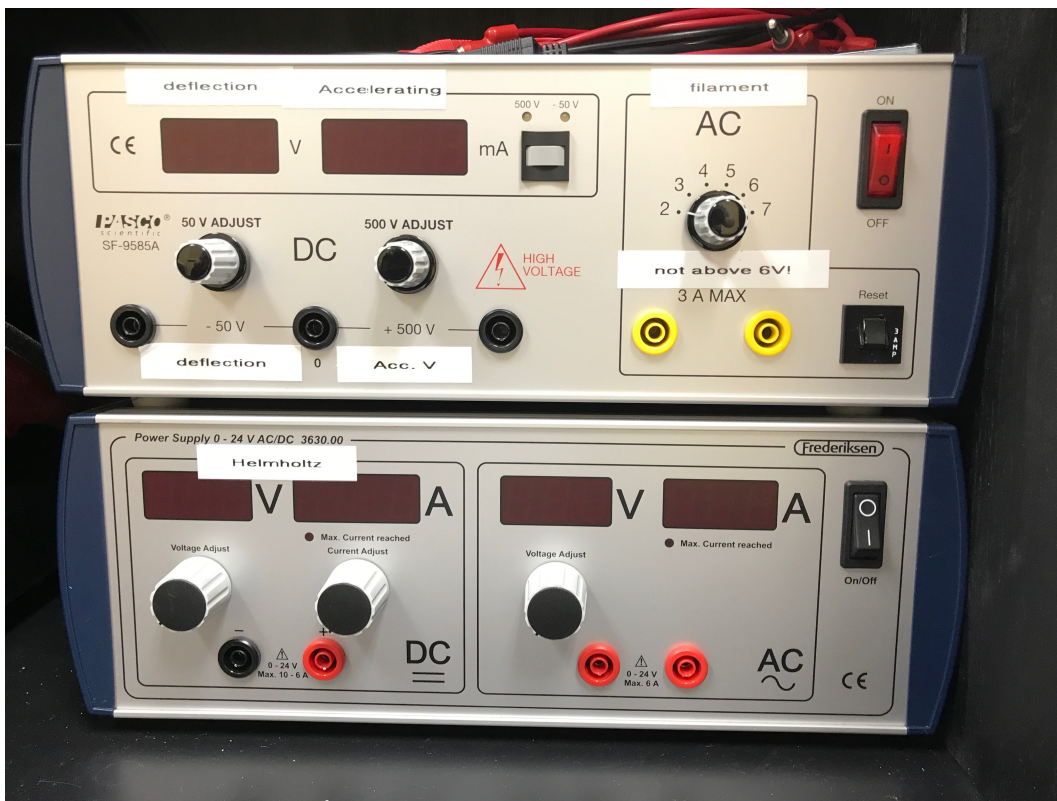


Figure 7.4: Power supplies. Top: Pasco SF-9585A high voltage power supply. Bottom: Frederiksen power supply.



Figure 7.5: Connections at the base of the e/m apparatus. Refer to this when connecting the power supplies. Left: right side of apparatus. Right: bottom front of apparatus.

- Cloth hood: for easier viewing of the electron beam in a lighted room.
 - Mirrored scale: for measuring the beam radius without parallax error.
2. Pasco SF-9585A high-voltage power supply (0–50/0–500 V DC, 0–7 V AC)
 3. Frederiksen power supply (0–24 V AC/DC)
 4. 7 banana cables
 5. lab notebook

7.5 Suggested procedure

7.5.1 Startup

1. Place the hood over the e/m apparatus if you will be working in a lighted room.
 2. Verify that your power supplies are plugged in, **but turned off**. Turn all knobs to their lowest positions.
 3. Refer to Fig. 7.4 and Fig. 7.5 to make the following connections between the power supplies and the e/m apparatus.
 4. Connect the positive terminal of the 500 V DC power supply (SF-9585A supply) to the positive terminal labeled “Accelerating Voltage” on the base of the e/m apparatus. Connect the negative terminal of the 500 V power supply to the negative terminal labeled “Accelerating Voltage”.
 5. Connect both terminals of the AC power supply (SF-9585A supply) to the terminals labeled “Filament” on the base of the e/m supply.
 6. Connect the positive terminal of the 24 V DC power supply (Frederiksen supply) to the red terminal of the front Helmholtz coil. Connect the black terminal of the front Helmholtz coil to the black terminal of the back Helmholtz coil. Finally, connect the red terminal of the back Helmholtz coil to the negative terminal of the 24 V DC power supply.
 7. Have the instructor check your wiring before proceeding.
1. Turn on the supplies, and adjust them to the following levels:
 - Filament current / Heater (SF-9585A supply, AC volts): 6 V AC. **CAUTION:** Do not at any time exceed 6 V, or you will burn out the filament and destroy the e/m tube!
 - Accelerating voltage / Electrodes (SF-9585A supply, 0–500 V DC): 150 to 250 V DC. The 0–500 V DC output uses the central and right-most plugs of the group of three plugs. The central plug is the ground. Note the small button near the voltage/current display to toggle the reading between 50 and 500 volt outputs.
 - Helmholtz coils (Frederiksen supply): 4–7 V DC, current below 2 A. Turn the current limit knob to about 3/4 of its full range. Note that you are using the DC output on the left-hand side of the supply!

2. Watch the ammeter on the Frederiksen supply and take care that the current does not exceed 2 A. If the current is too large, the beam will be near the edge of the tube, and your measurements will be distorted by refraction.
3. Wait several minutes for the cathode to heat up. When it does, you will see the electron beam emerge from the electron gun and it will be curved by the field from the Helmholtz coils. If it is not, *gently* turn the tube by the base (**do not touch the glass**) until it is. Do not take the tube out of its socket, as you rotate the tube the socket will turn. **Make sure the tube is properly seated in the socket.**

7.5.2 Measurement of e/m

In order to determine e/m accurately, you will need to make several measurements of the beam radius for varying V_{acc} . For your first measurement, adjust the current in the Helmholtz coils, the accelerating voltage, and tube position until you have a nice circular path of radius ~ 5 cm. Suggested starting parameters: $I_{\text{coil}} \approx 1.2$ A, $V_{\text{acc}} \approx 200$ V. **Do not exceed 250 V on the accelerating voltage.**

1. Align the rotational position of the tube to obtain a closed circular path. While doing this, qualitatively investigate the helical path of the beam when the beam is not aligned with the coils.
2. Read the current to the Helmholtz coils from the display on the Frederiksen supply and the accelerating voltage from the SF-9585A supply. Record these values in your laboratory notebook, following the example of the table provided below on page 60.
3. Carefully measure the radius of the electron beam. Look through the tube at the beam. To avoid parallax errors, move your head to align the electron beam with the reflection of the beam that you can see on the mirrored scale. Measure the radius of the beam as you see it on *both* sides of the scale, and average the results. Record your values, noting your estimated error.
4. Repeat the steps above for at least 10 accelerating voltages between 150–250 V.
5. Repeat the experiment with a different current through the Helmholtz coils (and the same 10 accelerating voltages).
6. When finished, reduce all voltages to zero in the reverse order you turned them on, and turn off the power supplies.

There are a few simple ways to improve your experimental results. The greatest source of error is in the velocity of the electrons. First, the non-uniformity of the accelerating field caused by the hole in the anode causes the velocity of the electrons to be slightly less than their theoretical value. Second, collisions with the helium atoms in the tube further rob the electrons of velocity. Since e/m is proportional to $1/r^2$, and r is proportional to v , experimental values of e/m are greatly affected by these two effects. One simple way to minimize these errors is to *measure the outside of the beam path*. A second rule of thumb is to keep the circle of the electron beam as large as

possible.^{iv} Note that both of these sources of error will tend to make your values of e/m *higher* than the accepted value.

7.5.3 Deflection of electrons by electric fields - OPTIONAL

You can also use the deflection plates to demonstrate how the electron beam is deflected in an electric field, and demonstrate that electrons are negatively charged. In the presence of an electric field E , the path of the beam can be shown to be a parabola, governed by the equation

$$y = \frac{eEx^2}{2mv^2} \quad (7.7)$$

where y is the linear deflection achieved over a linear distance x . Beyond the deflection plates, the trajectory is a straight line, since the electrons are moving in a field-free region of space. Clearly, the direction of the deflection will depend on the sign of the electron charge.

1. Set up the equipment as described above, *except*:
 - Perform these alterations *before* turning on any supplies.
 - Do not supply current to the Helmholtz coils (thus you can leave the Frederiksen supply turned off).
 - Connect the “Deflection Plates” on the e/m apparatus to the 0–50 V terminals on the SF-9585A supply. Note that you can stack the banana plugs on the central connector, which is common for the 0–50 V and 0–500 V outputs.
2. Apply 6 V AC to the filament/heater and 150–250 V for an accelerating voltage. Again, wait several minutes for the cathode to warm up.
3. When the electron beam appears, slowly increase the voltage to the deflection plates from 0 to approximately 50 V DC. Note the deflection of the beam. The beam should be bent toward the positively charged plate (the upper plate, if you followed the polarity conventions).
4. **Warning:** do not leave the beam on for long periods of time in this mode. The beam will eventually wear through the glass walls of the tube!

7.6 Data analysis

Using the analysis above, you should be able to determine e/m for each accelerating voltage at each current through the Helmholtz coils. Be sure to include error bars which properly take into account all experimental errors, correctly propagated. Make a plot, with error bars, of e/m versus accelerating voltage for each Helmholtz current, noting on the plot the position of the accepted value. You may also plot the ratio of your determined values to the accepted values if you choose.

^{iv}Note that if the voltage is too high, the radius measurement is distorted by the curvature of the glass at the edge of the tube.

You overall error should properly propagate the errors in the individual quantities that determine e/m . Estimate the accuracy in the voltages and currents (due to either meter precision or fluctuations) and the radius measurement. You may also repeat the analysis above taking into account the electrons' initial thermal energy (you may assume that only those electrons emitted directly toward the anode end up in the beam).

Determine the earth's magnetic field at the laboratory location^v Be sure to consider the alignment of the tube with respect to the earth's field. Make the necessary corrections, or justify *quantitatively* neglecting them. What other environmental sources of static magnetic fields might affect your results?

After considering all errors, report your final average value for e/m with appropriate significant figures and uncertainty.

7.7 Format of report

You may choose one of the following formats for your report:

1. two-page memo (excluding required plots)
2. group oral presentation
3. formal written lab report
4. research proposal

Further details and templates are provided for each of these formats. Keep in mind that over the course of all of the one-week experiments you must do each type of report.

7.8 Discussion and topics for your report

What determines the pitch of the helical path when the tube is not rotationally aligned? The radius?

Derive the formula for the field near the center of a Helmholtz coil. What is the advantage of this particular geometry?

If you did the optional deflection of electrons by electric fields section: How do you know that electrons are negatively charged from this experiment?

Based on the theoretical analysis and your knowledge of the apparatus, which quantity do you expect to be the dominate source of uncertainty? How could you improve the experiment?

^vSee, e.g., <http://www.ngdc.noaa.gov/geomag/magfield.shtml>.

Explain how a similar setup could be used to create a mass spectrometer for unknown charged particles.

Example Data Table

Table 7.1: *e/m* experiment, group X, 23 Aug 2010

V_{acc} (V)	$I_{\text{Helmholtz}}$ (A)	r (m)	e/m (A-s/kg)
200 ± 0.5	1.50 ± 0.01		
250 ± 0.5	1.50 ± 0.01	0.045 ± 0.001	
...			
200 ± 0.5	1.00 ± 0.01		
...			

The Photoelectric effect

8.1 Introduction

In the late 1800s, scientists ran into considerable difficulty in explaining the emission and absorption of light. Classical theory (Rayleigh-Jeans Law) predicted that the amount of light emitted from a black body would increase dramatically as the wavelength decreased, whereas experiment showed that it approached zero. This discrepancy became known as the “ultraviolet catastrophe.”ⁱ

Experimental data for the radiation of light by a blackbody also showed that the intensity of light at long wavelengths departed dramatically from the classically predicted values (Wien’s Law). In order to reconcile theory and experiment, Planck was forced to develop a new model for light called the quantum model. In this model, light is emitted in small, discrete bundles or *quanta*. In this experiment, you will study the relationship between classical and quantum theories of light emission. Analyzing the emission spectra from a mercury vapor lamp within the quantum model will allow a determination of the ratio of Planck’s constant to the electron charge, h/e . Using the known value of the electron charge e , Planck’s constant h can be determined.

8.1.1 Planck’s Quantum Theory

In 1901, Planck published his law of radiation. In it he stated that that an oscillator, or any physical system, had a discrete set of possible energy values or levels; energies between these values never occur. Planck went on to state that the emission and absorption of radiation is associated with transitions or jumps between two energy levels. The energy lost or gained by the oscillator is emitted or absorbed as a quantum of radiant energy, the magnitude of which is expressed by the equation

$$E = hf \tag{8.1}$$

where E is the radiant energy, f the frequency of the radiation (sometimes labeled ν), and h is a fundamental constant of nature. The constant h became known as Planck’s constant.

Planck’s constant was found to have a significance beyond relating the frequency and energy of light, it became a cornerstone of the quantum mechanical view of the subatomic world. In 1918, Planck was awarded a Nobel prize for introducing the quantum theory of light.

ⁱMuch of this procedure is reproduced from Pasco document 012-04049J, “ h/e Apparatus and h/e Apparatus Accessory Kit.” This partial reproduction is only for use in PH255 at the University of Alabama, as dictated by the copyright notice in 012-04049J.

8.1.2 The Photoelectric Effect

In photoelectric emission, light strikes a material, causing electrons to be emitted. The classical wave model of light predicted that as the intensity of incident light was increased, the amplitude and thus the energy of the wave would increase. This would then cause more energetic photoelectrons to be emitted. The new quantum model, however, predicted that higher frequency light would produce higher energy photoelectrons, independent of intensity, while increased intensity would only increase the number of electrons emitted (or the photoelectric current). In the early 1900s, several investigators found that the kinetic energy of the photoelectrons was dependent on the wavelength, or frequency, and independent of intensity, while the magnitude of the photoelectric current, or number of electrons, was dependent on the intensity as predicted by the quantum model. Einstein applied Planck's theory and explained the photoelectric effect in terms of the quantum model using his famous equation, for which he received the Nobel prize in 1921:

$$E = hf = K_{\max} + \phi_o \quad (8.2)$$

where K_{\max} is the maximum kinetic energy of emitted photoelectrons, and ϕ_o is the energy required to remove them from the surface of the material (also called the "work function"). E is the energy supplied by the quantum of light known as the photon.

8.1.3 h/e Experiment

A quantum of light, or photon, with energy hf is incident upon an electron in the cathode of a vacuum tube. The electron uses a minimum ϕ_o of its energy to escape the cathode, leaving it with a maximum energy of K_{\max} in the form of kinetic energy. Normally the emitted electrons reach the anode of the tube, and can be measured as a photoelectric current. However, by applying an opposing potential difference V between the anode and cathode, the electrons will be repelled from the anode and the photoelectric current can be stopped. K_{\max} can be determined by measuring the minimum reverse potential needed to stop the photoelectrons and reduce the photoelectric current to zero, known as the "stopping potential."ⁱⁱ The stopping potential V is thus the potential at which the photoelectrons' maximal kinetic energy is equal to their electrical potential energy,

$$K_{\max} = eV \quad (8.3)$$

Using Einstein's equation,

$$hf = eV + \phi_o \quad (8.4)$$

Solving for the stopping potential,

ⁱⁱIn the present experiment we measure the stopping potential directly with an amplifier circuit, rather than monitoring the photoelectric current. See the appendices for details.

$$V = \left(\frac{h}{e}\right) f - \frac{\phi_o}{e} \quad (8.5)$$

If we plot stopping potential (y) versus the frequency of incident light (x), the resulting graph should be linear with slope h/e and intercept $-\phi_o/e$. Coupling our experimental determination of h/e from the slope of the $V(f)$ graph with the accepted value for e , 1.602×10^{-19} C, we can determine Planck's constant h .

Our experiment will explore the dependence of stopping potential on the frequency (wavelength) of incident light. Incident light from an atomic vapor source will be split into discrete wavelengths using a diffraction grating and lens assembly. An individual wavelength will be chosen and aligned incident on a photodiode, and an amplifier circuit will use feedback to apply precisely the voltage required to nullify the resulting photocurrent. This voltage is the stopping voltage, and it can be measured on the output terminals of the amplifier. The photodiode and amplifier circuitry are contained in a single assembly, henceforth known as the “ h/e apparatus.” Below, we describe the circuitry briefly.

8.1.4 Theory of Operation

In this experiment, monochromatic light falls on a the cathode plate of a vacuum photodiode tube that has a low work function ϕ_o . Photoelectrons ejected from the cathode collect on the anode.

The photodiode tube and its associated electronics have a small capacitance which becomes charged by the photoelectric current. When the potential on this capacitor reaches the stopping potential of the photoelectrons, the current decreases to zero, and the anode-to-cathode voltage stabilizes. This final voltage between the anode and cathode is therefore the stopping potential of the photoelectrons.

To let you measure the stopping potential, the anode is connected to a built-in amplifier with an ultrahigh input impedance ($> 10^{13} \Omega$), and the output from this amplifier is connected to the output jacks on the front panel of the apparatus. This high impedance, unity gain ($V_{\text{out}}/V_{\text{in}} = 1$) amplifier lets you measure the stopping potential with a digital voltmeter.

Due to the ultrahigh input impedance, once the capacitor has been charged from the photodiode current it takes a long time to discharge this potential through some leakage. Therefore, a shorting switch labeled “PUSH TO ZERO” enables you to quickly bleed off the charge. However, the op-amp output will not stay at 0 volts after the switch is released, since the op-amp input is floating.

Due to variances in the assembly process, each apparatus has a slightly different capacitance. When the zero switch is released, the internal capacitance along with the user's body capacitance

coupled through the switch is enough to make the output voltage jump and/or oscillate. Once photoelectrons charge the anode, the input voltage will stabilize.

8.2 Objective

The goal of this experiment is to determine the ratio of Planck's constant to the electron charge, h/e . Using the known value of the electron charge e , Planck's constant h can be determined.

Hypotheses:

1. The stopping potential, and thus the electrons' maximal kinetic energy, should increase with the frequency of incident light.
2. The stopping potential should not depend on the *intensity* of incident light.
3. The stopping potential versus incident frequency should yield a linear graph of slope h/e .

8.3 Relevant Reading

Taylor[1], Ch. 2,3,8

Pfeffer & Nir[2], Ch. 2.1-2

http://en.wikipedia.org/wiki/Photoelectric_effect

8.4 Supplies

1. Digital voltmeter or multimeter
2. h/e apparatus
3. h/e accessories
 - Lens/grating assembly
 - Light aperture assembly
 - Support base and coupling bar assembly
 - Light block (for rear window of source)
4. Filters for Yellow, Green, Blue, Violet, and UV Hg emission lines
5. Graded intensity filter
6. lab notebook and USB drive for saving spectra

8.5 Suggested Setup & Alignment procedure

Before starting, examine Fig. 8.1 and the equipment list above to verify that all the required items are in present. Your first task is to make sure that your setup looks like that in the picture!

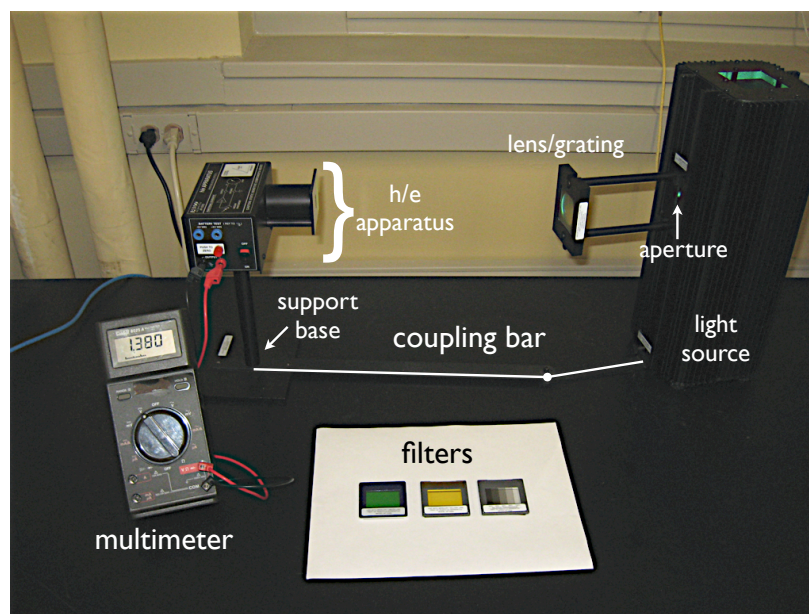


Figure 8.1: The photoelectric effect setup.

8.5.1 UV Safety

Ultraviolet radiation is emitted from low pressure mercury vapor discharges. The mercury vapor emits ultraviolet radiation when an electrical discharge is passed through it – most of the energy emitted is at a wavelength of 254 nm. This lies in the UV-C portion of the spectrum (180–280 nm). In the lamps used in this experiment, the 254 nm radiation will be absorbed by the outer glass tube encasing the lamp. If the outer glass tube is cracked or broken, this radiation can cause severe skin burns and eye inflammation. Inspect the tube for cracks before performing the experiment. If the glass bulb is broken, immediately turn the lamp off and ask the instructor to remove it to avoid possible injury.

In fluorescent lighting, the 254 nm radiation is used to excite a phosphor which coats the inside of the glass envelope of the lamp. The phosphor will re-emit at visible wavelengths (different phosphors produce different colors), and any UV-C which is not absorbed by the phosphor will be absorbed by the glass wall of the lamp. However, the mercury discharge will also emit at other wavelengths – notably at 365 nm, which lies in the UV-A (315 – 400 nm). This UV-A radiation may not be absorbed by the phosphor, and much of it will pass out through the lamp walls into the environment. For more information on UV radiation, see the Health Physics Society site at <http://www.hps.org/hpspublications/articles/uv.html>.

8.5.2 Setup

1. Insert the coupling bar assembly into the lower mounting groove of the Hg light source and secure in place with the thumbscrew.

2. The Hg source has two openings, allowing two simultaneous experiments. We will perform only one experiment, and for safety the rear opening (opposite side from the coupling bar) should be blocked. Verify that a light block is in place. If the light block is not present, a sheet of paper will suffice.
3. Slide the light aperture assembly into the center mounting groove on the front of the light source. Secure it in place by tightening the two thumb screws against the light source housing (finger tight only).
4. The lens/grating assembly mounts on the support bars of the light aperture assembly. Loosen the thumbscrew, slip it over the bars, and finger-tighten the thumbscrew to hold it securely. *The orientation is important! During the alignment procedure, you may need to turn the lens/grating assembly around to achieve sharp lines.*
5. Turn on the Hg lamp and allow it to warm up for a minimum of 10 minutes before making any measurements. In the mean time, you may proceed with setup and alignment. Check the alignment of the light source and aperture by looking at the light shining on the back of the lens/grating assembly. The light should shine directly on the center of the assembly; use the retaining screws on the aperture to adjust if necessary.
6. The support base should already be attached to the h/e apparatus. If it is not:
 - (a) Remove the screw from the end of the support base rod. Insert the screw through the hole in the support base plate and attach the rod to the support base plate by tightening the phillips screw.
 - (b) Place the h/e apparatus onto the support base assembly.
7. Place the support base assembly over the pin on the end of the coupling bar assembly (see Fig. 8.1).
8. While the h/e apparatus is off, connect a digital voltmeter to the “Battery Test” terminals of the h/e apparatus (see Fig. 8.2). Verify that the total voltage is $\gtrsim 17\text{ V}$. If it is not, ask the instructor for new batteries.
9. If the batteries are ok, connect the voltmeter to the “OUTPUT” terminals of the h/e apparatus. Press the “range” button until you have 3 decimal places on the measurement.

At this point, the setup is ready for operation. *Note that the vapor light sources can become quite hot!* The next step is to verify that the light from the source is diffracted into discrete lines and focused on the photodiode detector.

8.5.3 Alignment

1. Set the h/e apparatus directly in front of the Hg vapor source. By sliding the lens/grating assembly back and forth on its support rods, focus the light onto the white reflective mask of the h/e apparatus (Fig. 8.2). If you are not able to obtain a focused line, try turning the lens/grating assembly around.

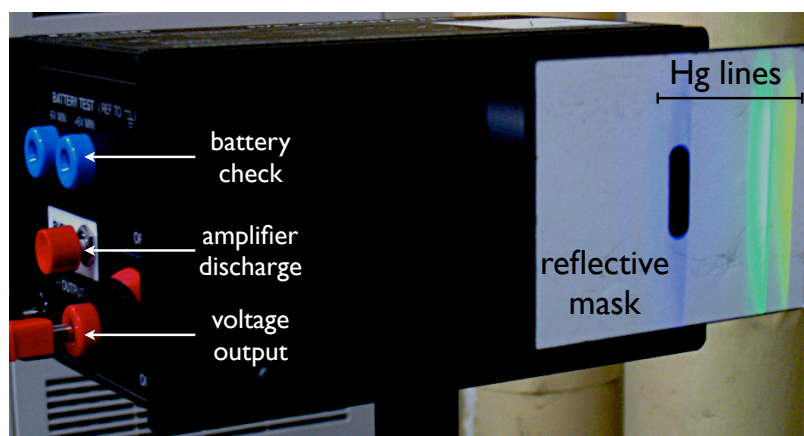


Figure 8.2: Projecting the Hg lines onto the reflective mask. Note that the cylindrical light shield is closed here, so we do not know if the blue Hg line falls on the photodiode detector!

2. Roll the cylindrical light shield of the apparatus (behind the white reflective mask) out of the way to reveal the white photodiode mask inside the apparatus (Fig. 8.3). Rotate the h/e apparatus about its support rod until the image of the aperture is centered on the window in the photodiode mask.
3. As above, slide the lens/grating assembly back and forth to achieve the sharpest possible image of the aperture on the window in the photodiode mask.
4. Turn the power switch for the h/e apparatus on.
5. Rotate the h/e apparatus *about the pin on the coupling bar assembly* until one of the colored maxima in the first order series shines directly on the slot in the white reflective mask.
6. Rotate the h/e apparatus on its support base so that the *same* spectral maximum that falls on the opening in the white reflective mask also falls on the window in the photodiode mask.ⁱⁱⁱ
7. It is crucial that only **one** color falls on the photodiode window, there must be no overlap from adjacent spectral maxima.
8. Replace the cylindrical light shield when you are satisfied.

8.5.4 Making Measurements

Once you have selected your spectral line, making a measurement of the stopping potential is straightforward:

1. Press the “push to zero” button on the side panel of the h/e apparatus to discharge any accumulated potential in the unit’s electronics. This will assure that the apparatus records only the potential of the light you are measuring. Note that the output voltage will drift with the absence of light on the photodiode.

ⁱⁱⁱThe white reflective mask is made of a fluorescent material which allows you to see the ultraviolet line as a blue line, and it also makes the violet line appear more blue. You see the actual colors of light if you hold a white non-fluorescent material in front of the mask (your hand will fluoresce slightly ...).

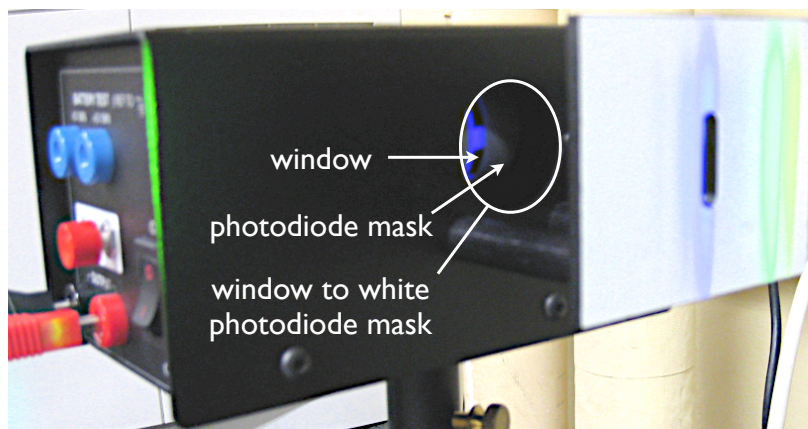


Figure 8.3: *Aligning the reflective mask aperture and photodiode mask with the incident Hg line. Note that this picture has been taken after rotating the cylindrical light shield to the open position. The focused blue Hg line falls on the window in the photodiode mask, the system is properly aligned.*

2. Read the output on your voltmeter. It is a direct measurement of the stopping potential for the photoelectrons.
3. Repeat the steps above a few times. Monitor the stopping potential for 20-30 seconds after zeroing the apparatus to note any initial transient behavior before “settling” on the actual stopping potential.

For some spectral lines, or when varying the intensity of incident light, you will need to use a light filter. The apparatus includes several color filters meant for use with the Hg vapor lamp, you can find their characteristics on page 71. The intensity filter has markings to indicate the percentage of transmitted light through each region of the filter. To use the filters:

1. The yellow, green, and variable transmission filters have magnetic strips and mount on the outside of the white reflective mask of the h/e apparatus.
2. The blue, violet, and UV filters (OPTIONAL) may be placed directly in front of the aperture, or affixed to the white reflective mask using a magnetic mount from the optics component shelf. Do not use tape!
3. The green and yellow filters are necessary, use them when you are investigating the yellow and green spectral lines of the Hg lamp. These filters limit higher frequencies of light from entering the h/e apparatus. This prevents ambient light from interfering with the lower energy yellow and green light and masking the true results. It also blocks the higher frequency ultraviolet light from the higher order spectra which may overlap with the lower orders of yellow and green (though this may not be visible due to reduced intensity).
4. The blue, violet and UV filters (OPTIONAL) can be used to ensure spectral purity of the blue and UV lines if there is a concern about overlap of the blue, violet, and UV lines. The violet filter is not recommended, as it results in considerable attenuation of even the violet line.

5. The variable transmission filter consists of computer-generated patterns of dots and lines that vary the intensity (not the frequency) of the incident light. The relative transmission can be set to 100, 80, 60, 40, and 20%

8.6 Experimental Procedure

According to the photon theory of light, the maximum kinetic energy K_{\max} of photoelectrons depends only on the frequency of incident light, and is independent of intensity. Thus, the higher the frequency of light, the greater its energy.

In contrast, the classical wave model of light predicted that K_{\max} would depend on light intensity. The brighter the light, the greater its energy.

In this experiment, you will investigate both assertions. In the first portion of the experiment, you will select two spectral lines from a Hg source and investigate the maximum energy of photoelectrons as a function of intensity. In the second portion of the experiment, you will select different spectral lines and investigate the maximum energy of photoelectrons as a function of the frequency of light.

8.6.1 Experiment I: Wave vs. Quantum Model of Light

1. Set up the equipment as described in Sect. 8.5.
2. Adjust the h/e apparatus so that only one of the spectral colors falls upon the opening of the mask of the photodiode. If you select the green or yellow line, place the corresponding colored filter over the white reflective mask on the h/e apparatus. You may also use the filters for the Blue or UV lines if you choose.
3. Place the variable transmission filter in front of the white reflective mask (and over the colored filter, if one is used) so that the light passes through the section marked 100% and reaches the photodiode. Record the voltmeter reading (stopping potential) in the table at the end of this section. Record the approximate error in your measurement.
4. Press the instrument discharge button, release it, and observe approximately how much time is required to return to the recorded voltage. Monitor the potential for at least a full minute to be certain the stable stopping potential has been reached.
5. Move the variable transmission filter so that the next section is directly in front of the incoming light. Record the new voltmeter reading, and approximate time to recharge after the discharge button has been pressed and released.
6. Repeat the previous step until you have tested all five sections of the filter.
7. Repeat the procedure for a second color in the spectrum.

8.6.2 Experiment II: Energy, Frequency, and Wavelength

1. Set up the equipment as described in Sect. 8.5.

2. You should be able to see five colors in two orders of the mercury light spectrum. Use a sheet of white paper to view the whole set of lines at once if necessary.
3. Adjust the h/e apparatus so that only one of the spectral colors from the first order lines falls upon the opening of the mask of the photodiode.
4. For each color in the first order, measure the stopping potential and record it in the table below (including an estimate of the error in your measurement). If you select the green or yellow line, place the corresponding colored filter over the white reflective mask on the h/e apparatus. You may also use the filters for the Blue or UV lines if you choose.
5. Repeat the process for the second order lines (OPTIONAL).

8.6.3 Data Tables

Note 1 THz = 10^{12} Hz.

Table 8.1: *Stopping potential and charging time vs. intensity*

Color	% Transmission	Stopping potential (V)	Approx. charge time (s)
Color #1:	100		
	80		
$\lambda =$	60		
	40		
	20		
Color #2:	100		
	80		
$\lambda =$	60		
	40		
	20		

Table 8.2: *Stopping potential vs. wavelength*

First order color	Wavelength (nm)	Frequency (THz)	Stopping potential (V)
yellow			
green			
blue			
violet			
UV			
Second order color	Wavelength (nm)	Frequency (THz)	Stopping potential (V)
OPTIONAL			
yellow			
green			
blue			
violet			
UV			

8.7 Analysis and Discussion

8.7.1 Stopping potential vs. intensity

1. Describe the effect that passing different amounts of the same colored light through the variable transmission filter has on the stopping potential and thus the maximum energy of photoelectrons, as well as the charging time after pressing the discharge button
2. Describe the effect that different colors of light had on the stopping potential and thus the maximum energy of photoelectrons.
3. Defend whether this experiment supports a wave or quantum model of light based on your results.
4. Explain why there is a slight drop in the stopping potential as the light intensity is decreased.^{iv}
5. Plot stopping potential versus intensity for each line, including error bars on the vertical axis.

8.7.2 Stopping potential vs. wavelength

1. Determine the wavelength and frequency of each spectral line.
2. Plot a graph of stopping potential versus frequency, including error bars on the vertical axis.
3. Determine the best-fit slope and y -intercept of the graph. Interpret the results in terms of the h/e ratio and the ϕ_o/e ratio. Using the accepted value for e , calculate h and ϕ_o . Get the accepted value of the work function from the instructor.
4. In your discussion, report your values and discuss your results with an interpretation based on a quantum model for light.

Visible Spectrum

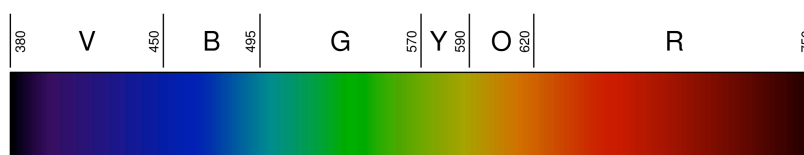


Figure 8.4: Linear representation of the spectrum of visible light. From http://en.wikipedia.org/wiki/Visible_spectrum. Numbers are wavelengths in nanometers.

Lamp and Filter Reference Data

^{iv}The impedance of the zero gain amplifier is very high ($\sim 10^{13} \Omega$) but not infinite, so some charge leaks off. Thus charging the apparatus is analogous to filling a bath tub with different water flow rates while the drain is partly open.

Table 8.3: Dominant Hg emission lines

Color	f (THz)	λ (nm)
yellow	518.7	578 [†]
green	549.0	546.1
blue	687.9	435.8
violet	740.9	404.7
ultraviolet	820.3	365.5

[†] The yellow line is a doublet of 578 and 580 nm.

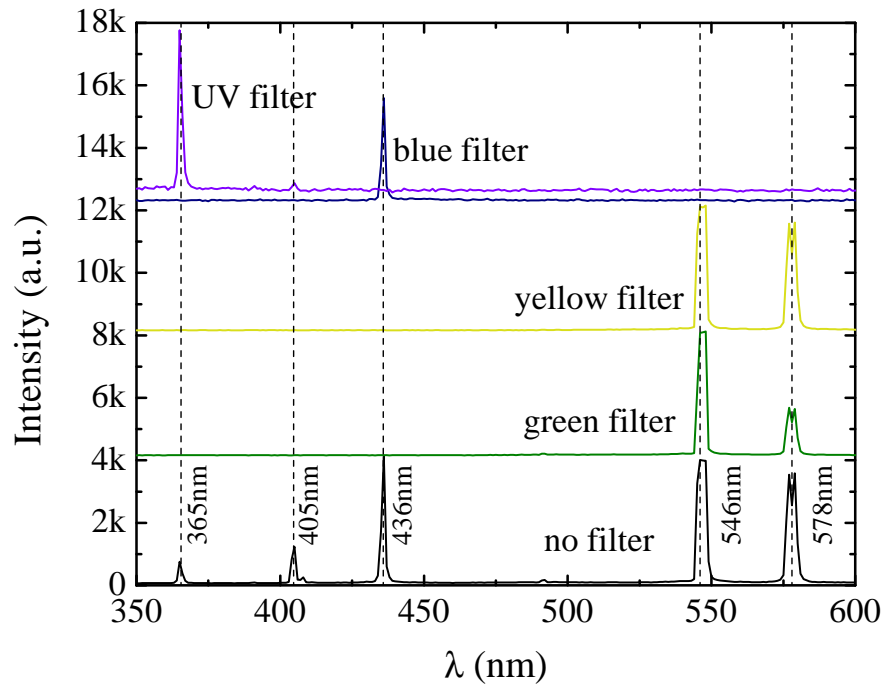


Figure 8.5: Intensity of the Hg high pressure lamp as a function of wavelength with and without various filters. The vertical dashed lines are reference data, except the yellow 578 nm line which was determined experimentally with the Ocean Optics USB-600 spectrometer. Note that the 578 nm line is actually a doublet of 578 nm and 580 nm. The intensity of the UV- and blue-filtered spectra have been multiplied by ten times for visibility, and all spectra have been vertically separated for clarity.

Observation of Atomic Spectra

9.1 Introduction

When gases are placed in a tube and subjected to a high-voltage electric discharge, the electrons in the atoms can be excited to higher energy levels within the atoms; when they return to their original levels electromagnetic radiation is emitted. In this experiment a vapor (such as helium or hydrogen) is placed in an electric discharge tube and a high voltage is placed across the tube. The excited emission may look almost white or have a characteristic color depending on the vapor inside, but it is in reality composed of a number of different colors or wavelengths of visible light. You will use a USB-controlled optical spectrometer to carefully measure the emission lines and their relative intensities.

In the second stage of this laboratory, you will apply your knowledge of spectroscopic measurements to study the thermal emission characteristics from two very important sources: the sun, and normal household light bulbs. Comparing these spectra will help to illustrate the concepts behind black-body radiation, as well as solar and atmospheric physics and color composition. You will be able to estimate the temperature of the surface of the sun and the light bulb filament, and by comparison with your atomic spectra from the first stage, establish the likely presence of H, and He in the sun.

9.1.1 Atomic Spectra

9.1.2 Hydrogen Spectrum: Balmer Series

When gases are placed in a tube and subjected to a high-voltage electric discharge, the electrons in the atoms can be excited to higher energy levels within the atoms; when they return to their original levels electromagnetic radiation is emitted. The resulting light intensity versus wavelength for an excited gas should thus contain discrete peaks, corresponding to discrete atomic transitions unique to individual atoms. Some of this radiation may be in a wavelength region that is visible to the human eye. Similarly, when light passes through a gas, photons whose energies correspond to these transitions will be absorbed, leading to a *loss* of intensity at the corresponding wavelengths.

In this experiment, a vapor (such as helium or hydrogen) is placed in an electric discharge tube and a high voltage is placed across the tube. The excited emission may look almost white or have a characteristic color depending on the vapor inside, but it is in reality composed of a number of different colors or wavelengths of visible light. For the sake of discussion, we will focus on the hydrogen atom.

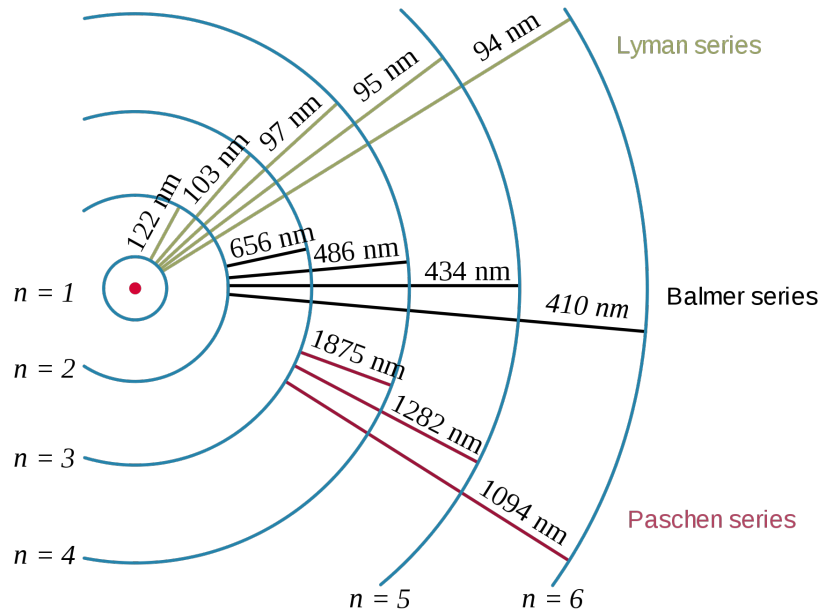


Figure 9.1: In the simplified Bohr model of the hydrogen atom, the emission lines correspond to an electron jump from a higher energy level n to a lower n' with the emission of a photon corresponding to the energy difference between the two levels. For the visible Balmer series, $n'=2$ and n runs from 3 upward. From http://en.wikipedia.org/wiki/Hydrogen_spectrum.

A *series* in the emission spectrum of excited hydrogen gas is the set of spectral lines observed when various excited states of the hydrogen atom decay into a common final state, illustrated schematically in Fig. 9.1. In the case of the Balmer series, which contains several wavelengths in the visible range, this final state is the $n = 2$ state of hydrogen. The quantity n is the so-called principal quantum number of the hydrogen atom. The energy levels of the hydrogen atom are given by the formula

$$E_n = -\frac{E_i}{n^2} \quad n \in \mathbb{N}_1 = \{1, 2, 3, \dots\} \quad E_i = \text{ionization energy} = 13.6 \text{ eV} \quad (9.1)$$

Essential to the derivation of emission and absorption spectra is the postulate that a single light quantum, more commonly called a photon, of energy $\Delta E_n = hf = hc/\lambda$ is emitted when the atom makes a downward transition from an excited state to a lower state (here h is Planck's constant, c is the speed of light, f the frequency of emitted light, and λ the wavelength of emitted light). The energy of the light emitted from a transition between two energy levels E_n and $E_{n'}$ is thus equal to the photon energy,

$$hf = \frac{hc}{\lambda} = \Delta E_n = E_n - E_{n'} = E_i \left(\frac{1}{n'^2} - \frac{1}{n^2} \right) \quad (9.2)$$

Hence, for the case of the Balmer series where the final state $n'=2$ is the same for every line, we obtain

$$hf = \frac{hc}{\lambda} = \Delta E_n = E_n - E_{n'} = E_i \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \quad (9.3)$$

Other series include the Lyman series (final state $n = 1$, emission in the UV) and the Paschen series (final state $n = 3$, emission in the infrared). Some of the lines from the Paschen series can be observed with the USB-controlled spectrometer you will use. Often the spectral lines are labeled α , β , γ , δ , etc., starting with the longest wavelength line.

For historical reasons, the formula for hydrogen emission lines is often given in terms of the principle quantum numbers, the emission wavelength, and the Rydberg constant R_H :

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n'^2} - \frac{1}{n^2} \right) \quad (9.4)$$

You should be able to derive this equation using the equation of the energy levels in a hydrogen atom and show how R_H is related to E_i and fundamental constants. You should also be able to sketch the energy level diagram of hydrogen and draw in the transitions you observe in this experiment. Your measurements of emission lines will ultimately give you an experimental value for R_H , which you can compare to accepted values.

9.1.3 Helium Spectrum

The emission spectrum from helium turns out to be considerably more complex than hydrogen. This does not bode well for elements of increasing sophistication, of course. A full understanding of the spectrum of complicated atoms cannot proceed without a detailed discussion of spin and angular momentum, which is somewhat beyond the scope of this course. However, we can make some general comments about the main features of the helium emission spectrum.

One can construct an energy level diagram for He similar to that of H. However, the fact that we now have two electrons means that we must consider the interactions between them, and several multi-electron atom features are already apparent in He. If we wish to study only lower-level excitations of the He atom, we presume that one electron is in its $1s$ ground state, while the other is excited into a higher energy level. When both electrons are in the $1s$ ground state, their spins are antiparallel by Hund's first rule.ⁱ However, once excited, the electron in the upper level now has two possibilities with regard to its spin: it may be excited into a state with spin antiparallel to the ground state electron ($S=0$, singlet-state *para-helium*), or it may be excited into a state parallel to the ground state electron ($S=1$, triplet-state, *ortho-helium*). It is observed that the ortho-helium states (parallel spins) lie lower in energy than the para-helium states, for a few reasons which are rather too detailed to explain here. The main point is this: if in the H atom we had a single possible transition, in the helium transition there are at *least* two similar transitions, since we must

ⁱI.e., the term with maximum multiplicity lies lowest in energy.

consider the extra degrees of freedom provided by the spin quantum number. To this we must also add the spin-orbit interaction, which further splits some of these lines into even *more* lines. Long story short: helium is already a bit complicated.

For the purposes of this laboratory, we will simply consider para- and ortho-helium to be two different states of helium. Our helium discharge can be considered to be made up of a mixture of para- and ortho-helium, and the observed spectra should contain a mixture of the para- and ortho-spectra. A reference spectra is provided in the appendix, and an energy level diagram is depicted below in Fig. 9.2.

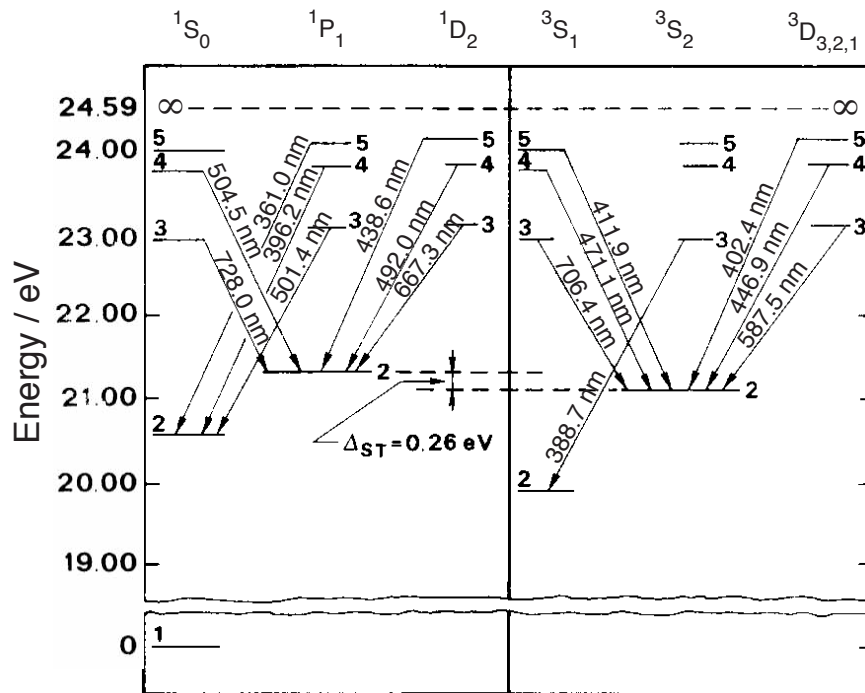


Figure 9.2: Energy level diagram for singlet ($S = 0$) para-helium (left) and triplet ($S = 1$) ortho-helium (right). The corresponding emission wavelengths are also given in table 9.2. From [9].

In fact, the Bohr model of the hydrogen atom can be “rescued” by introducing another parameter, the so-called “quantum defect correction” δ_{nl} . The defect correction essentially parameterizes the degree of deviation from the Rydberg formula, and is related to the interaction of the electron whose transitions produce the spectrum with the inner core electrons. These interactions are not present for hydrogen. It can be considered approximately constant for levels of different n but with the same orbital momentum quantum number l (i.e., for the same series). For different series, it takes on different values: it is highest for the S seriesⁱⁱ and close to zero for the D series. Using the

ⁱⁱThis is spectroscopic notation, which you are not expected to be familiar with. For our purposes, it merely characterizes a set of energy levels in Fig. 9.2. We have three types of levels: S , P , and D (not the same as the s , p , and d used in atomic orbital notation).

defect correction, we can use the following empirical relationship for the D series:

$$\frac{1}{\lambda} = -R_{He} \left[\frac{1}{n_1^2} - \frac{1}{(n_2 - \delta_{3p})^2} \right] \quad (9.5)$$

Here R_{He} is the Rydberg constant for He, $n_1 = 3, 4, 5$, $n_2 = 2$, and δ_{3p} is the (dimensionless) defect correction for the para- or ortho- 3P state. Thus, a plot of $1/\lambda$ versus $1/n_1^2$ should give a straight line of slope $-R_{He}$ and intercept $R_{He}/(2 - \delta_{3p})^2$, allowing a determination of both R_{He} and δ_{3p} . Beyond identifying lines in your spectra due to transitions in para- and ortho- helium states, you should attempt a determination of these two constants for both ortho- and para- He. Accepted values are $\delta_{3p} = -0.009$ and 0.063 for para- and ortho-helium, with

9.1.4 Black-body Radiation

“Blackbody radiation” or “cavity radiation” refers to an object or system which absorbs all radiation incident upon it and re-radiates energy which is characteristic of this radiating system only, not dependent upon the type of radiation which is incident upon it. The radiated energy can be considered to be produced by standing wave or resonant modes of the cavity which is radiating.

The amount of radiation emitted in a given frequency range should be proportional to the number of modes in that range. The best of classical physics suggested that all modes had an equal chance of being produced, and that the number of modes went up proportional to the square of the frequency.

From the assumption that the electromagnetic modes in a cavity were quantized in energy with the quantum energy equal to Planck’s constant times the frequency, Planck derived a radiation formula. The average energy $\langle E \rangle$ per “mode” or “quantum” is the energy of the quantum times the probability that it will be occupied (the Einstein-Bose distribution function):

$$\langle E \rangle = \frac{hf}{e^{hf/k_B T} - 1} \quad (9.6)$$

where f is the frequency of radiation, T the temperature of the source, and k_B is Boltzmann’s constant. This average energy times the density of such states, expressed in terms of either frequency or wavelength gives the energy density, the Planck radiation formula in terms of wavelength or frequency:

$$S_f = \frac{8\pi h}{c^3} \frac{f^3}{e^{hf/k_B T} - 1} \quad (9.7)$$

$$S_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/k_B T \lambda} - 1} \quad (9.8)$$

Here c is the speed of light. The Planck radiation formula is an example of the distribution of energy according to Bose-Einstein statistics. An example of an approximately black body spectrum is

shown Fig. 9.3, the spectrum of an incandescent bulb at various applied voltages (and thus various powers).

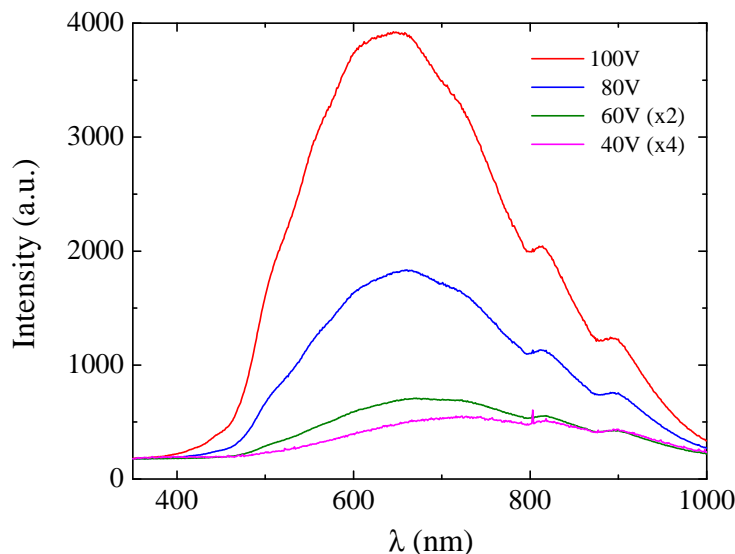


Figure 9.3: Spectrum of a 60 W soft white incandescent bulb at various ac supply voltages, exhibiting approximately black body behavior.^[3]

The above expressions are obtained by multiplying the density of states in terms of frequency (or wavelength) times the photon energy times the Bose-Einstein distribution function with normalization constant $A=1$. To find the radiated power per unit area from a surface at this temperature, multiply the energy density by $c/4$. The density above is for thermal equilibrium, so setting inward=outward gives a factor of $1/2$ for the radiated power outward. Then one must average over all angles, which gives another factor of $1/2$ for the angular dependence which is the square of the cosine.

From our point of view, the interesting point is that the peak of a blackbody spectrum as a function of wavelength can be uniquely related to the temperature of the source through Wien's displacement law:

$$\lambda_{\max} = \frac{b}{T} \quad (9.9)$$

where the constant b , known as Wien's displacement constant, is equal to $2.8978 \times 10^{-3} \text{ m} \cdot \text{K}$. Thus, finding the peak wavelength will tell you the source temperature, and we will use this technique to estimate the sun's temperature and the temperature of the filament in an incandescent bulb. Similar methods are used in infrared imaging to determine temperatures remotely.

9.1.5 Solar Spectrum

The spectrum of the Sun's solar radiation is close to that of a black body with a temperature of about 5800 K. About half that lies in the visible short-wave part of the electromagnetic spectrum and the other half mostly in the near-infrared part. Some also lies in the ultraviolet part of the spectrum. In addition to the black body spectrum, however, several other features are readily noticeable, the most prominent of which are narrow bands or "notches" in the spectrum corresponding to discrete absorptions. These absorptions have two origins: first, one has absorption in the outer layers of the sun, due to the radiation from the interior exciting atoms in the outer layers; second, one has to consider absorption of solar radiation by our own atmosphere.

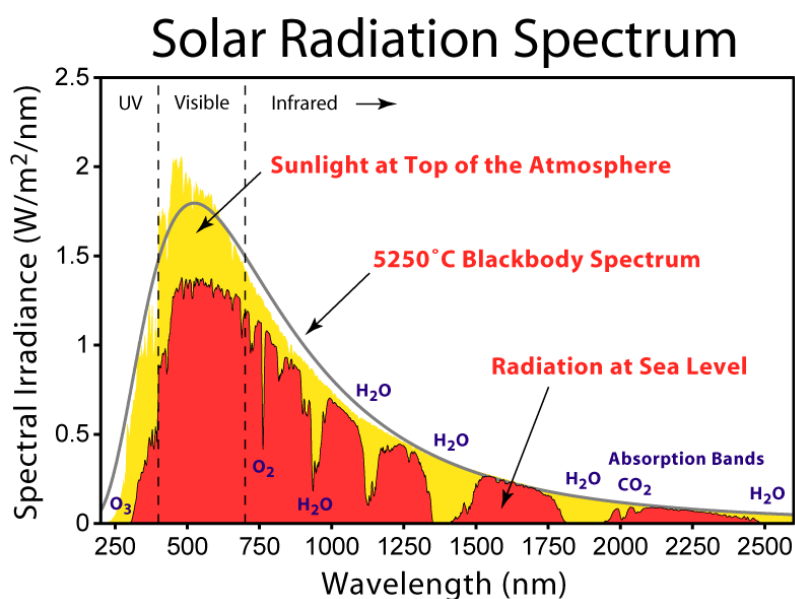


Figure 9.4: Solar radiation spectrum, showing the black body contribution and solar/atmospheric absorption. From <http://en.wikipedia.org/wiki/Sunlight>.

The prominent absorption lines in the solar spectrum are known as *Fraunhofer lines*, named for the German physicist Joseph von Fraunhofer. In all, he mapped over 570 lines, and designated the principal features with the letters A through K, and weaker lines with other letters. Modern observations of sunlight can detect many thousands of lines. Your quick observations should yield dozens, many of which can be correlated with your measured H and He spectra to provide a strong indication that these elements are present in the sun's outer layers. Figure 9.4 shows an example spectrum, exhibiting black body behavior and solar/atmospheric absorption.

It was later discovered by Kirchhoff and Bunsen that each chemical element was associated with a set of spectral lines, and deduced that the dark lines in the solar spectrum were caused by absorption by those elements in the upper layers of the Sun. Some of the observed features are also caused by absorption in oxygen molecules in the Earth's atmosphere.

9.1.6 Ocean Optics USB spectrometer

The Ocean Optics Red Tide Spectrometer (Fig. 9.5) is a preconfigured, off-the-shelf spectrometer where all of the optical bench options such as grating, and entrance slit size are already selected. It has a wavelength range of 350–1000 nm and uses a detector with 650 active pixels; that’s 650 data points in one full spectrum, or one data point per nanometer. Data programmed into a memory chip on each Red Tide includes wavelength calibration coefficients, linearity coefficients, and the serial number unique to each spectrometer. The spectrometer operating software simply reads these values from the spectrometer, eliminating the calibration procedures normally associated with optical spectrometers. Figure 9.5 also shows a diagram of how light moves through the optical bench of the spectrometer, along with a brief explanation of the components.[10] Note that the optical bench has no moving parts.

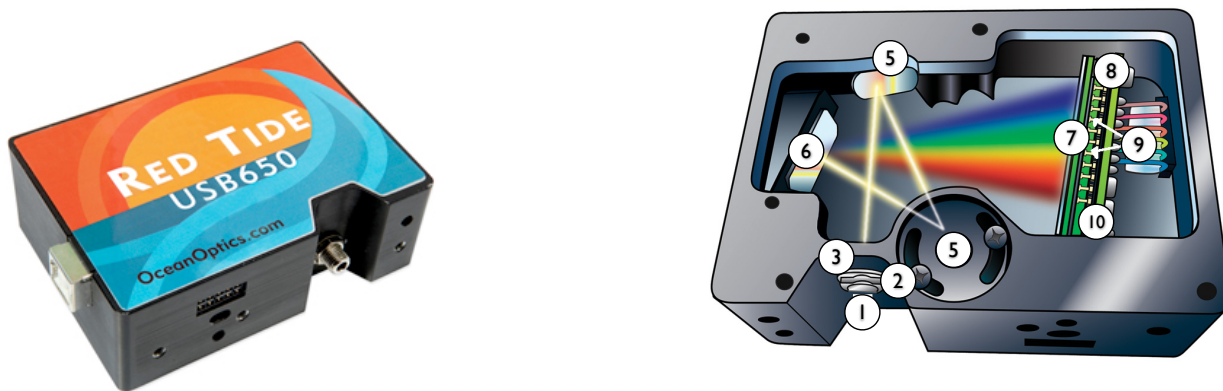


Figure 9.5: **Left:** USB-650 “Red Tide” spectrometer. **Right:** Components of a typical Ocean Optics USB-series spectrometer. See text for explanation of numbered components. Adapted from the Ocean Optics operating instructions manual for the USB-4000 spectrometer. Image credits: Ocean Optics.[10]

1. **SMA connector:** Secures the input fiber to the spectrometer.
2. **Slit:** A dark piece of material containing a rectangular aperture, which is mounted directly behind the SMA Connector. The size of the aperture (from 5 to 200 μm) regulates the amount of light that enters the optical bench and controls spectral resolution.
3. **Filter:** Restricts optical radiation to pre-determined wavelength regions.
4. **Collimating Mirror:** Focuses light entering the optical bench toward the grating. Light enters the spectrometer, passes through the SMA Connector, Slit, and Filter, and then reflects off the Collimating Mirror onto the Grating.
5. **Grating:** Diffracts light from the Collimating Mirror and directs the diffracted light onto the Focusing Mirror.
6. **Focusing Mirror:** Receives light reflected from the Grating and focuses first-order spectra onto the detector plane.

7. **Detector Collection Lens:** An optional component that attaches to the Detector to increase light-collection efficiency. It focuses light from a tall slit onto the shorter Detector elements.
8. **Detector** Collects the light received from the Focusing Mirror or L4 Detector Collection Lens and converts the optical signal to a digital signal. Each pixel on the Detector responds to the wavelength of light that strikes it, creating a digital response. The spectrometer then transmits the digital signal to the Oceanview application.
9. **OFLV Filters:** OFLV Variable Longpass Order-sorting Filters block second- and third-order light. Optional upgrade; note present in the current setup.
10. **UV4 Detector Upgrade:** The detector's standard window is replaced with a quartz window to enhance spectrometer performance (< 340 nm). Optional upgrade; not present in the current setup.

With the present configuration, the spectrometer can measure intensities within a wavelength range of approximately 350 to 1000 nm. See the Ocean Optics manual provided for additional specifications operational instructions.

The Red Tide Spectrometer connects to a computer via the USB port. When connected through a USB 2.0 or 1.1 port, the spectrometer draws power from the host computer, eliminating the need for an external power supply. The Red Tide is controlled by the Oceanview software, a completely modular, Java-based spectroscopy software platform that operates on Windows, Macintosh and Linux operating systems. The operation of the Oceanview software is straightforward, and instructions on its use are integrated into the laboratory procedure below.

9.2 Objective

In this experiment you will observe and measure the discrete wavelengths of different colors of light emitted by atoms. You will first observe light emitted from excited hydrogen atoms, and compare that spectra to more complicated atoms, i.e., He and Hg. You will also characterize the spectra from continuous thermal sources (approximate black bodies) to ascertain the source temperature. From a solar spectra, you should also be able to identify solar and atmospheric absorption (giving indications of the solar and atmospheric composition).

Hypotheses: The absorption and emission of light by atoms should occur at discrete energies, whereas the emission of light by a thermal source should be continuous. The hydrogen spectrum can be well-characterized by the Bohr model and Rydberg formula, while more complicated atoms require a more sophisticated model. Real thermal sources display approximate black body behavior, accompanied by discrete emissions.

9.3 Relevant Reading

Pfeffer & Nir[2], 2.1.1-7, 3.2.1

http://en.wikipedia.org/wiki/Black_body

<http://en.wikipedia.org/wiki/Sunlight> (spectra part only)

http://en.wikipedia.org/wiki/Fraunhofer_line

http://en.wikipedia.org/wiki/Color_temperature (beginning only)

9.4 Supplies & Equipment

1. USB drive or other portable mass storage
2. H, He, Ar, Ne, and Hg vapor discharge tubes
3. Additional vapor discharge lamp of your choosing
4. Vapor lamp housing & power supply
5. Incandescent and compact fluorescent bulbs
6. Bulb holder
7. Variable transformer, 0–140 V
8. Ocean Optics USB-650 “Red Tide” spectrometer and PC
9. lab notebook and USB drive for saving spectra

9.5 Suggested procedure

9.5.1 USB-650 Setup

The USB-650 spectrometer should already be plugged in to a USB port on the adjacent PC, and the input fiber optic cable should be running through a hole in the “dark box.” Inside the dark box you should also find a discharge lamp power supply and a lab stand and clamp to keep the input fiber in place, i.e., pointed at the source. Basically, the setup should look something like Fig. 9.6 below.

(In or near the dark box should also be an incandescent bulb holder and a variable transformer, not shown in the picture.) If elements of the setup appear to be missing or improperly set up, please notify the instructor. A few words on the elements of the setup:

- **USB-650:** this little box is the entire spectrometer. It should be plugged into a USB port on the rear of the PC, and the optical input fiber should run through a hole in the dark box.
- **Input fiber:** a fiber optic cable used to couple incident light into the spectrometer. Basically: point it at the source! At its heart it is a glass fiber, and it is fragile.
 1. Remove the plastic cover from the SMA 905 Connectors gently. Pulling the connector away from the fiber when removing the cover will permanently damage the fiber.

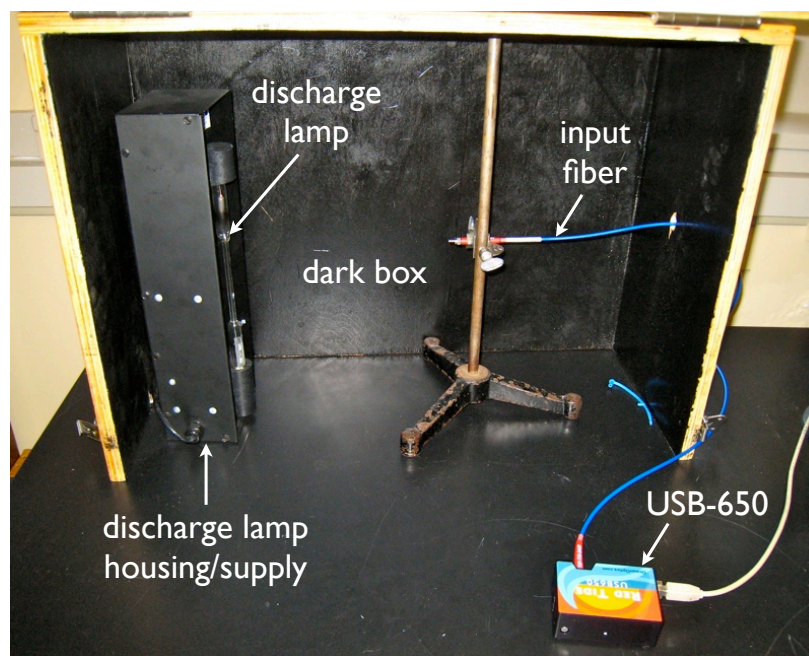


Figure 9.6: *The spectrometer setup.*

2. Inspect the fibers periodically to ensure that the fibers are transmitting light. Broken fibers stop transmitting light. Visually inspect the fibers for light transmission from time to time.
 3. **Avoid coiling the fiber too tightly.** While the momentary bend radius of a fiber is typically 200 times the diameter of the fiber, the maximum bend radius of a fiber that you fix in place is 400 times the diameter of the fiber (e.g. 16 cm for a 400 μm fiber). Bending the fiber past this threshold causes attenuation and can cause permanent damage to the fiber.
 4. **Avoid bending the fiber in sharp angles.**
 5. Cover the SMA 905 connectors with the supplied caps when the fiber is not in use.
 6. Clean the fiber ends periodically with lens paper and distilled water, alcohol, or acetone. Avoid scratching the surface.
- **Dark box:** just what it sounds like. The front of the box may be closed to create a completely dark environment for the spectrometer to operate in, which will eliminate any ambient light from polluting your spectra. Close the lid carefully, and be sure that the optical fiber does not get caught!
 - **Discharge lamp:** a glass tube with two electrodes, filled with gas. Handle with care, they are fragile and will break if dropped. After several minutes of operation, they can become quite hot as well!
 - **Discharge lamp housing/supply:** the discharge lamps operate at 5000 V dc. Never, ever remove or insert lamps while the power is on, and be extremely careful not to touch the housing electrodes even when the supply is off. When removing lamps that have been in use,

check that they are not too hot to handle. If you only replace tubes with the power off, and avoid touching the lamp or surrounding region during operation, there is no danger of shock.

9.5.2 Oceanview Software

The Oceanview software will assist you in acquiring a spectrum of emitted light intensity as a function of wavelength. To begin, start the “Oceanview” program from the icon on the desktop. Click “okay” if there is a splash screen and click the “Acquisition Group Window” button on the left. Lock the acquisition group window in place with the “Restore” icon on the upper right corner of the acquisition group window, see Figure 9.7. You should see a window like the one in Fig. 9.7.

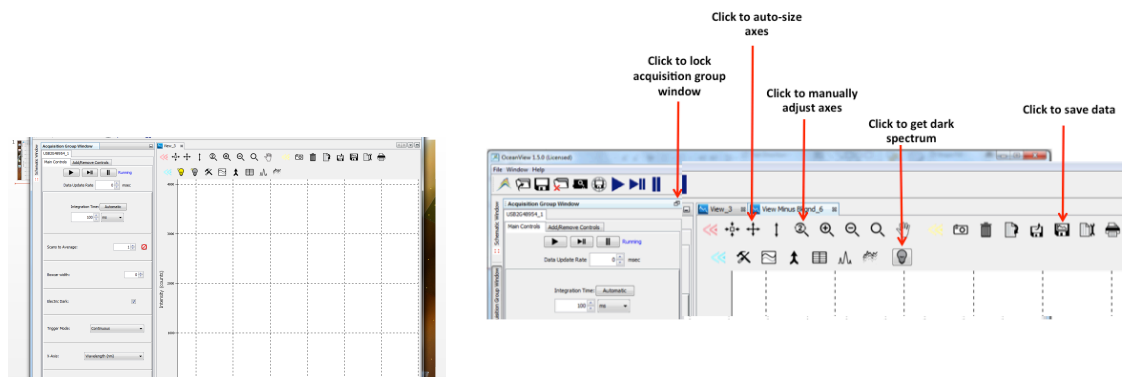


Figure 9.7: Left: Oceanview software after opening and clicking the “Acquisition Group Window” button. The spectrometer should already be acquiring a spectrum in real time, shown as a red line. Right: Oceanview software icons.

For basic emission spectroscopy, for example to observe the spectrum of a hydrogen discharge source, we only need to set up the acquisition parameters and store a reference spectrum with no incident light. First, set the following parameters:

1. Set the integration time to 100 ms. This is the time taken by the CCD detector to collect incoming photons. Longer integration times lead to better signal-to-noise ratios, but longer acquisition times. If the integration time is too high the spectrum will “saturate” by exceeding the maximum count of 4095. No harm is done, but to see the full trace the integration time must be reduced, or the light source attenuated.
2. Set the Scans to Average to 1. This is the number of discrete acquisitions taken and averaged together to yield a single resulting spectrum on the screen. The signal to noise ratio (S/N) improves with for larger numbers of scans n , as \sqrt{n} . Keep in mind that the acquisition time grows as n however.
3. Set the boxcar average to 0. This parameter sets the boxcar smoothing width. It averages the counts of a group of adjacent detector elements. For example, a value of 3 averages the 3 points (or bins) to the right and 3 points to the left. The final result would be a smoother spectrum and improved S/N ratio. But if it is too high, a loss in spectrum resolution would result. Normally the boxcar average is not required.

You may need to adjust these parameters during your actual measurements to improve S/N or avoid saturation. Typically, you want to adjust the integration time to yield a maximum number of counts at any wavelength to be 1000–3000, but less than 4000. Increase the number of averages until you are satisfied with the S/N. Before making your measurements, only one more thing needs to be done, recording a reference “dark” spectrum. This is simply a spectrum with no light source present to verify the spectrum baseline.

1. Make sure the spectrometer input fiber is inside the black wooden box (through the hole on the right side). Do not bend the fiber excessively, it can easily be broken!
2. Make sure all light sources are turned off, and *carefully* close the lid on the black wooden box. Tighten the latches. The real-time spectrum should now be a flat line near zero counts.
3. Click on the darkened light bulb to acquire and store a dark spectrum. If you wish, click on the disk icon (see Figure 9.7) to save this reference spectrum. The icon with the wrench next to it can be used to change the data format if needed: check that “ASCII with header data” is the format chosen. **Data will be saved by default in C:/Users/student/oceanview/records** . The file saved is a text file which can be opened later in Excel for further analysis. You will need to rename your spectrum files to something descriptive.
4. The window labeled “View Minus Bkgnd” will now show all spectra with the dark spectrum subtracted off; use this window for all of the following measurements.

You are now ready to acquire emission spectra. *Carefully* open the dark box.

9.5.3 Measurement of the Hydrogen spectrum

When using the discharge lamps, there are three important safety points. First, do not insert or remove bulbs while the power supply is on. Second, do not touch the bulb or power supply contacts while the bulb is mounted, even if you think the supply is off! Finally, the bulbs can become quite hot after operation, especially the hydrogen bulb. Allow sufficient time for them to cool before removing.

1. Verify that the discharge tube supply is turned off.
2. Carefully insert the H spectral tube in the discharge tube housing. The contacts are spring-loaded, so gently press the bottom of the bulb into the socket, allowing the top of the bulb to fit.
3. Situate the discharge tube housing in the black box on the left side, such that the bulb faces the right side. The cord should go through a notch in the left side of the box.
4. Situate the spectrometer input fiber on the far right side of the box, pointing at the discharge tube. You may need to adjust the height of the input fiber. **Do not bend the fiber. Do not tighten the clamp on it beyond what is necessary to keep the fiber from moving.** The fiber is fragile, and can be broken by either action.
5. Your setup should now look like Fig. 9.6.

6. Turn on the discharge source, you should see the hydrogen tube glowing.

You are now ready to acquire spectra. From the previous section, the spectrometer should already be acquiring data in real-time and measuring in “View Minus Bkgnd” mode. If so, you should see a few high, sharp peaks in your spectrum.

1. Adjust the integration time and the position the input fiber such the peak intensity is below 4000 to avoid saturation of the detector. You may want to fix the integration time at 10 or 50 ms and then slowly move the fiber stand closer until you observe a good spectrum which does not saturate the detector.
2. Close the dark box and acquire a spectrum. You can adjust the axes with the crossed-arrow icon (auto-adjust, recommended) or the magnifying glass with the “2” in it (manual adjust). You may want to average several or many spectra together for better signal to noise ratio (e.g., 5–10 averaged spectra).
3. Save one of these spectra by clicking on the disk icon.

At this point, you have a nice overview of the hydrogen spectrum, displaying all the primary peaks. However, there may be (i.e., there are) smaller peaks “hiding” below your current resolution limit. The primary problem is the large difference in intensity of some of the peaks – keeping the largest peaks below overload intensity means that some of the smaller ones are barely visible.

This is not just a problem of the plot scale: while the integration time for the measurement is sufficient to collect data for the larger peaks, the peaks of much lower intensity will have an insufficient S/N for proper identification using the same integration time. The problem is a generic one: we are trying to measure over a large dynamic range, and that usually means that the measurement suffers at one extreme or the other. One simple way to minimize this problem is simply to take two spectra. We already have one at low integration times, suitable for measuring the most intense peaks (at the expense of not seeing the smaller ones). Now we will just take a second spectra with much longer integration time suitable for measuring the weaker peaks (at the expense of “cutting off” the larger ones by saturating the spectrometer).

1. Adjust the integration time until one of the smaller peaks easily visible in the previous scan now has ~ 3500 counts.
2. The actual value of integration time will depend on how you have positioned the fiber relative to the source, but it may need to be as high as a few seconds, and probably about ten times higher than your previous scan.
3. You should now see a few peaks in the spectra that were not visible before. You can also adjust the number of scan averages to improve the S/N. You should see at least 6 lines from the Balmer series (see Table 9.1).
4. The acquisition times will now be several seconds long, and you will be sensitive to very low light levels. Be sure that you have tightly closed the dark box.

5. When you are satisfied, save this “zoomed” spectra.

You can first identify the major spectral peaks in the “overview” scan, and then use the “zoom” scan to identify any smaller peaks. Keep in mind that if peaks in the zoomed spectra have a flat top, they have overloaded the spectrometer, and you should not use them for peak identification (use the overview scan).

9.5.4 Helium spectra

Once you have completed the measurement for a hydrogen lamp, you should complete the same measurements for the He and Hg sources. Turn off the discharge lamp source and carefully replace the bulb. **Do not access the bulb when it is turned on. The bulb may have become quite hot during the experiment, be careful when removing it!**

9.5.5 Heavier elemental spectra

As a point of comparison, choose another elemental gas (Ar, Ne, Hg, etc.) and record spectra.

9.5.6 Incandescent Bulb Spectrum

Now screw an incandescent 60 W bulb into the holder inside the dark box.

1. Plug the source into the variable transformer, and make sure the transformer dial is set to 0.
2. Plug the transformer into the power strip at the base of the table. By varying the output voltage of the transformer from 0–140 V, you are varying the power to the bulb and hence the brightness.
3. Set the transformer to 120 V, it should glow brightly. Note: read the actual voltage value off the meter, not the dial!
4. Start the spectrometer acquisition, you should observe a broad spectrum reminiscent of black body radiation.
5. Adjust the integration time and the position the input fiber such the peak intensity is below 4000 to avoid saturation of the detector. You may want to fix the integration time at 10 or 100 ms and then slowly move the fiber up and down on its stand until you observe a good spectrum which does not saturate the detector.
6. Close the dark box and lock the lid with the clamps on the sides and acquire a spectrum. You may want to average several or many spectra together for better signal to noise ratio (e.g., 5–15 averaged spectra).
7. Without disturbing the setup, acquire spectra for several different bulb powers, noting the voltage and current. You should see a much lower intensity at all wavelengths, but also a shift of the peak in the spectrum to higher wavelengths.

9.5.7 Solar Spectrum

1. Carefully unscrew the clamp on the input fiber and remove it from the dark box.
2. Place the stand on top of the dark box and replace the fiber. **Do not tighten the clamp on the fiber beyond a very gentle “finger tight” or you will damage it!**
3. Point the input fiber at the sky through the window. Note the rough appearance of where you are pointing the fiber (e.g., cloudy, white, gray, blue). Note the date and time (as you would for *any* measurement ...).
4. Set the integration time for the spectrometer to 100 msec with 3 averages.
5. Start the spectrometer acquisition, you should observe a broad spectrum reminiscent of black body radiation. You may need to rescale the plot. If the spectrum “flatlines” on the top, you have overloaded the detector. Halve the integration time, and repeat until the spectra does not have a flat top indicating saturation.
6. When the intensity levels are high ($\sim 1000-3000$ max) but not saturated (≥ 4000), you are ready. Increase the number of averages to at least 10 until you are satisfied with the signal to noise ratio. Remember, you are trying to tell the difference between discrete absorptions in your spectrum and noise!
7. Save the spectrum.

9.5.8 Compact Fluorescent Bulb Spectrum

Also acquire a spectrum for a the compact fluorescent light bulb, using *only* a setting of 120 V on the variable transformer. As with the other measurements, position the input fiber such that your maximum intensity for any line is below 4000 to avoid saturating the detector.

9.6 Analysis and Discussion

Note: The spectrometer often shows weak peaks at 777, 804, and 845 nm which are not due to the light sources, but characteristic of the spectrometer itself. Do not worry if you cannot identify these lines.

9.6.1 Hydrogen Spectrum:

Identify all of the observed peaks and calculate the frequency and photon energy for each peak. Assign for each peak the atomic transitions responsible for each peak (from which level to which level). How do your values compare to reference data? Are there expected transitions you do not observe? Make a plot of $1/\lambda$ versus $1/n^2$. Its slope should be R_H . From R_H , calculate the ionization energy of hydrogen and both compare with the accepted values.

9.6.2 Helium Spectrum:

Identify all of the observed peaks and calculate the frequency and photon energy for each peak. Using the reference spectrum in the Appendix, try to assign for each peak the atomic transitions responsible for para- and ortho-helium.

Extra credit (5%): Using the theory outlined in the introduction, calculate the Rydberg constant and defect corrections for helium. Why is the helium spectrum so much more complicated than the hydrogen spectrum?

9.6.3 Heavier element spectrum:

Can your data be explained in the same manner as the hydrogen or helium spectra? Can you index some or all of the observed spectral lines to known atomic energy level transitions, or at least known spectral lines? Locate a reference spectra and compare your results.

9.6.4 Solar spectrum:

Using the position of the main peak in your spectrum, and assuming the sun is a black body radiator, calculate the apparent surface temperature of the sun. Does it agree with accepted values? Away from the peak to higher (IR) and lower (UV) wavelengths, the intensity falls off more quickly than expected for a black body radiator. Comment on possible reasons for the discrepancy (hints: Raleigh scattering, atmospheric and window glass absorption).

Finally, in addition to the overall black-body-like behavior, you should observe a number of discrete absorption lines and bands in the spectrum, known as Fraunhofer lines. You should be able to locate the principle H, He, and Hg emission lines in the solar spectrum, strong evidence that these three elements are present in the sun. Try to identify some of the other main absorptions. What elements' and molecules' absorptions might you expect to find? You should be able to discern bands due to absorption by common atmospheric constituents (hint: try searching for "Fraunhofer lines" online).

9.6.5 Incandescent Bulb Spectrum

Plot all of your spectra together, and determine the peak emission wavelength for each voltage setting. What trend do you expect, and what do you observe? Using the peak wavelength, estimate the bulb's temperature as a function of applied voltage and make a plot. Assuming a constant filament resistance for the voltages used, what behavior might you expect (hint: conservation of electrical and photon energy)?

(optional:) The integral of your spectrum gives the total incident power through the wavelength range studied. How should this vary with bulb temperature? With applied voltage (assuming

constant resistance)? Estimate or calculate the area under each curve and plot it as a function of voltage.

Compare your spectra qualitatively to the solar spectrum. Can you understand why incandescent bulbs at particular powers are favored for indoor lighting? Why is “color temperature” used to characterize such lighting sources?

9.6.6 Compact Fluorescent Bulb Spectrum

Can you identify any of the lines present (hint: phosphors coat the bulb glass)? Can you comment on why some people find fluorescent lighting “harsh” or “unnatural” compared to incandescent or solar lighting? Why does the bulb appear white, despite the presence of discrete emission wavelengths? Does the concept of a “color temperature” make sense for this source?

Reference Spectral Data

Table 9.1: *Hydrogen emission lines, UV-Vis-NIR[11]*

λ (nm)	I (a.u.)	Assignment
383.538	5	Balmer $9 \rightarrow 2$
388.905	6	Balmer $8 \rightarrow 2$
397.007	8	Balmer $7 \rightarrow 2$
410.174	15	Balmer $6 \rightarrow 2$
434.047	30	Balmer $5 \rightarrow 2$
486.133	80	Balmer $4 \rightarrow 2$
656.272	120	Balmer $3 \rightarrow 2^\dagger$
656.285	180	Balmer $3 \rightarrow 2^\dagger$
954.597	5	Paschen $8 \rightarrow 3$
1004.94	7	Paschen $7 \rightarrow 3$

[†] Spin-orbit split lines, $2p_{1/2} \rightarrow 1s$ and $2p_{3/2} \rightarrow 1s$

Table 9.2: Selected Helium emission lines, UV-Vis-NIR[11-13]

λ (nm)	I (a.u.)	Assignment	
		(spectroscopic)	(orbital)
361.3	4	$5^1P_1 \rightarrow 2^1S_0$	$1s3p \rightarrow 1s2s$ (p)
388.9	64	$3^3P_{2,1,0} \rightarrow 2^3S_1$	$1s2p \rightarrow 1s2s$ (o)
396.5	26	$4^1P_1 \rightarrow 2^1S_0$	$1s2p \rightarrow 1s1s$ (p)
402.6	2	$5^3D_{3,2,1} \rightarrow 2^3P_{2,1,0}$	$1s3d \rightarrow 1s2p$ (o)
412.1	8	$5^3S_1 \rightarrow 2^3P_{2,1,0}$	$1s3s \rightarrow 1s2p$ (o)
438.8	12	$5^1D_2 \rightarrow 2^1P_1$	$1s3d \rightarrow 1s2p$ (p)
447.2	19	$4^3D_{3,2,1} \rightarrow 2^3P_{2,1,0}$	$1s3d \rightarrow 1s3p$ (o)
471.3	4	$4^3S_1 \rightarrow 2^3P_{2,1,0}$	$1s3s \rightarrow 1s2p$ (o)
492.2	5	$4^1D_2 \rightarrow 2^1P_1$	$1s3d \rightarrow 1s2p$ (p)
501.6	25	$3^1P_1 \rightarrow 2^1S_0$	$1s3p \rightarrow 1s1s$ (p)
504.8	15	$4^1S_0 \rightarrow 2^1P_1$	$1s2s \rightarrow 1s2p$ (p)
587.6	72	$3^3D_{3,2,1} \rightarrow 2^3P_{2,1,0}$	$1s3d \rightarrow 1s2p$ (o)
667.8	23	$3^1D_2 \rightarrow 2^1P_1$	$1s3d \rightarrow 1s2p$ (p)
706.6	32	$3^3S_1 \rightarrow 2^3P_{2,1,0}$	$1s3s \rightarrow 1s2p$ (o)
728.1	18	$3^1S_0 \rightarrow 2^1P_1$	$1s3s \rightarrow 1s2p$ (p)

[†] (o) denotes ortho-helium, (p) denotes para-helium

Table 9.3: Selected Mercury emission lines, UV-Vis-NIR[11, 13]

λ (nm)	I (a.u.)	Assignment
365.02	2800	
365.48	300	
366.29	80	
404.66	1800	
407.78	140	
433.92	250	
434.75	400	
435.83	4000	
546.07	1100	
567.59	160	
576.68	240	
578.97	100	
579.07	280	
580.38	140	
614.95	1000	
671.64	160	
690.75	250	
708.19	250	
709.19	250	

Table 9.4: USB-650 Specifications[10]

Physical	
Detector Type	Sony ILX511 CCD
No. of elements	2048 pixels (650 active)
Pixel size	14 μm x 200 μm
Pixel well depth	$\sim 62,500$ electrons
Sensitivity	75 photons/count at 400 nm
Optical Bench	
Design	$f/4$, asymmetrical crossed Czerny-Turner
Focal length	42 mm input; 68 mm output
Entrance aperture	25 μm wide slit
Fiber optic connector	SMA 905
Spectroscopic	
Wavelength range	350–1000 nm
Entrance aperture	25 μm -wide slit
Optical resolution	~ 2.0 nm FWHM
Signal-to-noise ratio	250 : 1 (at full signal)
A/D resolution	12 bit
Dark noise	3.2 RMS counts
Dynamic range	2×10^8 ; 1300 : 1 for a single acquisition
Integration time	3 ms to 65 s (15 s typical max)
Stray light	$< 0.05\%$ at 600 nm; $< 0.10\%$ at 435 nm
Corrected linearity	$> 99.8\%$

Visible Spectrum

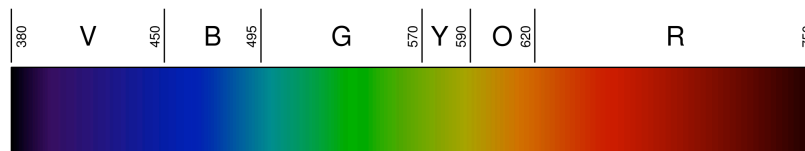


Figure 9.8: Linear representation of the spectrum of visible light. From http://en.wikipedia.org/wiki/Visible_spectrum. Numbers are wavelengths in nanometers.

Compton Scattering

10.1 Introduction

Compton scattering refers to the scattering process

$$\gamma + X \rightarrow \gamma + X$$

where γ represents a photon and X represents an element of matter such as an electron, atom, nucleus, etc. It was first observed by Arthur Compton in 1923, for which he won the 1927 Nobel Prize in physics because it showed that the photon, the quantum of light, has definite energy *and* momentum as would be expected for a particle. The behavior of light in diffraction and interference experiments, together with its behavior in phenomena such as the photoelectric effect and Compton scattering, empirically demonstrates the wave–particle duality of light.

In the language of collisions, this is a two–body collision and because the initial–state particles are the same as the final–state particles, there is a well–defined relationship between the initial and final energies of the photon and the angle θ at which the photon is scattered relative to its initial direction

In this experiment, we will look at photon scattering from a target in a regime where the photon scatters dominantly from individual atomic electrons, i.e. $X = e$. The energy transferred to the electron is typically large compared to the binding energy of the electron so that the electron can be well–approximated as being initially at rest. In that case, if we let ϕ be the angle which the direction of the scattered electron makes with respect to the direction of the initial photon, the conservation of energy and momentum implies the relations

$$p_i = p_f \cos \theta + p_e \cos \phi$$

$$0 = p_f \sin \theta + p_e \sin \phi$$

$$E_i = E_f + K_e$$

where E_i is the energy of the incident photon, E_f is the energy of the scattered photon, p_i and p_f are the respective momentum magnitudes, p_e is the magnitude of the scattered electron’s momentum, and K_e is the kinetic energy of the scattered electron.

What is the connection between the momentum p and kinetic energy of a particle K and its speed v ? If we assume Newtonian mechanics

$$p = mv$$

$$K = \frac{1}{2}mv^2$$

while if we assume the theory of special relativity,

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$K = E - mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

Assuming Newtonian mechanics and applying energy and momentum conservation, we can find the following relationship between the initial photon energy, final photon energy, and the scattering angle (see full derivation in the Appendix):

$$\frac{E_i}{E_f} (1 - b) + \frac{E_f}{E_i} = 2 \cos \theta - b \quad (10.1)$$

where

$$b \equiv \frac{2mc^2}{E_i}$$

Similarly, assuming the theory of special relativity (see full derivation in the Appendix):

$$\frac{E_i}{E_f} = 1 + a(1 - \cos \theta) \quad (10.2)$$

where

$$a \equiv \frac{E_i}{mc^2}$$

In this experiment, where m is to be taken as the mass of the electron, you will check which, if any, if any of the above predictions are consistent with your observations of Compton scattering from electrons.

10.2 Experimental Method

An intense beam of 662 keV photons from a strong (~ 0.7 Ci) radioactive ^{137}Cs source will be directed on a small target. A NaI detector, coupled to a pulse height digitizer, will be used to detect the scattered photons at several different angles with respect to the direction of the beam.

10.3 Procedure

Ground rule never to be violated: whenever somebody is in the detector room the entrance door must remain open. This will prevent inadvertent release of the beam.

Figure 10.1 show the main part of the experiment.

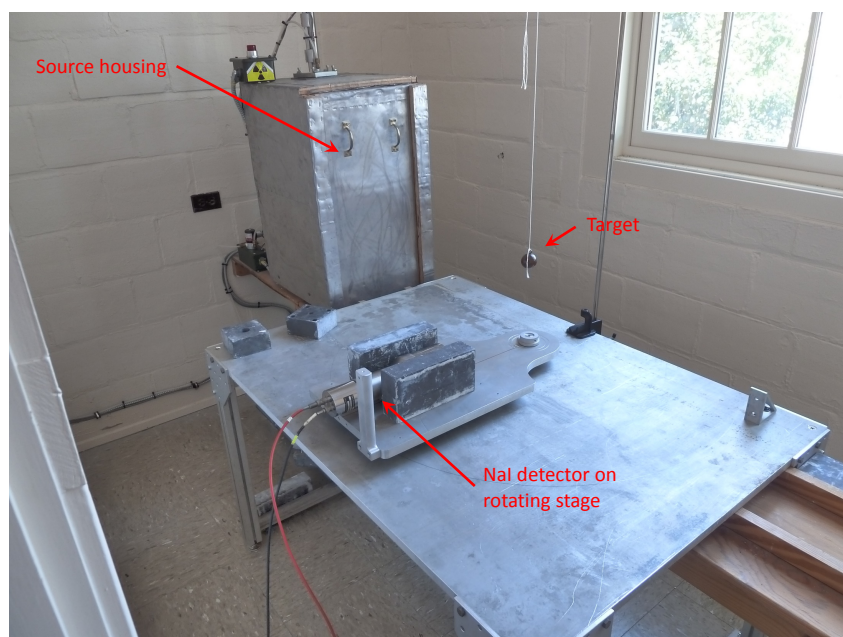


Figure 10.1: Overview of the experiment setup.

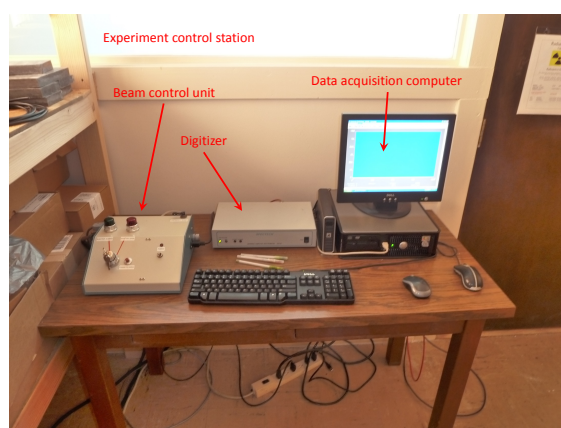


Figure 10.2: The data acquisition system (UCS 30) interfaced to a PC and beam control unit.

1. Set up the experiment as shown in Figures 10.1 and 10.2. The red cable should run between the HV output connector on the rear of the UCS 30 to the SHV connector on the base of the NaI detector. The black cable should run between the input connector on the rear of the UCS 30 to the BNC connector on the rear of the NaI detector.
2. Switch on the Geiger counter and make sure that it is working by holding its probe near a disk calibration source. You will use the Geiger counter to verify that the beam is off whenever you enter the detector room. When sitting at the control table keep the Geiger counter turned on to verify that there is no beam near you. You will verify the absence of the beam beam means of this counter whenever you manipulate the target. The red warning light on the beam control unit and over the door will tell you whether the beam is on or not. The door into the detector room has an interlock, automatically shutting off the beam whenever the door is opened. The Geiger counter, thus, only serves as an additional safety check. **Nobody will be present in the detector room whenever the beam is turned on.** You will now activate the beam control:
 - (a) The beam is released by pneumatic shutter. Gas pressure is supplied by the gas bottle behind the experiment control table. Open the block-off valve on top of the bottle to energize the piston. Turn the valve indicated in figure 10.3 counter clock wise to



Figure 10.3: Gas bottle powering the pneumatic cylinder used to release the beam.

open. Don't forget to close the gas bottle when you're done with the experiment. No adjustments need to be done to the operating pressure. The pressure regulator has been blocked.

- (b) Next power up the beam control unit shown in figure 10.4.
3. Operation of the beam control unit needs an access key provided by the instructor. To power the the control unit toggle its power switch. A read power light will come on after the unit has been activated. To operate the beam shutter insert the key. The green light will be on when the shutter is closed and no beam is present. There is an additional red warning light over the door. When the beam is off this light is off too and it is safe to enter the detector room.



Figure 10.4: Beam control unit used to release the γ -ray beam.

It is always good practice to take the Geiger counter with you when entering the detector room to verify the beam is actually off. To activate the beam switch the key to the “Shutter Open” position. This only enables opening. To actually open the shutter push the “Press to Open” button. The red “Shutter Open” light will now come on in addition to the beam on indicator over the door into the detector room. **You must not enter the detector room when the beam is open. You must never activate the beam with anybody being present in the detector room.** Normal counting time is 3 minutes per run. Always shut down the beam by turning the key to the “Shutter Closed” position after you completed a data taking run. As a safety measure the shutter will close automatically 5 minutes after opening. As an additional safety measure the door into the detector room has been equipped with an interlock. The beam will shut down automatically whenever the door is opened. One last point: the beam operation needs electrical power. In case of a power failure you can shut off the beam by depressing the “Emergency Close” valve. It will bleed the gas pressure which will close the shutter.

There are two places where it is safe to stand when the ^{137}Cs source is open: behind the source housing and in the room with the PC and UCS 30 data acquisition and control box is. The collimator mounted on the front of the opening in the source housing attenuates the 662 keV gammas which are not emitted toward the hole in the center of the collimator. Some gammas do penetrate the collimator, comprising the so-called “halo” particles surrounding the relatively intense core of the main beam. Take training from the instructor on how to work safely with the ^{137}Cs source. Make sure that you heed this training throughout the process of data collection.

4. Turn on the power to the SpecTech UCS 30 box.
5. If you have not already done so, log into the PC.
6. Start the UCS 30 program and carry out the following operations.

- (a) Under the Mode menu, select PHA (amp in).
 - (b) Under the Settings menu, select Energy Calibration, and then select Uncalibrate if it is available. The horizontal axis of the spectrum will then be displayed in channels, not energy. You will do the energy calibration offline.
 - (c) Under the Settings Menu, select Amp/HV/ADC, and then set the HV voltage to approximately 1100 V, set the HV polarity to positive, click the radio button for HV on, set Amp In Polarity to negative, Coarse Gain to 4, Fine Gain to 1.3, and LLD (lower discriminator threshold) to about 7.
7. Take data for the energy calibration of the NaI detector using the ^{137}Cs , ^{57}Co and ^{133}Ba radioactive sources, which you will find in the red case containing 8 disk sources. Before taking the final data for calibration, adjust the PMT HV and/or Fine Gain, if necessary, so that the ^{137}Cs full absorption peak is near 900 channels. Example ^{137}Cs and ^{133}Ba pulse height spectra are shown in Figure 10.5. Take data long enough so that the pulse height spectra are smooth in the region of the photo-peaks (statistical fluctuations small compared to the changes in number of events from one channel to the next).
 8. Once you have taken the calibration data, select Presets under the *Settings* menu and set the live time (LT) to 180 s. This ensures that all runs will have exactly the same length and will make subtraction of target-out data from target-in data straightforward. If it turns out that you cannot get good statistics in 180 s, increase the LT preset accordingly. 300 s is the longest integration time you may choose. After this the γ -ray beam will shut down automatically. While the UCS 30 circuitry is digitizing the signal from the PMT, any further events arriving from the PMT are ignored; the DAQ is said to be “dead” during this interval. The livetime is thus the time during which the UCS 30 is ready to accept and digitize a signal from the NaI detector.
 9. Check that the target is aligned with the ^{137}Cs beam. Make any adjustments to the target location as needed. For each of at least five different scattering angles between 30 degrees and 160 degrees, orient the NaI detector so that it is facing the target and ~ 50 cm away from it. The predictions from classical mechanics and special relativity differ the most at large scattering angles, so choose at least two of the angles to be larger than 120 degrees. Be sure that the detector is well-shielded from the beam halo (mainly a problem for scattering angles less than 90 degrees). When rotating the detector, take care not to stress any signal or power cables. The NaI detector is fragile, so handle it accordingly.
 10. Measure each angle and then take a sequence of three runs:
 - (a) Target-in run. Open the source shutter, then start the run. Once the run has stopped upon reaching the preset limit, close the source shutter, and save the file with an appropriate name. Opening and closing of the beam shutter is performed by means of the control unit in the data taking room, as explained before.

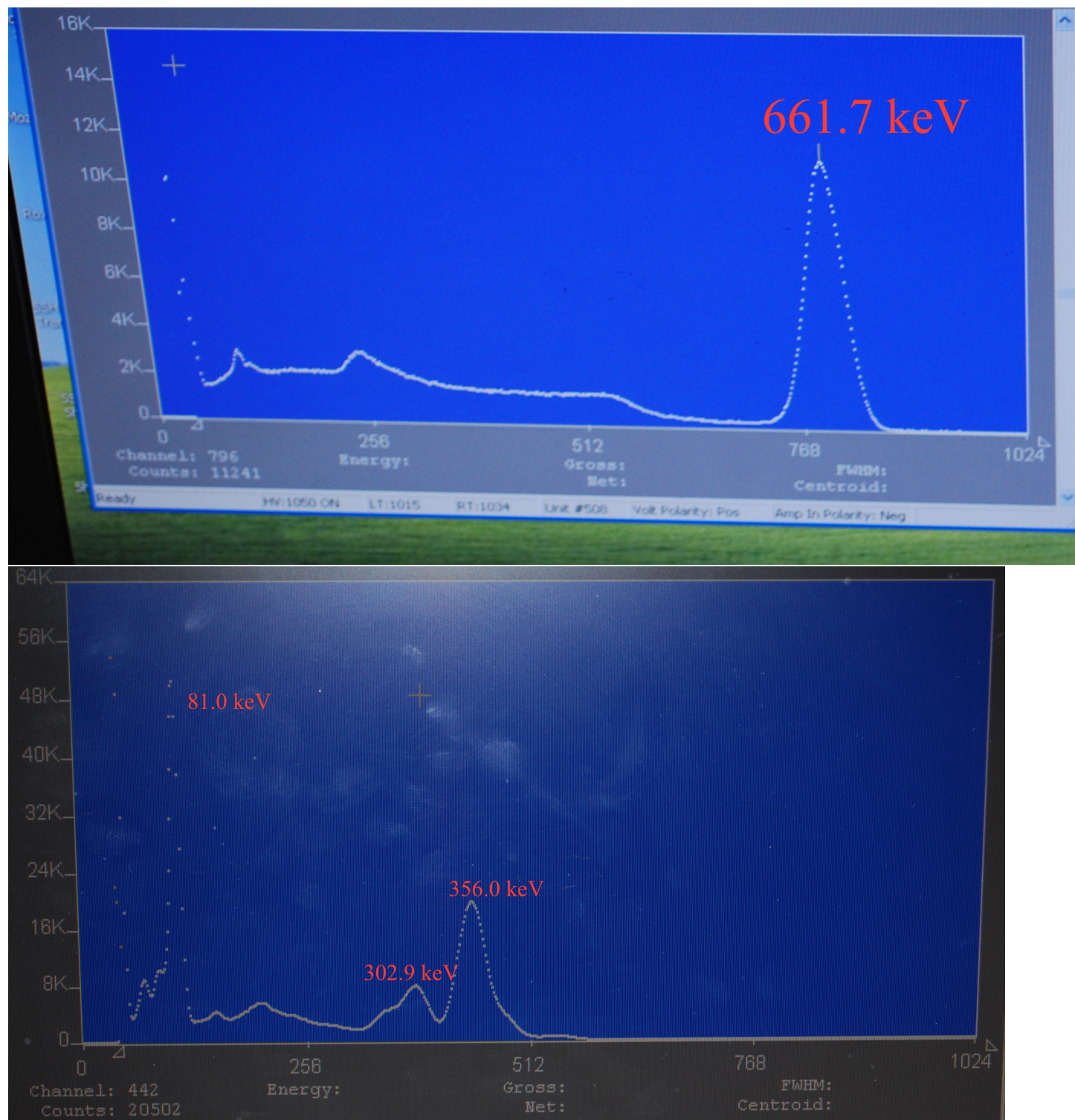


Figure 10.5: Illustrative pulse height spectra for the Cs-137 source (upper) and Ba-133 source (lower). The energies of the gammas associated to the more prominent photopeaks are indicated.

- (b) Target-out run. With the beam shut off, enter the detector room and pull the target aside, e.g. by hanging the string to which the target is attached over the clamp on the ring stand. You can use these data to subtract background from the target-in data and see the full-absorption peak more clearly.
 - (c) Calibration run with ^{137}Cs disk source to check gain stability. This does not have to run longer than necessary to check that the 662 keV photopeak position has remained the same within 5–10 channels.
11. Repeat the energy calibration to check the stability of the NaI detector.
 12. Document the experimental setup, by making a sketch and/or taking photos.

10.4 Analysis

1. Use your energy calibration data to find the relationship between energy and channel number.
 - (a) Make a plot of counts versus channel number for each disk source and turn in with your report.
 - (b) For each of the following energies 81.0 keV, 302.9 keV, 356.0 keV, and 661.7 keV, estimate the center channel of the corresponding photopeak in the pulse height spectra you took for energy calibration. Tabulate the energies and corresponding center channels. What is the uncertainty in the determination of the center channels? Note that some photopeaks are somewhat distorted in shape by the presence of other peaks nearby.
 - (c) Plot the channel number versus energy. Are these points consistent with lying along a straight line? If so, use the method of linear regression to fit a straight line to get a linear equation for the channel number in terms of energy. Use the EXCEL function LINEST for this. It will give the the parameters of the straight line and estimate their errors. If the calibration data is not described well by a straight line, consult with your instructor.
 - (d) Solve the equation obtained in the last step for the energy so that you have a linear equation for the energy in terms of the channel number. This is the energy calibration which you will use to reconstruct the energy from the observed channel number corresponding to the center of a photopeak.
2. For each scattering angle, subtract the target-out spectrum from the target-in spectrum. Include in your report the plot of the difference versus channel number for the largest scattering angle and for the smallest scattering angle. This subtraction can either be performed using EXCEL or the UCS 30 program. If you want to try the latter ask your instructor for help.

3. Convert the peak channel number for each scattering angle to energy. Make a table of scattering angles and corresponding energies.
4. If classical physics is correct, plotting $\frac{E_i}{E_f}(1 - b) + \frac{E_f}{E_i}$ versus $\cos \theta$ will yield points lying along a straight line with slope 2. Make this plot from your experimental data. Are these data consistent with the classical hypothesis? Explain.
5. Plot $\frac{E_i}{E_f}$ versus $1 - \cos \theta$. Are the data consistent with lying along a straight line as predicted by special relativity? If so, use your data to fit for the rest energy of the electron. Is your measurement of the rest energy consistent with the accepted value? Explain.

Derivation of Classical and Relativistic Compton Scattering Predictions

In this section we show the full derivation of Eq. 10.1 and Eq. 10.2. In both cases we assume a photon scatters off an electron initially at rest, imparting some energy to the electron which then has a final nonzero velocity. For simplicity we assume the incoming photon is along the x-axis. The recoil angle of the scattered photon with respect to the x-axis is denoted θ and the recoil angle of the scattered electron with respect to the x-axis is denoted ϕ ; we do not assume that these angles are equal. Figure 10.6 shows the scattering geometry.

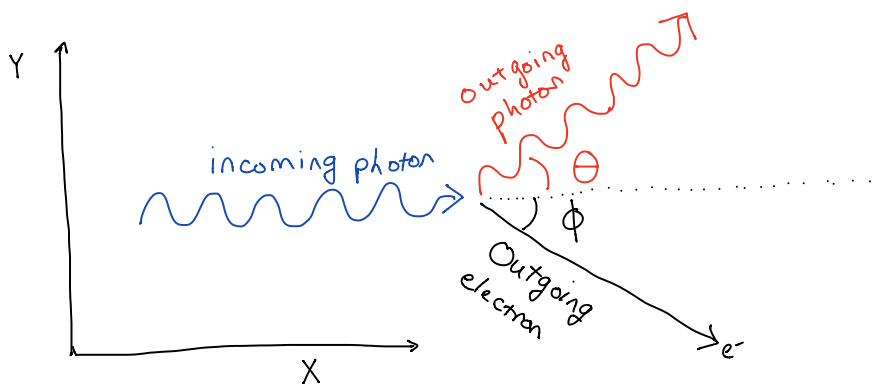


Figure 10.6: The geometry of the Compton scattering assumed for both the classical and relativistic cases.

10.4.1 Classical Prediction

This section shows the derivation of Eq. 10.1, which assumes classical definitions of energy and momentum. The electron energy is purely kinetic, $E = \frac{1}{2}mv^2$. There is no rest energy. The electron momentum $p = mv$. Since the electron is initially at rest, both the electron energy and momentum are zero prior to the scattering. We denote the initial energy of the photon to be E_i and the final energy of the photon to be E_f . The initial momentum of the photon is $p_i = \frac{E_i}{c}$ and the final momentum of the photon is $p_f = \frac{E_f}{c}$.

Conservation of energy requires

$$E_i = E_f + \frac{1}{2}mv^2$$

Conservation of momentum along the x axis requires

$$p_i = p_f \cos \theta + mv \cos \phi$$

Conservation of momentum along the y axis requires

$$p_f \sin \theta = mv \sin \phi$$

Now we eliminate ϕ :

$$m^2v^2 \cos^2 \phi = (p_i - p_f \cos \theta)^2 = p_i^2 - 2p_i p_f \cos \theta + p_f^2 \cos^2 \theta$$

$$m^2v^2 \sin^2 \phi = p_f^2 \sin^2 \theta$$

$$m^2v^2(\sin^2 \phi + \cos^2 \phi) = p_f^2(\sin^2 \theta + \cos^2 \theta) + p_i^2 - 2p_i p_f \cos \theta$$

$$m^2v^2 = p_f^2 + p_i^2 - 2p_i p_f \cos \theta$$

Now convert photon p to $E = pc$:

$$m^2v^2 = \frac{E_f^2}{c^2} + \frac{E_i^2}{c^2} - 2\frac{E_f E_i}{c^2} \cos \theta$$

Eliminate v using conservation of energy, $E_i - E_f = \frac{1}{2}mv^2$, meaning that $m^2v^2 = 2m(E_i - E_f)$:

$$2m(E_i - E_f) = \frac{E_f^2}{c^2} + \frac{E_i^2}{c^2} - 2\frac{E_f E_i}{c^2} \cos \theta$$

Divide through by $\frac{E_i}{c^2}$:

$$\frac{2mc^2}{E_i}(E_i - E_f) = \frac{E_f^2}{E_i} + E_i - 2E_f \cos \theta$$

Divide through by E_f :

$$\frac{2mc^2}{E_i} \left(\frac{E_i}{E_f} - 1 \right) = \frac{E_f}{E_i} + \frac{E_i}{E_f} - 2 \cos \theta$$

Define $b \equiv \frac{2mc^2}{E_i}$ and rearrange the equation to get Eq. 10.1:

$$\frac{E_i}{E_f}(1 - b) + \frac{E_f}{E_i} = 2 \cos \theta - b$$

10.4.2 Relativistic Prediction

This section shows the derivation of Eq. 10.2. In this case, the electron has a rest energy mc^2 prior to the collision, and a total energy equal to the rest energy plus the kinetic energy after the collision. The total energy E is related to the momentum p as follows:

$$E^2 = p^2 c^2 + m^2 c^4$$

Note that the momentum of the electron is still zero prior to the scattering. Therefore the equation we derived above using conservation of momentum is still valid, except we will express the final electron momentum as p instead of mv . We still use p_i , p_f , E_i and E_f to denote the initial and final momentum and energy of the photon.

$$p^2 = p_f^2 + p_i^2 - 2p_i p_f \cos \theta$$

Now consider conservation of energy in the relativistic regime. We will call the final energy of the electron E , whereas the initial energy of the electron is just mc^2 .

$$E_i + mc^2 = E_f + E \Rightarrow E = E_i - E_f + mc^2$$

Inserting this into the equation for total energy:

$$p^2 = \frac{E^2}{c^2} - m^2 c^2 = \frac{1}{c^2} (E_i - E_f + mc^2)^2 - m^2 c^2$$

$$p^2 = \frac{1}{c^2} (E_i^2 - 2E_i E_f + E_f^2 + 2mc^2 E_i - 2mc^2 E_f + m^2 c^4) - m^2 c^2$$

Using the momentum conservation equation above with $E_i = p_i c$ and $E_f = p_f c$:

$$p^2 = \frac{E_f^2}{c^2} + \frac{E_i^2}{c^2} - 2 \frac{E_i E_f}{c^2} \cos \theta$$

Combining the two above equations for p^2 and eliminating terms that sum to zero, and rearranging:

$$mE_i = mE_f + \frac{E_i E_f}{c^2} - \frac{E_i E_f}{c^2} \cos \theta$$

Divide by mE_f :

$$\frac{E_i}{E_f} = 1 + \frac{E_i}{mc^2} - \frac{E_i}{mc^2} \cos \theta$$

Define $a \equiv \frac{E_i}{mc^2}$ to get Eq. 10.2:

$$\frac{E_i}{E_f} = 1 + a(1 - \cos \theta)$$

Muon Lifetime

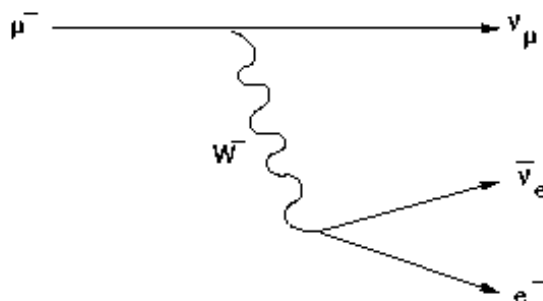
Introduction

The muon, conventionally denoted by the Greek letter μ , is an unstable elementary particle. Its rest mass is $105.7 \text{ MeV}/c^2$. Together with the electron and tau, it comprises the three known charged leptons appearing in the Standard Model of elementary particle physics. (Leptons are distinguished by not having strong interactions with other particles, only electromagnetic and weak interactions in addition to the gravitational interaction.) What is conventionally called the muon particle has charge $-e$ (denoted μ^-) while the muon anti-particle has charge $+e$ (denoted μ^+).

Muons are produced mainly as a result of cosmic ray interactions in the earth's atmosphere and at high-energy particle accelerators. In cosmic ray interactions, a cosmic ray (proton, neutron, gamma, nucleus) collides with an atmospheric gas molecule, the result of which is typically the production of a large number of hadrons (protons, neutrons, kaons, pions, etc.) propagating in the same general direction as the parent cosmic ray. The kaons and pions are unstable and eventually decay; many of the decay final states include muons. Because both particles and anti-particles are produced in cosmic ray interactions, the produced muons come in both signs of charge. The flux of muons at sea level is about 1 per second per 100 cm^2 ; in rough terms, if you hold your hand out flat, on average a muon passes through it every second.

The muon is unstable because of the existence of particles into which it can decay. In the case of the muon, according to the Standard Model and as illustrated below, it can couple to a muon neutrino through the emission of a virtual W^- gauge boson, which in turn decays into an electron and electron anti-neutrino. That is,

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$



The probability per unit time that a muon decays, known as the decay constant λ , can be calculated from the Standard Model starting from the quantum mechanical amplitude for the above process and is found to be

$$\lambda = \frac{1}{32} \frac{(m_\mu c^2)^5}{(m_W c^2)^4} \frac{g^4}{192\pi^3 \hbar} (1 + \Delta q)$$

where m_μ is the muon mass, m_W is the mass of the W gauge boson, Δq is a constant, and g is the weak interaction coupling constant. Note the strong dependence (to the fifth power) of the decay constant on the muon mass. In the quantum mechanical amplitude for muon decay, there is a factor of the coupling constant g for each W vertex, so that the amplitude is proportional to g^2 . When the modulus of the amplitude is squared to calculate the decay probability, the factor of g^4 is obtained. At energies low compared to the rest energy of the W boson, the factor (“propagator”) appearing in the amplitude due to the propagation of the virtual W from the $\mu\nu_\mu W$ vertex to the $W e\bar{\nu}_e$ vertex is proportional to $\frac{1}{m_W^2 c^4}$, so that λ is inversely proportional to m_W^{-4} . That the W is almost 100 times heavier than the proton and its mass appears to the fourth power in the denominator accounts for the fact that the interaction leading to muon decay is weak compared to the electromagnetic and strong interactions. The decay constants in the case of decays mediated by the electromagnetic or strong interactions are 10–15 orders of magnitude larger. One can show that the mean muon lifetime τ is $\frac{1}{\lambda}$, so that measurement of the mean muon lifetime is directly sensitive to fundamental quantities, namely the muon mass, the W mass, and the weak coupling constant g .

Given the decay constant λ , the probability that a muon decays between time t and $t + dt$, where dt is an infinitesimal interval, is

$$\lambda dt$$

Conversely, the probability that the muon does not decay between time t and time $t + dt$ is

$$1 - \lambda dt$$

Suppose a muon exists at some instant, which we will define as $t = 0$. Let $P(t)$ be the probability that the muon then survives until time t . The probability $P(t+dt)$ that the muon survives until time $t+dt$ is

$$P(t + dt) = P(t)(1 - \lambda dt)$$

In other words, the probability that the muon survives until time $t + dt$ is the product of the probability it survives until time t and the probability it does *not* decay between t and $t + dt$. Re-arranging the above equation, we find that we have an equation we can use to solve for P :

$$P(t + dt) - P(t) = -\lambda P dt$$

$$dP = -P\lambda dt$$

$$\frac{dP}{P} = -\lambda dt$$

Integrating both sides

$$\ln P = -\lambda t + B$$

where B is an integration constant. Therefore,

$$P = e^{-\lambda t+B} = e^{-\lambda t} e^B = e^B e^{-\lambda t} = C e^{-\lambda t}$$

where $C = e^B$ is the integration constant. We can use the boundary condition $P(0) = 1$ to determine that $C = 1$, so

$$P(t) = e^{-\lambda t}$$

Not unexpectedly, we have obtained the exponential decay law. Casting it in a more familiar form, if we observe N_0 muons altogether, the mean number $N(t)$ surviving to time t is

$$N(t) = N_0 e^{-\lambda t}$$

One can also show that the mean survival time, or in other words the mean muon lifetime, is

$$\tau = \frac{1}{\lambda}$$

We can therefore also write

$$N(t) = N_0 e^{-t/\tau}$$

In this experiment, you will observe how many muons decay in different intervals. We expect that the number of muons in each interval will fall exponentially with time, which can be shown as follows. The expected number ΔN of muons which decay in an interval Δt between t and $t + \Delta t$ is just the difference between the number of muons which survive to time t and the number which survive to time $t + \Delta t$:

$$\Delta N = N_0 e^{-\lambda t} - N_0 e^{-\lambda(t+\Delta t)} = N_0 e^{-\lambda t} (1 - e^{-\lambda \Delta t})$$

Because you will divide the muon decay times into intervals of constant size, the quantity $N_0 (1 - e^{-\lambda \Delta t})$ is a constant, which we will denote as A . Then

$$\Delta N = A e^{-\lambda t}$$

that is, the number of muon decays per interval falls exponentially with increasing time, as claimed. If you evaluate the natural logarithm of both sides,

$$\ln \Delta N = -\lambda t + \ln A$$

you see that if we plot $\ln \Delta N$ versus t , the points should lie along a straight line with slope $-\lambda$.

11.1 Experimental Method

A block of plastic scintillator optically coupled to a photomultiplier tube (PMT) is used to detect charged particles passing through it. The (analog) output of the PMT is sent to a discriminator, which produces an output logic pulse if the PMT signal is above threshold. The logic pulse is used to either start a clock or stop a clock if it is already running. The time between the start signal and stop signal is recorded. A muon which stops in the scintillator and then decays will produce two correlated signals: the first signal due to the energy it deposits in the scintillator as it is stopping and the second signal due to the energy deposited by the electron after the muon decays. Such a pair of time-correlated signals is often called a delayed coincidence. The scintillation light pulses due to the energy deposits in the scintillator are very short (tens of ns) compared to the typical time between the start and stop pulses.

11.2 Procedure

i

The apparatus is comprised of four scintillator blocks arranged in a vertical stack, each connected to a PMT (labeled 0, 1, 2 and 3). The PMTs are connected to a power distribution unit (PDU) which controls their operating voltage. The data from the PMTs are read out by a Quarknet [14] card attached to the control computer by USB. The quarknet card is attached to GPS antenna to record the time. The quarknet card is attached to GPS antenna to record the time. The apparatus is shown in Figure 11.1.

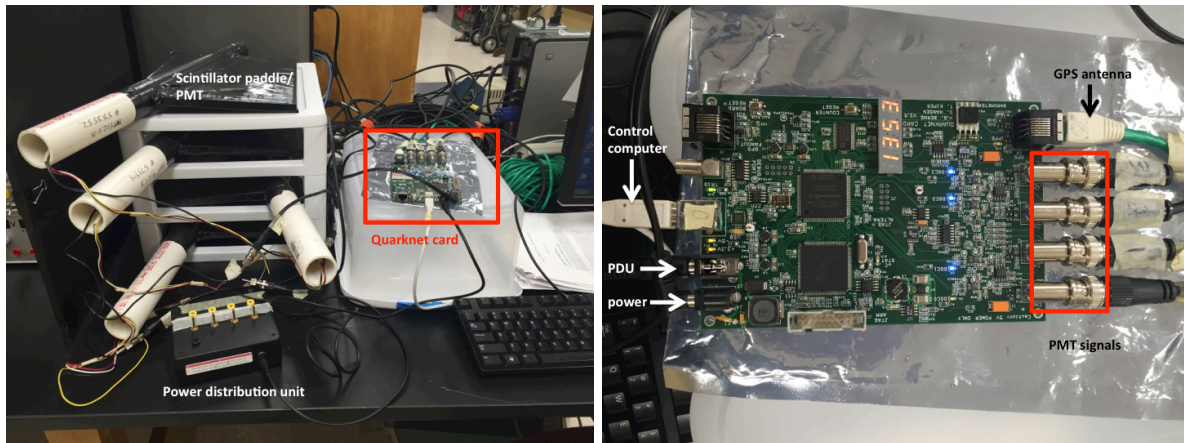


Figure 11.1: Left: overview of the experiment. Right: Quarknet control card and locations of connections to other parts of the apparatus.

Each PMT/scintillator combination is referred to as a *channel*. Each channel is recorded by the Quarknet card when the signal read by the PMT exceeds a certain level called a *discriminator threshold*. Figure 11.2 shows a sketch of a typical PMT pulse. You can see the rate at which each

ⁱThanks to Daniel Castillo (University of Alabama, class of 2014) for his assistance in setting up and documenting the Quarknet data collection. This section is based on his notes.

channel is reading out in the blue lights on the Quarknet card labeled DSC0, DSC1, DSC2 and DSC3. Random noise from radioactive decays, throughgoing muons and other processes cause signals in each channel at a fairly high rate, which increases as the operating voltage of the PMT increases. (The operating voltage is controlled by the potentiometer.) In order to reduce the data rate, in this experiment we require a *coincidence* between three of the four channels within a window of $10\ \mu\text{s}$. The window allows time for the muon (lifetime $2.2\ \mu\text{s}$) to decay and produce the second signal. The Quarknet card is not capable of determining how many signals are in a single channel, only whether the channel was hit with the coincidence window. Therefore, after coincidence data is collected, a software program is run to search for channels which have two signals in them: one from the muon stopping and one from the muon decaying. Sometimes a *veto* is used, meaning an event is rejected if a certain channel or channels are hit. We do not use a veto in this experiment.

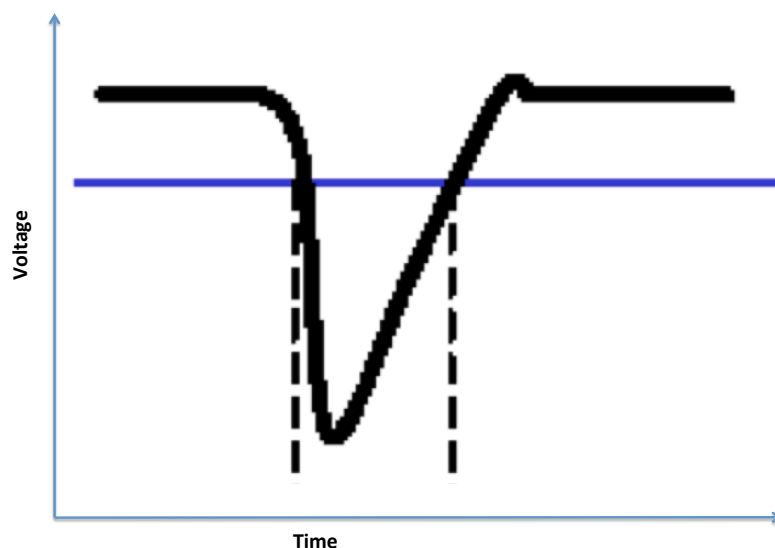


Figure 11.2: Sketch of a typical PMT pulse. The blue line denotes the discriminator threshold, and the area within the vertical dashed lines is the time over threshold.

11.2.1 Connecting with Hyperterminal

The event data from the Quarknet card are collected using the HyperTerminal program. HyperTerminal is a preinstalled terminal emulator on Windows machines. Any other terminal emulator should work just as well, but this documentation will assume use of HyperTerminal, any changes in functionality from using a different terminal emulator will need to be solved by the user.

Connect the power cable to the quarknet card. If there is not a HyperTerminal shortcut on the computer desktop, it can be found in the start menu by looking in Programs>Accessories>Communications.

After opening HyperTerminal, you will be prompted to choose a name for the session; the name does not matter as you will not be saving the connection upon leaving HyperTerminal, but you should choose a suitable name like "MuonLifetime." After you choose a name for the connection, you will need to choose a port to connect to. If you are unsure about which port the DAQ board is connected to, right click on My Computer and go to Properties>Hardware, click on "Device Manager", and click on the [+] button next to Ports. The DAQ board should show up in the list of Ports as something like "Silicon Labs CP210x USB to UART Bridge (COM4)." After choosing the correct port to connect to, click Ok. Then you will need to choose some parameters for the connection with the Quarknet board. A screenshot is shown in Figure 11.3 of the correct parameters to use when connecting to the Quarknet board.

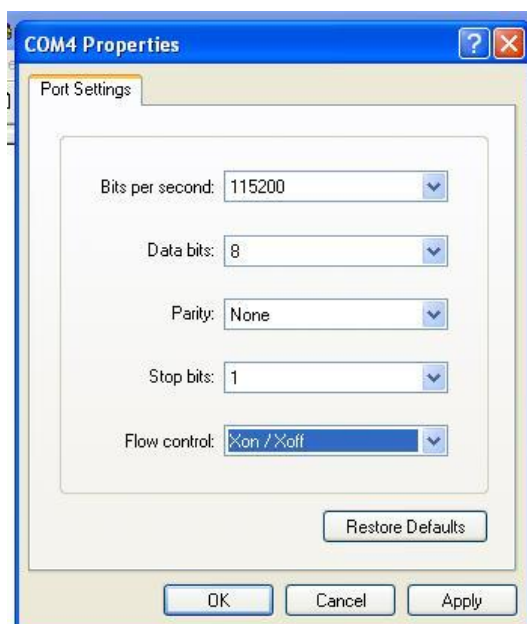


Figure 11.3: Connection parameters for the Quarknet board connection in HyperTerminal.

11.2.2 PDU Settings

You will now need to determine the correct voltage settings for the PMT, which are controlled by the PDU. The PDU provides an output of 0.3 V to 1.8 V. The actual operating voltage of the PMT is 1000 times the voltage delivered by the PDU. The PDU voltage for each channel is changed by turning the knob corresponding to that channel: clockwise to increase voltage and counterclockwise to decrease. To monitor the PDU voltage, use a voltmeter with the voltage connection to the yellow port on the PDU for the channel you are measuring, and the common connection to the black port on the PDU.

11.2.2.1 Correct Voltage Settings: the Plateau Method

Each PMT has a unique optimal operating voltage. In order to determine this, we will measure the rate of hits on each PMT as a function of the voltage on the PDU, referenced against another PMT where the voltage is held steady, and determine the voltage at which the rate of coincident hits becomes stable. An example of a plateau plot is shown in Figure 11.4. In this case, channel 0 was the reference channel and the voltage on channel 1 was varied. The rate of hits on channel 0 is steady and the rate on channel 1 rises with the voltage, but the coincidence rate between the two channels becomes steady at about 700 mV. A voltage slightly above this value is chosen as the operating voltage of the PMT.

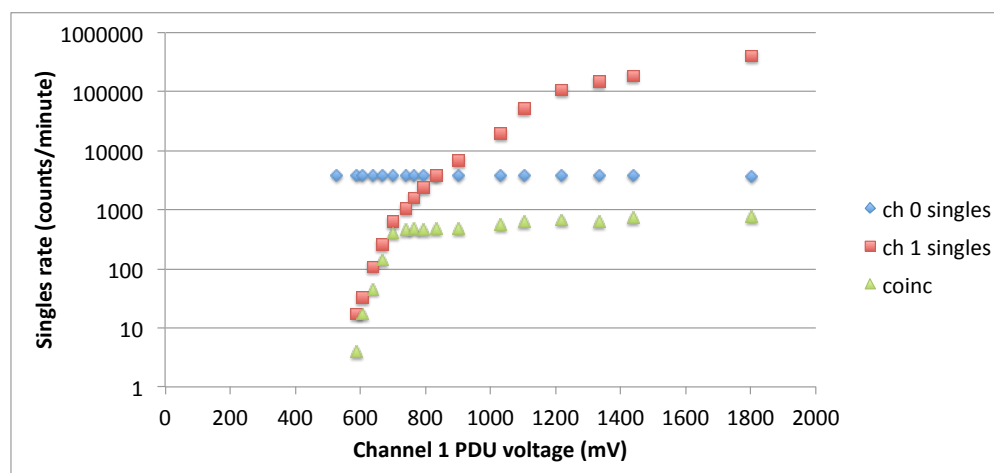


Figure 11.4: Example plateau curve for channel 1. Channel 0 is the reference channel

To measure to plateau for channel 0, pick a neighboring channel as a reference channel, in this case channel 1. Commands to be entered in Hyperterminal are denoted in boldface.

- First disable any event counting with the command **CD**.
- Turn the knobs on the PDU all the way counterclockwise for all four channels.
- Set the discriminator threshold of all 4 counters to 700 mV with the command **TL 4 700**. There are two ways you can verify that the threshold voltage is set correctly:
 - 1) type the command **TL** into Hyperterminal, and you should see the following:
TL L0=700 L1=700 L2=700 L3=700
 - 2) You can check the threshold directly by attaching fine-tipped probes to the voltmeter and measuring the voltage between the TPVTHX (where X is 0, 1, 2, or 3 depending on which channel you check) and the DEBUG PORT. The locations of the ports on the Quarknet card are indicated in Figure 11.5.
- Set the coincidence level to 0 (only one channel required to be hit) and activate the channel 1 only with the command **WC 00 02** (see Table 11.1 for commands for each channel).

- Increase the PDU voltage for channel 1 (the reference channel) until the rate of hits on channel 1 is somewhere between 2000 and 3000 hits per minute. To see what the rate of hits on channel 1 is, type the command **ST 3 1** which resets all the counters and then displays the counts on each channel once per minute. After one minute, you should see something like this:

```
DS 00000000 000009f8 00000000 00000000 000009f8
```

“DS” is the command to display the counter rates. The next 4 numbers are the hits counted on each channel (0, 1, 2, 3) in the last minute in *hexadecimal* format. The last number is the number of coincident events counted, which is in this case is the same as the number of events on channel 1. You can use the HEX2DEC and DEC2HEX commands in Excel to translate hex to decimal values and vice versa. So in this example, there are $9f8 = 2552$ hits per minute.

- Once the voltage level is set on Channel 1, record the value in your lab notebook and then reset the coincidence condition to require 2 channels, 0 and 1, and activate channels 0 and 1 with this command: **WC 00 13**. See Table 11.2 for the commands for each channel pair.
- Increase the voltage on channel 0 until you just start to see hits. There is a blue LED next to each channel that begins to light up when there are any hits.
- Enter the command ST 3 1. You will see something like this:

```
DS 00000035 00000368 00000000 00000000 0000000C
```

The first column after DS is the rate of hits per minute on channel 0, followed by channel 1, channel 2, channel 3 and the coincidence rate. In this example, channel 0 is the channel for which you are trying to determine the plateau, channel 1 is the reference channel (which should maintain a fairly constant rate throughout the exercise) and channels 2 and 3 are not active, and therefore read 0. The coincidence rate is what is used to determine the plateau as seen in Figure 11.4

- Increase the voltage in increments of 25 mV or so and record the results of the command **ST 3 1** each time. Make a plateau plot and determine the correct PDU setting for channel 0.
- Repeat to make plateau plots for channels 2 and 3, keeping channel 1 as the reference channel. Then set channel 0 to its plateau voltage and use it as the reference channel, and measure the plateau on channel 1.

11.2.3 Capturing Data

Once you enter the correct parameters and connect to the Quarknet board, you should see the data stream in hyperterminal. If you do not see any text in the hyperterminal screen, try typing “CE”

Table 11.1: *WC settings for reading out single channels, i.e. to set the rate for the reference channel.*

Single Channel	command
0	WC 00 01
1	WC 00 02
2	WC 00 04
3	WC 00 08

Table 11.2: *WC settings for collecting pairs of channels in coincidence*

Channel pair	command
0,1	WC 00 13
1,2	WC 00 16
1,3	WC 00 1A

and Enter. Once you can see the data stream and you know you are successfully connected to the DAQ board, type "CD" to disable the counters and the data stream should stop. This is not necessary, but it is easier to enter the Quarknet card settings when the data stream is disabled. Before you continue, it's a good idea to start capturing text from HyperTerminal so that all the settings you input can be read at the beginning of the data file. In the HyperTerminal window, choose Transfer>Capture Text... and browse for the folder set up in My Documents>muonDecay_data, enter the filename, and press "Open." To keep the folder of data most organized, name the data file using the format YYYYMMDD_AB.txt where AB are your first and last initials.

You communicate with the Quarknet card via the HyperTerminal using text commands.

Table 11.3: *Some Quarknet commands and their meanings*

Command	Function
H1, H2	help menu
DG	Display GPS Info, Date, Time, Position and Status
DC/WC	Display/Write settings of control registers
DT/WT	Display/Write settings of time measurement chips (TMC)
V1	View setup registers
TL	Set threshold value
SA	Save setup

Enter the following commands in HyperTerminal, after TextCapture is enabled as described above:

1. WC 00 2F (this enables all 4 channels and sets a condition that three channels must register hits in coincidence)
2. WC 01 70 (this sets a veto width to the maximum value, although veto is not used in this experiment)
3. WC 02 FF

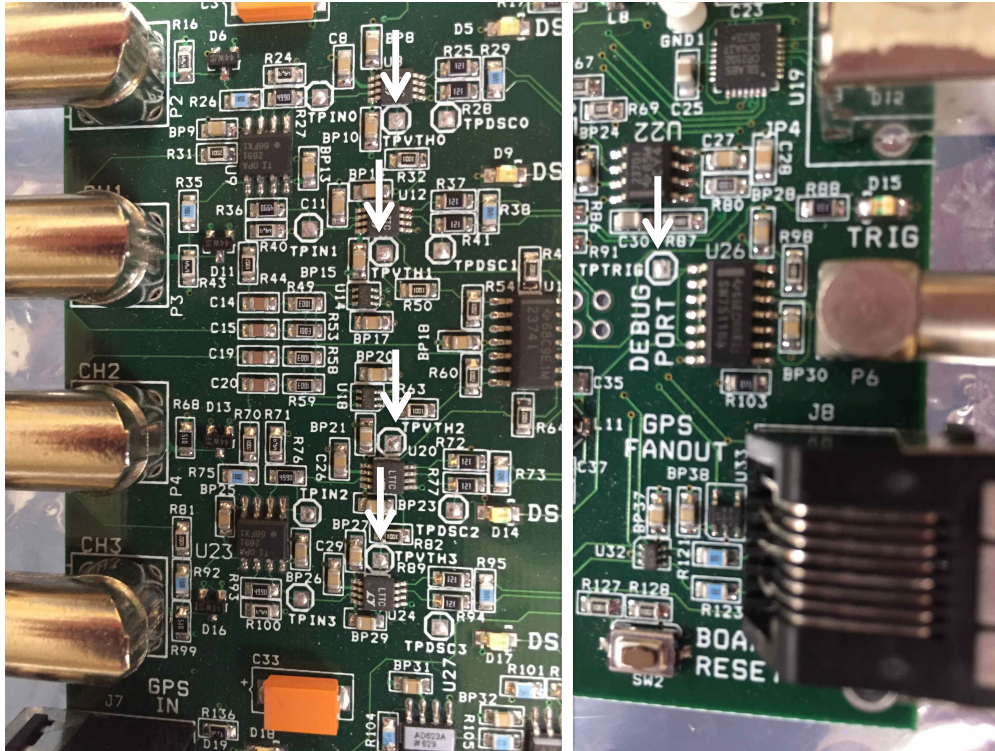


Figure 11.5: Location of the TPVTH ports (left) and the DEBUG port (right).

4. WC 03 07 (this command and the command above set the coincidence gate window to about 20 microseconds)
5. DG (this shows the status of the GPS connection)
6. DC (this shows the current value of the control registers, should be 2F 70 00 04)
7. WT 01 00
8. WT 02 04 (this and the command above set the read/write delay to 40 ns)
9. DT (this shows the current value of the delay registers, should be 00 00 04 00)
10. V1 (this displays the thresholds and gate widths)
11. TL 4 700 (this sets the threshold to 700 mV)
12. ST 3 5 (this sets the status line to read out the counts every 5 minutes)
13. SA 1 (this saves the setup)

Once all the settings for the experiment have been set up, type the command CE into the terminal to display the counters, and **leave HyperTerminal open** to collect the data for at least a few days or longer for the muon lifetime experiment. When you have collected all the data you want to analyze type CD to disable the counters and end text capture in HyperTerminal by choosing Transfer>Capture Text>Stop. You may now close HyperTerminal without saving any connection settings.

11.2.4 Extracting Decay Lengths

The data is in hexadecimal format. A python program on the Windows control computer will look for events where two signals are observed in the same channel (the muon stopping and then decaying into an electron), and extract the time difference between them in decimal form. This will form the basis of your decay time analysis.

To run the program, open the Windows command prompt and type `cd "My Documents", cd muDecay`. Next, type the command `"python muDecay.py"` This will start the data analysis program and ask for a data file to analyze. Browse to the data file you just collected in HyperTerminal and select "Open." Depending on how large the data file is, it may take a few minutes for the program to complete. Once the program is finished running, there should be a file in the `muonDecay_data` directory which has the same name as the data file with `"_decays"` appended to the end of the filename. This file contains the decay length of each decay event and can be imported into any text processing or data analysis program that can do histograms.

11.3 Analysis and Questions

1. Plot your plateau curves on a log-y plot.
2. Using Excel or a custom program you've written, count the number of candidate decays in each $0.5 \mu\text{s}$ interval in the range $0\text{--}20 \mu\text{s}$. Correct the number of counts for the random background coincidences, which will be in the tail end of the distribution at large time differences. Present the results in a table. Also make a log-y plot of number of candidate events versus time. Include error bars (which will be statistical in nature: what is the correct error to use in this case?)
3. All PMTs afterpulse, that is, a secondary pulse will sometimes follow the primary pulse produced by scintillation light. Afterpulses are usually associated with collisions between gas molecules and the electron cloud produced by PMT amplification. (The interior of the PMT is ideally a vacuum, but in reality it contains a small amount of gas.) The collisions excite or ionize the molecules, after which the ions can be accelerated and collide with the photocathode or one of the dynodes, producing additional electrons which are then amplified or the molecules can de-excite, producing photons which can then be absorbed by the photocathode to produce more photoelectrons. Afterpulses typically occur within a fraction of a microsecond after the primary pulse. For measurements of the muon lifetime, afterpulsing gives rise to fake delayed coincidences, the primary pulse due to the deposition of energy by a muon passing through the scintillator and the afterpulse mimicking the energy deposit due to the muon decay. Is there evidence for afterpulsing in your data? Explain.
4. Delayed coincidences can also occur due to two independent pulses randomly occurring closely in time with each other. Two muons passing through the scintillator in quick succession is one such way this could happen. The probability of a so-called accidental delayed coincidence

will typically be very small in this experiment but not necessarily negligible. Is there any evidence for accidental delayed coincidences in your data? Explain.

5. Using the method of least squares, obtain an estimate of the muon lifetime from your data. (Note that the measurement uncertainties should be time-dependent.) Also estimate the uncertainty in your estimate. Explain how you dealt with the effects of any afterpulsing and delayed coincidences in applying the method.
6. Compare your measured value for the muon lifetime with the accepted value. Are they consistent within uncertainties? Explain.

Planck's Constant from LEDs

In this experiment you will determine the quantity hc/e by analyzing the current-voltage characteristics and light output of light-emitting diodes (LEDs).

12.1 Introduction

Max Planck (1858-1947) was an early pioneer in the field of quantum physics. Around 1900 Planck developed the concept of energy quantization to explain the spectral distribution of blackbody radiation.[15] This idea is fundamental to the quantum theory of modern physics. Planck received a Nobel Prize for his work in the early development of quantum mechanics in 1918. Planck proposed that atoms absorb and emit radiation in discrete quantities given by

$$E = nhf \quad (12.1)$$

where n is an integer known as a quantum number, f is the frequency of vibration of the atom, and h is now known as Planck's constant. The smallest discrete amount of energy radiated or absorbed by a system results from a change in state whereby the quantum number n of the system changes by one.

In 1905 Albert Einstein (1879-1955) published a paper[16] in which he used Planck's quantization of energy principle to explain the photoelectric effect. The photoelectric effect involves the emission of electrons from certain materials when exposed to light and could not be explained by classical models. Einstein assumed that the electrons absorbed one quantum of electromagnetic energy at a time and that the energy of this quantum (photon) is

$$E = hf = \frac{hc}{\lambda} \quad (12.2)$$

where f is the frequency of the light and λ is its wavelength. In Einstein's model, owing to the discrete nature of light, an electron would only be ejected if the photon energy was greater than the energy binding the electron to the metal. Einstein received the Nobel Prize in Physics for this work in 1921. Niels Bohr (1885-1962) used Planck's ideas on the quantization of energy as a starting point in developing the modern theory for the hydrogen atom. Robert Millikan made the first measurement[17] of Planck's constant in 1912. The best current value for Planck's constant is[18]

$$h = 6.62606896(33) \times 10^{-34} \text{ J} \cdot \text{s} = 4.13566733(10) \times 10^{-15} \text{ eV} \cdot \text{s} \quad (12.3)$$

The two digits in parentheses denote the standard uncertainty in the last two digits of the value. In this experiment, you will use the current-voltage relationship of a set of light emitting diodes

(LEDs) to measure Planck's constant.

12.1.1 Light-Emitting Diodes (LEDs)

An LED is a semiconductor device that emits electromagnetic radiation at (typically) optical and infrared frequencies. The device is a junction between p -type and n -type semiconductors, known as a p - n diode, usually GaAs-, GaP- or SiC-based materials. They emit light only when an external applied voltage is used to bias the diode above a minimum threshold value. The gain in electrical potential energy delivered by this voltage is sufficient to force electrons to flow out of the n -type material, across the junction barrier, and into the p -type region. This threshold voltage for the onset of current flow across the junction and the production of light is V_0 .

The emission of light occurs after electrons enter into the p -type region (and holes into the n -region). In the p -type region, the electrons are a small minority surrounded by many holes (in this context, essentially the anti-particles of the electrons) and they will quickly find a hole to recombine with. In this process, the electron relaxes from the excited state (conduction band) to the ground state (valence band) while the hole does the opposite. The diodes are called light-emitting because the energy given up by the electron as it relaxes is predominantly released as a photon. Above the threshold value, the current and light output increase rapidly with the bias voltage across the diode.

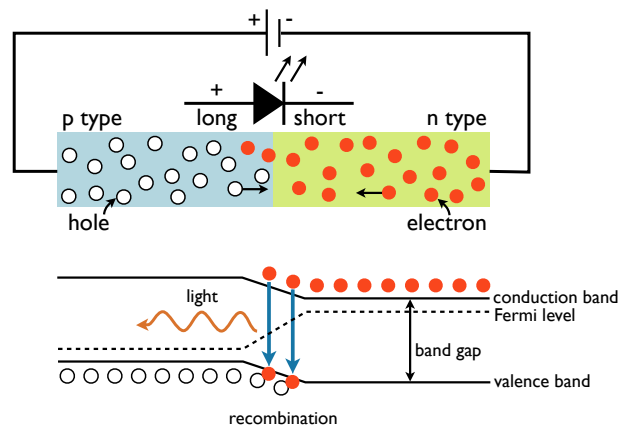


Figure 12.1: The electrons and holes flow over a barrier at the p - n interface when they acquire sufficient potential energy through the applied bias voltage.

The most important energy scale is set by the so-called 'band gap' E_g , the energy difference between the conduction and valence bands. This band gap is a property of the materials that make up the diode and can be tuned over a wide variety of energies from the infrared to the ultraviolet. All electrons and holes, once excited by the applied voltage V_0 , release the same energy E_g when they relax, and thus only photons of a specific energy are emitted.

Based only on these facts, we know enough to predict the minimum voltage required for light emission. If only photons of a specific energy (and hence, a specific color/wavelength/frequency) are emitted, then regardless of the mechanism of conduction in the diode, we should expect nothing to happen until the electrical potential energy supplied to the LED is equal to the energy of the photons emitted. In other words, we imagine that the potential energy of each charge injected into the LED is converted completely to light energy. Right at the threshold voltage, the electrical energy is just equal to the photon energy, which should in turn be equal to the band gap. The quanta of energy or photon has an energy $E = hf$, and an electron at a potential V_o has a potential energy eV_o , so the relation between the photon energy and the threshold voltage V_o , is given by conservation of energy:

$$eV_o = E_g = hf = \frac{hc}{\lambda} \quad (12.4)$$

where f and λ are the frequency and wavelength of the emitted photons, c is the speed of light, and e is the electronic charge. The quantity hc/e can thus be determined by separate measurements of the threshold voltage and the emission wavelength:

$$\frac{hc}{e} = \lambda V_o \quad (12.5)$$

There are two ways to analyze this data: you can average the values of λV_o , or you can plot V_o vs. $1/\lambda$, in which case the slope of the plot will be hc/e . The latter will account for any systematic offsets.

The accepted value of hc/e is $1.2398 \times 10^{-6} \text{ eV} \cdot \text{m}$. Since the speed of light is defined exactly in the SI system, determining hc/e is equivalent to determining h/e (which you can compare to your photoelectric experiment, Sec. 8). Using a known value for the electron charge (for example, obtained from your Millikan experiment later this semester, Sec. 15), you can determine Planck's constant h by itself.

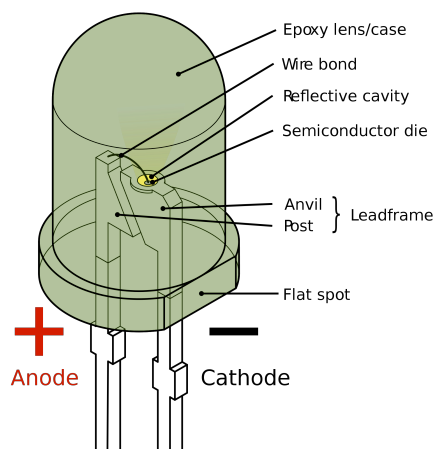


Figure 12.2: Schematic of a 5 mm round LED, one of the most common types of LEDs. From [http://commons.wikimedia.org/wiki/File:LED,_5mm,_green_\(en\).svg](http://commons.wikimedia.org/wiki/File:LED,_5mm,_green_(en).svg)

12.1.2 “Bamalab” boxes

In this experiment, you will be using the ‘Bamalab’ software and hardware to measure the current-voltage characteristics of several LEDs in order to determine their threshold voltages. The Bamalab box is a computer controlled, USB-driven box that has four basic functions: (1) sourcing current, (2) sourcing voltage, (3) measuring current, (4) measuring voltage.

Current sourcing: current must always have a closed path to flow through. When using the current source ($+I_{\text{out}}$, $-I_{\text{out}}$), imagine that current flows from the red positive terminal ($+I_{\text{out}}$), through your device, back to the black negative terminal.

Voltage sourcing: the red terminal is the positive voltage, the black is the negative voltage. Applying a potential difference of, e.g., 1 Volt means that $V_{\text{red}} - V_{\text{black}} = 1 \text{ V}$.

Voltage measurement: the same polarity convention applies - red is positive, black is negative. Measuring a potential difference of, e.g., 1 Volt means that $V_{\text{red}} - V_{\text{black}} = 1 \text{ V}$.

Current measurement: the same polarity convention applies - red is positive, black is negative. Measuring current is slightly different, however: current must flow into the $+I_{\text{in}}$ terminal and back out of the $-I_{\text{in}}$ terminal to be measured, thus the current meter is in series with your device.

12.2 Objective

In this experiment, you will learn how to perform basic current-voltage characteristics for electrical devices and measure the emission wavelength of a monochromatic light source.

Hypothesis: Comparing the threshold (“turn-on”) voltage for light-emitting diodes to their emitted wavelength, one can determine hc/e .

12.3 Preparatory Questions

You should touch on these questions in your report.

1. A light-emitting diode (LED) emits blue photons of wavelength 480 nm. What would be the minimum voltage you would expect to apply to the LED before it emits light?
2. The wavelength of a green light is $5.5 \times 10^{-7} \text{ m}$. What is the approximate frequency of this kind of light? What is the photon energy?
3. Why can't we use incandescent or fluorescent lights for this experiment?

12.3.1 Relevant Reading

Taylor[1], Ch. 2, 3,8

Pfeffer & Nir[2], Ch. 2.2.1-3

12.4 Supplies

1. ‘Bamalab’ electrical supply / measurement box
2. 6 banana cables
3. 4-5 LEDs (infrared, red, green, blue, UV)
4. lab notebook and USB drive for saving spectra
5. (optional) Ocean Optics USB-600 ‘Red Tide’ spectrometer or diffraction grating

12.5 Suggested procedure

12.5.0.1 Starting the Bamalab Software

Start the software by double clicking the “Shortcut to Bamalab” icon on the desktop.

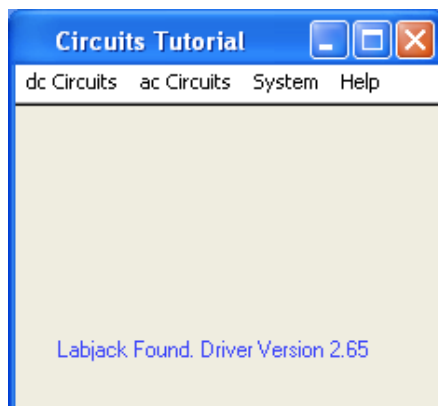


Figure 12.3: *Introductory screen after opening the ‘Bamalab’ software.*

12.5.0.2 Checking your Bamalab box

Occasionally, components of your Bamalab box may stop functioning, or at least you may have reason to believe so. Fortunately, there is a simple procedure for verifying your Bamalab box. This procedure presumes that the previous stage - starting the software and verifying the USB connection - has been successfully completed.

Voltage input and output First, open the “multimeter” panel under the “dc Circuits” tab in the software. This should bring up a panel like the one shown in Fig. 12.4. From the “Source” pull-down menu, select “Voltage,” and also select “Voltage” from the “Measure” pulldown menu. After

performing each action, the middle portion of the window should change to reflect your choices.

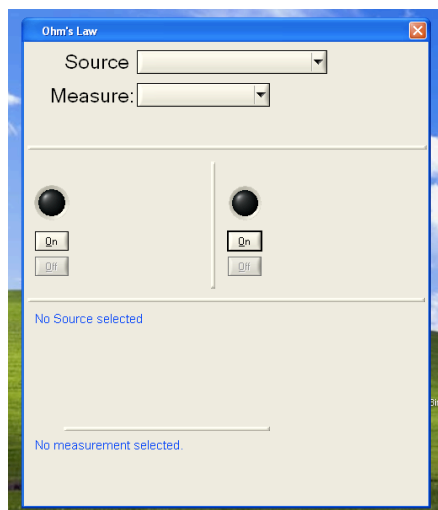


Figure 12.4: *Multimeter panel.*

Next, using two “banana” cables, connect the voltage input and output together. Do this by connecting “ $+V_{in}$ ” on one side of the box to “ $+V_{out}$ ” on the other side, and similarly connect “ $-V_{in}$ ” to “ $-V_{out}$.” Now click the “on” button under “Voltage Output” portion of the screen. The circle just above the “on” button should now be green. Adjust the voltage output dial to some moderate value, perhaps 1.5 or 2 V. You are now supplying voltage from the V_{out} terminals.

In the “Voltage measurement” portion of the screen, click “on.” The circle above the “on” button should turn red. Now you are measuring the voltage at the V_{in} terminals, which are connected directly to the V_{out} terminals. If everything is functioning properly, the voltage measurement should be very close to your output voltage. Finally, change the voltage output dial to some new value (say, 3 V). You should see the “voltage measurement” indicators change to follow the change in output, in real time. Variations of 10–15% between output and input are tolerable. If the above is successful, your voltage sourcing and measuring are OK!

Current input and output If you have just performed the previous step, turn off the voltage output and measurement. On the multimeter screen, elect to source and measure current now. The output and measurement regions of the screen should change after you do this. Using two “banana” cables, connect the current input and output together. Do this by connecting “ $+I_{in}$ ” on one side of the box to “ $+I_{out}$ ” on the other side, and similarly connect “ $-I_{in}$ ” to “ $-I_{out}$.” Operating the current source also requires you to flip a small switch on the side of the box. Make sure this switch is set to “on.”

Now click the “on” button under “Current Output” portion of the screen. The circle just above the “on” button should now be green. Adjust the current output dial to some moderate value, perhaps 3 or 5 mA. You are now sourcing current from the I_{out} terminals. Next, in the “Current measurement” portion of the screen, click “on.” The circle above the “on” button should turn red. Now you are measuring the current passing through the I_{out} terminals, which are connected directly to the I_{out} terminals. If everything is functioning properly, the current measurement should be very close to your output current. Finally, change the current output dial to some new value (say, 2 mA). You should see the “current measurement” indicators change to follow the change in output, in real time. Again, variations of 10–15 % between output and input are tolerable. If the above is successful, your current sourcing and measuring are OK!

12.5.0.3 Testing the LEDs

Check that the LEDs are operational before performing the $I(V)$ sweep in the next section. Use the white LED tester; if you do not have this, check with the instructor.

1. Using the 50 mA slot on the tester, place the positive (longer) end of the LED in the upper hole and the negative end of the LED in the lower hole.
2. Press the “Test” button. You should see the green “Batt” light turn on, and you should see your LED illuminate. The infrared LED can be checked in a dark room with the camera on a smartphone.
3. Consult your instructor for a replacement if the LED does not light up.

12.5.0.4 Performing the $I(V)$ sweep

From the “dc Circuits” menu on the Bamalab main screen, select “Current vs. Voltage.” That should bring up a screen like the one in Fig. 12.5 below.

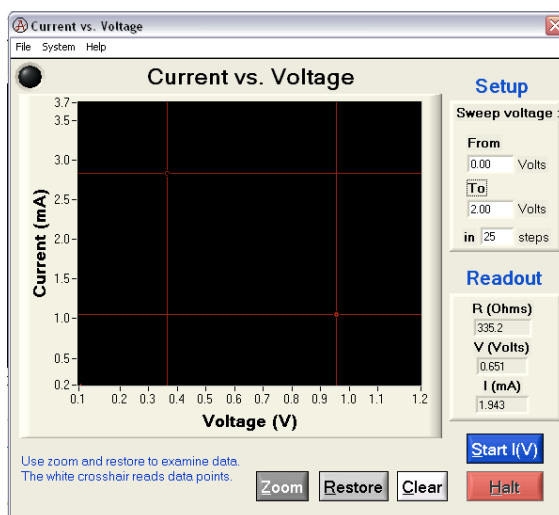


Figure 12.5: Current-voltage panel.

1. Connect the positive end of your LED (longer leg) to $+V_{\text{out}}$. Using a second wire, connect $+V_{\text{out}}$ and $+V_{\text{in}}$ together.
2. Using a third wire, connect the negative side of the LED to $+I_{\text{in}}$, and with a fourth wire, connect $+I_{\text{in}}$ to $-V_{\text{in}}$.
3. Finally, connect $-I_{\text{in}}$ to $-V_{\text{out}}$. This should give you the circuit in Fig. 12.6, which applies a voltage to LED and ammeter in series and measures the current through and voltage across the LED.
4. In the “Current vs. Voltage” panel you should already have open, sweep the voltage from 0–4 Volts, in 50 steps or so. You should see a curve which is zero or close to it at low voltage values, then rises rapidly as voltage increases.
5. Once you have performed this coarse $I(V)$ scan, clear the graph and perform a more detailed scan starting just below the threshold voltage (where the current begins to rise rapidly) up to about $3\text{--}4\text{ V}$.¹ The main point is to capture the threshold behavior and a portion of the rapid rise
6. When you have a suitable $I(V)$ characteristic, save the data (File/Save) to your USB memory device, and measure the rest of your LEDs.

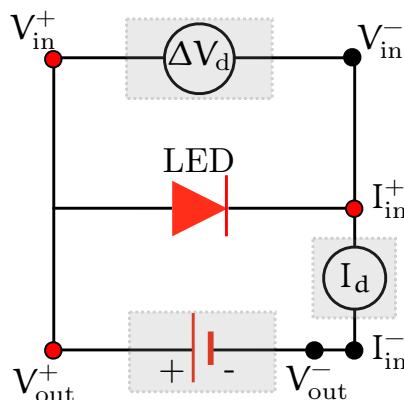


Figure 12.6: Circuit for measuring $I(V)$ for a light-emitting diode, with Bamalab box connections labeled. Shaded components are functions performed by the Bamalab box.

12.5.0.5 Wavelength Determination

The wavelength determination can proceed in three possible ways. First, if you are pressed for time you may use the tabulated wavelengths in the Appendix on page 126. Do not consider this to be skirting your laboratory duties: you will perform optical spectroscopy in later experiments! You can use the color-wavelength charts in the laboratory or in the Appendix to this section as a

¹If you notice the current dropping to zero above a certain voltage, you are probably exceeding the current limit on the Bamalab box, which is about 8 mA. In this case, reduce the maximum voltage until your maximum current is below 8 mA.

rough verification that the table matches your particular LEDs. Keep in mind that you have **two experiments** to perform during this laboratory period.

Second, *if you have time*, you may use the Ocean Optics USB-650 optical spectrometer. The spectrometer can very quickly (i.e., in a few minutes) measure the emission spectrum (intensity versus wavelength) for all of your LEDs. If you have already performed the atomic spectroscopy experiment, and the apparatus is free, this is the fastest and most accurate method. There is a very good chance that another group is already using the spectrometer, in which case you might bargain with them for use of the spectrometer for 5 minutes or so . . .

Third, if you have already performed the optics experiment, you may use a diffraction grating and spectroscope or projection screen to relatively quickly (~ 15 min) measure the LED wavelengths. The instructor can help you if you wish to use this method and have *not* already performed the optics experiment. Again, *if you have time*. If you have not already performed the second experiment (e/k), you may want to simply use the tabulated results.

12.6 Data analysis

12.6.1 Threshold voltage

An operational definition of the threshold voltage could be that value of the bias voltage when the current reaches 0.01 mA. Extrapolate your $I(V)$ curves to where they cross 0.01 mA current and use that as the working value of V_o .

Another way to determine the threshold voltage is to plot $\log(I)$ (y) versus V (x), which should give roughly linear behavior near the threshold. If you fit the straight-line behavior just above threshold to a linear trendline, you can use the resulting best-fit line equation to determine the threshold voltage, either as defined above, or by finding where the line intercepts the x axis.

Finally, you can just plot $I(V)$ and perform a linear extrapolation of the curve (i.e., do a linear fit and find the x intercept). Over a range of $\sim 0.05 = 0.1$ V above the perceived threshold, the characteristic is quite linear for most LEDs, and this procedure can give quite accurate results. The main point is that you find a reasonable and systematic way to determine the threshold voltage, and use the *same* procedure in analyzing all of your data.

12.6.2 Determination of h/e

From the discussion above, you should be able to determine h/e from your data (given that c is a constant). Do this for all of your LEDs, taking care to include estimated uncertainties in threshold

voltage and LED wavelengths.ⁱⁱ Find h/e for each LED. Using the accepted value for e , determine h for each LED and the average result for all of your LEDs together (making sure to propagate the uncertainty correctly). Since you have 3 or 4 measurements of Planck's constant, you may wish to find the mean and standard deviation of the mean to quote the uncertainty (see the introduction to the counting statistics experiment, Sec. 5).

Then, plot V_o vs. $1/\lambda$ and determine hc/e from the slope. Use the uncertainty from the linear regression calculation. Which method is more accurate?

12.7 Discussion and topics for your report

In addition to the data analysis outline above, below are some questions and topics you may wish to address in your report.

1. Is your value for h reasonable, i.e., does the range of the value plus or minus the uncertainty coincide with the accepted value? If not, why?
2. What is the product of the threshold voltage and the emission wavelength? What should it be?
3. Why is it problematic to determine the threshold voltage from the $I(V)$ equation governing a diode?
4. How could you make your threshold determination more accurate? Presume you have use of an optical spectrometer or light meter.
5. How is this experiment related to the photoelectric experiment you have/will perform?
6. What materials might be used to give the colors for your particular LEDs?
7. How can you create interior white lighting with discrete LED sources?

Visible Spectrum

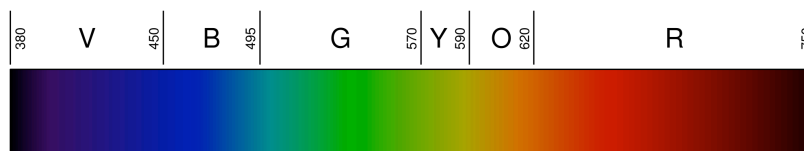


Figure 12.7: Linear representation of the spectrum of visible light. From http://en.wikipedia.org/wiki/Visible_spectrum. Numbers are wavelengths in nanometers.

LED Wavelengths

The table below lists the known emission wavelengths for the LEDs provided in the laboratory, as measured by the same USB-650 Ocean Optics spectrometer you will use in later experiments. If

ⁱⁱFor wavelength uncertainty, you may use the full width at half maximum of the quoted wavelengths in the Appendix on page 126.

you do not have sufficient time to verify these measurements yourself with the spectrometer or a diffraction grating, at least make a rough verification with the color-wavelength charts in the lab or the one above. A printed and verified color chart in the laboratory is preferred, owing to the wide variety in the quality of color reproduction with monitors and printers.

Table 12.1: *LED Wavelengths*[†]

Type	Peak λ (nm)	FWHM [‡] (nm)
Infrared	931.7	40.7
Red	635	20
Yellow	593	14
Green	510	36
Blue	453	32

[†] Wavelengths should be verified for your particular LEDs.

[‡] FWHM = full width at half maximum

Polarization and Diffraction of Light

13.1 Introduction

In the first portion of this experiment, Malus' Law of Polarization is verified by showing that the intensity of light passed through 2 polarizers depends on the square of the cosine of the angle between the 2 polarization axes. Adding a third polarizer makes clear the vector nature of polarization.

In the second portion of this experiment, a diffraction grating consisting of a series of closely-spaced vertical lines is used to measure the wavelength of the laser light used in the first experiment.

13.1.1 Polarization

In the first experiment, Laser light ($\lambda = 650 \text{ nm}$) is passed through two polarizers. As the second polarizer (the analyzer) is rotated, the relative light intensity is recorded as a function of the angle between the axes of polarization of the two polarizers. The plot of light intensity, as measured by a calibrated photocell, versus the relative angle between the polarizers will be compared with theoretical results.

A polarizer only allows light which is vibrating in a particular plane to pass through it. This plane forms the "axis" of polarization. Unpolarized light vibrates in all planes perpendicular to the direction of propagation. If unpolarized light is incident upon an "ideal" polarizer, only half of the light intensity will be transmitted through the polarizer.

The transmitted light is polarized in one plane. If this polarized light is incident upon a second polarizer, the axis of which is oriented such that it is perpendicular to the plane of polarization of the incident light, no light will be transmitted through the second polarizer, as depicted in Fig. 13.1. However, if the second polarizer is oriented at an angle not perpendicular to the axis of the first polarizer, there will be some component of the electric field of the polarized light that lies in the same direction as the axis of the second polarizer, and thus some light will be transmitted through the second polarizer.

If electric field after passing through the first polarizer at angle θ is \vec{E}_o , then the amplitude passing through the second polarizer will be the projection onto the angle of the second polarizer. That is, only the component of \vec{E}_o parallel to the second polarizer set at angle φ will pass, giving an electric field amplitude $E_o \cos(\theta - \varphi)$ after passing through both polarizers. Since the intensity of light varies as the square of the electric field amplitude, the light intensity after passing through both polarizers is given by

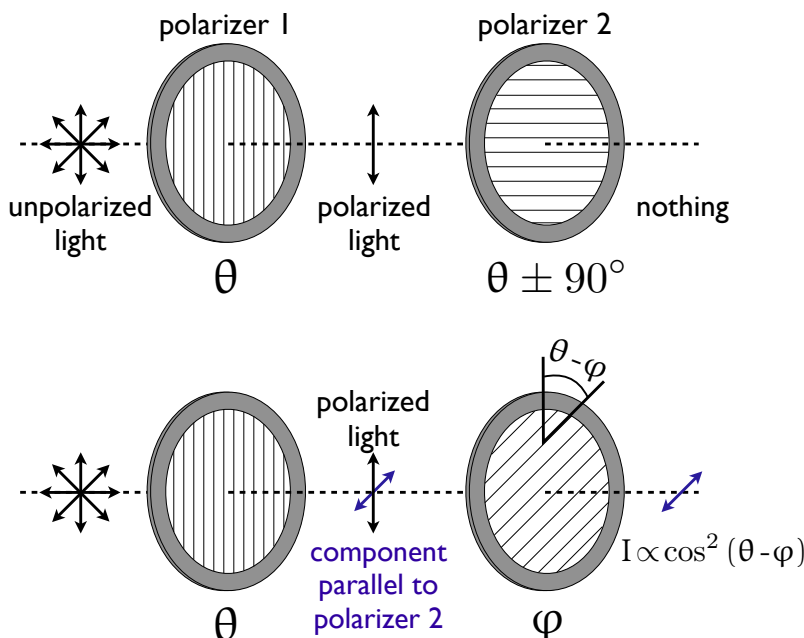


Figure 13.1: Light transmitted through two polarizers. If the polarizers are aligned 90° offset, essentially no light passes through both polarizers. If the polarizers have a relative angle $\theta - \phi$, the intensity after both polarizers vary as the cosine of the relative angle squared.

$$I = I_o \cos^2(\theta - \phi) \quad (13.1)$$

This is known as *Malus' law*, and we see that the transmitted intensity will vary from zero when the polarizers have a relative angle of $\pm 90^\circ$ to I_o when the polarizers are aligned at 0 or 180° .ⁱ In retrospect, we can use Malus' law in this form to find the transmission through a single polarizer. A beam of unpolarized light can be thought of as containing a uniform mixture of linear polarization at all possible angles. Since the average value of $\cos^2 \theta$ is $1/2$, the transmitted intensity for unpolarized light through a single polarizer is $1/2$.

Real polarizers do not perfectly block light with polarization orthogonal to their polarization axis. The ratio of the transmission of the unwanted (90°) component to the desired (0°) component is called the extinction ratio, and varies from around $1 : 500$ for Polaroid sheets (which we will use) to about $1 : 10^6$ for Glan–Taylor prism polarizers. In practice, some light is also absorbed by the polarizer, and the actual transmission will be somewhat lower. Simple Polaroid-type polarizers may have a transmission of $\sim 38\%$, whereas some birefringent prisms can be considerably higher, nearly reaching the ideal limit of $1/2$. Thus, even for crossed polarizers with a relative angle of $\pm 90^\circ$, even though the theoretical transmission is exactly zero some light will be transmitted.

In the next phase of this experiment, you will explore a curious phenomena when three polarizers

ⁱWhen the polarizers have a relative angle of $\pm 90^\circ$, they are said to be *crossed*.

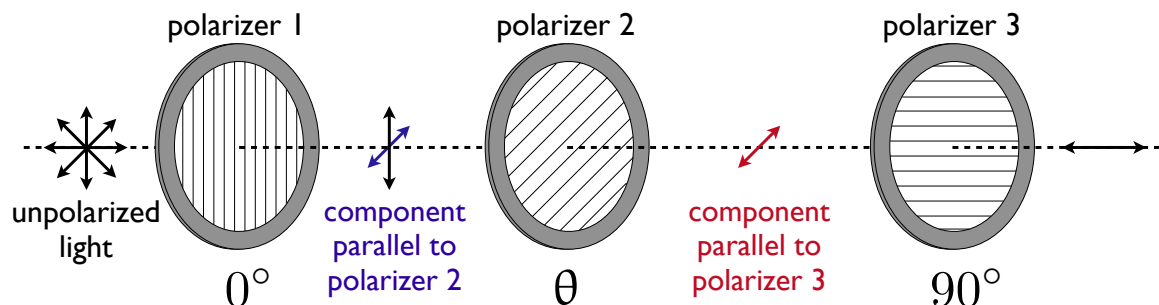


Figure 13.2: Light transmitted through three polarizers.

are placed in series, depicted in Fig. 13.2. The first and third have a relative angle of 90° while the middle polarizer has an angle θ relative to the first (and therefore an angle $90 - \theta$ relative to the third). If the intensity after passing through the first polarizer is I_1 , and the intensity after passing through the second polarizer is I_2 , then Malus' law gives:

$$I_2 = I_1 \cos^2 \theta \quad (13.2)$$

The intensity after the third polarizer is found similarly, using the relative angle between the second and third polarizers:

$$I_3 = I_2 \cos^2 (90 - \theta) = I_1 \cos^2 \theta \cos^2 (90 - \theta) \quad (13.3)$$

Using the trigonometric identities $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$, and $2 \cos \alpha \sin \alpha = \sin 2\alpha$ we can simplify this result:

$$I_3 = \frac{1}{4} I_1 \sin^2 2\theta \quad (13.4)$$

This is the curious aspect: if we had only the first and third polarizers, which are crossed, the transmitted intensity would be zero. By inserting an intervening polarizer at an intermediate angle, we recover a non-zero intensity! This clearly demonstrates the vector nature of the electric (and magnetic) field. If the intervening polarizer is placed with an angle of 45° , we see that the transmitted intensity is $1/4$ the original intensity.

13.1.2 Photoresistors

A photoresistor, also called a light-dependent resistor or cadmium sulfide (CdS) cell, is a resistor whose resistance decreases with increasing incident light intensity. Photoresistors are made of a high resistance semiconductor. If the incident light is of sufficient energy to excite electrons in the semiconductor from the valence to conduction band, the resulting free electron and its hole partner are available to conduct electricity, thereby lowering the resistance. In extrinsic devices, impurities (or dopants) are added which introduce energy levels nearer the conduction band. This means the electrons may be excited with even less energy, and lower energy (longer wavelength) photons can

also influence the resistance. Cadmium sulfide, the usual photoresistor material, has a bandgap of 2.42 eV, corresponding to green light of wavelength 512 nm. Thus, CdS photocells have greatest sensitivity to green light, and their spectral response in fact closely mimics that of the human eye. A typical photoresistor is shown in Fig. 13.3.

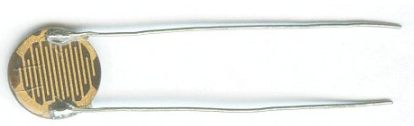


Figure 13.3: A light-dependent resistor. From <http://en.wikipedia.org/wiki/Photoresistor>.

In the present experiment, you will shine a laser beam through the polarizers under study and measure the intensity of the transmitted light by measuring the resistance of a photoresistor.

One complication is that the response of a photocell is a non-linear function of incident light intensity. That is, if the incident light intensity doubles, the resistance will almost certainly not change by a factor of 2. You will need to calibrate your photocell by first varying the intensity of the laser in a known fashion using a variable transmission filter, and measuring the resistance versus percent transmission. Once this characteristic is known, subsequent measurements of resistance (using the same source of illumination) can be converted into accurate measurements of transmission percentage.

Typically, for a photoresistor the resistance R exhibits a power law characteristic as a function of incident intensity. If the full (unblocked) intensity of the source is I_o , and the resistance at that intensity is R_o , the behavior should follow the form

$$\frac{R}{R_o} = a \left(\frac{I}{I_o} \right)^b \quad (13.5)$$

where $a \sim 1$ and $b \sim -0.5$ are constants to be determined for your photoresistor. After measuring your $R(I)$ characteristic, you will perform a least-squares regression to determine the coefficients for the power law above. This will calibrate your photocell and allow relative intensity to be accurately determined from a resistance measurement.

13.1.3 Diffraction Grating

You may want to review the basics of single- and multiple-slit diffraction before reading this section, which can be found in essentially any introductory electricity and magnetism or optics text.

A diffraction grating is an optical component with a regular pattern, in this case many thousands of parallel lines. The form of the light diffracted by a grating depends on the structure of the

elements and the number of elements present, but all gratings have intensity maxima at angles θ_n which are given by the grating equation

$$d(\sin \theta_n + \sin \theta_i) = n\lambda. \quad (13.6)$$

where θ_i is the angle at which the light is incident, d is the separation of grating elements and n is an integer which can be positive or negative. The light diffracted by a grating is found by summing the light diffracted from each of the elements, and is essentially a convolution of diffraction and interference patterns. In the present experiment, you will measure the diffraction angle and use the known grating spacing to determine the wavelengths of the lasers used in the preceding experiments.

13.2 Objective

Verify the dependence of transmitted light intensity on the angle between two or three polarizers placed in series. This demonstrates the basic functionality of a liquid crystal display (LCD), where the central polarizer is a twisted nematic liquid crystal whose polarization can be controlled by the application of a small voltage. Polarization also plays a crucial role in some newer 3D movie technologies.

Verify that the wavelength of light may be measured by diffraction through a grating of closely-spaced lines. You will use this technique in later spectroscopy labs.

Hypothesis: The intensity of light passing through two polarizers is proportional to the square of the cosine of the relative angle between the polarizers' axes. A simple extension of this model gives the intensity after passing through three polarizers. The position of diffracted monochromatic light beams after passing through a grating of a known spacing of vertical lines may be used to determine the wavelength of the light.

13.3 Preparatory Questions

You should touch on these questions in your report.

1. For 3 polarizers, what is the angle between the middle polarizer and the first polarizer to get the maximum transmission through all 3 polarizers?
2. For an incident wavelength of $\lambda = 632 \text{ nm}$, what do you expect the angle θ to be for the first diffracted beam with a diffraction grating of 600 lines/mm?

13.3.1 Relevant Reading

Taylor[1], Ch. 2, 3

Review of basic wave optics (e.g., Serway, Halliday & Resnick, etc.)

<http://en.wikipedia.org/wiki/Polarizer>

[http://en.wikipedia.org/wiki/Polarization_\(waves\)](http://en.wikipedia.org/wiki/Polarization_(waves))

<http://en.wikipedia.org/wiki/Diffraction>

13.4 Supplies

1. Red diode ($\lambda \approx 650$ nm) and green ($\lambda = 543$ nm) laser
2. 3 Polarizers (with angular scale)
3. 1 Variable transmission filter
4. 48 mm or 22 mm convex lens
5. 1 Diffraction grating
6. 1 Diffraction optics wheel
7. 3 Magnetic optical component holders
8. Magnetic optical rail
9. 1 Photoresistor, mounted to optical component holder
10. Digital voltmeter
11. Safety goggles for red & green lasers
12. 2 banana cables
13. lab notebook

13.5 Suggested procedure

The first task is to set up the optical rail system and collect the relevant components listed above.

13.5.1 Setup

1. Once you have the required items, place the laser on one end of the magnetic optical rail. You may choose either the red or green laser for the polarization experiment. The red diode laser has built-in positioning, which may be more convenient.
2. At all times, BE CAREFUL NOT TO DIRECTLY VIEW THE BEAM OR ITS DIRECT REFLECTION. Use the green safety goggles with the red laser, and vice-versa.
3. Plug the laser in to the nearest outlet, taking care that the cord is secure and will not be disturbed during your measurements – aside from the obvious safety issue of tripping over loose cords, vibrations in the cord will be coupled into your optical system and make your readings more variable. Vibration isolation is important! Do not turn on the laser yet.
4. Place the photodiode in its component holder on the opposite end of the rail.
5. Connect the photoresistor to the digital voltmeter using the banana cables (polarity is not important), again taking care to secure the cables against vibrations.

6. Turn on the laser. Do not, under any circumstances, stare into the beam or point it into your eye directly. Though the lasers we are using are not dangerous per se, extended exposure can still cause vision impairment. Align the laser spot such that it falls directly on the center of the photocell.
7. Turn on the digital voltmeter, and set it for a resistance (“ohms”) measurement. Set the voltmeter range at $20\text{ k}\Omega$, and *do not change the range throughout the experiment.*ⁱⁱ The resistance of your photoresistor under full illumination of the laser should be approximately $0.5\text{--}1\text{ k}\Omega$
8. You may wish to use a short focal length lens to focus and “steer” the beam onto the photoresistor; a 48 mm convex lens works well. If you choose to use a lens, leave it in place for **all** measurements to maintain your calibration.

13.5.2 Calibrating the Photoresistor

1. Place the variable transmission filter in the path of the laser beam using an optical component holder, such that the beam passes through the 0% transmission region. Place the filter as close to the photoresistor as possible, as the scattering by the filter will “spread” the beam.
2. Record the resulting resistance of the photoresistor.ⁱⁱⁱ
3. Move the variable transmission filter such that the 20% transmission window intercepts the beam, and record the resistance. Repeat for 40, 60, 80, and 100% transmission and complete the table of transmission and the corresponding photoresistance below. Include the estimated uncertainty in the resistance.
4. Make a plot of your resistance (y) versus transmission (x) data in, e.g., Excel and perform a power-law regression (termed a “trendline” in Excel). Alternatively, you may make a table of $\log R$ versus $\log T$ and perform a linear regression, which can be performed on most handheld calculators. Do not include the 0% transmission point in your trendline analysis ... think about that for a minute.
5. Verify that your fit function predicts approximately the correct resistance for a given transmission percentage, and note the fit parameters and R^2 value.

If you wish to obtain a more accurate calibration, there are two simple suggestions: first, use a second transmission filter in series to have more calibration points; second, shield as much ambient light as possible, both during the calibration and following experiments. For the latter suggestion, shielding the region surrounding the detector is sufficient, e.g., using a small box or tube.

ⁱⁱAt a fixed range, the voltmeter performs a resistance measurement by sourcing a known current and measuring the resulting voltage. Lower resistance ranges use higher currents. Since your semiconductor photoresistor is a non-ohmic device, changing the current will change the resistance value. Thus, if you change the range during your measurements, you will not be able to compare all of your data together.

ⁱⁱⁱYou will also be measuring the ambient light as well, which we are assuming is constant. For this to be true, you should be careful not to read the resistance while, e.g., your shadow is covering the photoresistor. Make each measurement in the same way as much as possible.

Table 13.1: Photoresistor calibration

Transmission	R (k Ω)
0.20	
0.40	
0.60	
0.80	
1.00	

13.5.3 Two Polarizers

Now you are ready to verify Malus' law.

1. Place a single polarizer in the beam path. Leave the transmission filter in place near the photodiode such that the beam passes through the now-calibrated 100% transmission region for all measurements.^{iv}
2. Verify that the beam is still falling on the photoresistor.
3. Record the photodiode resistance. This corresponds to the intensity after passing through a single polarizer, i.e., I_o .
4. Place the second polarizer in the beam path, such that each polarizer about 1/3 of the way along the space between the laser and photoresistor. Again, leave the transmission filter in place at 100%.
5. Set the both polarizers to 0° .
6. Align the elements by checking that the beam *reflected* from each element falls near the center of the previous element.
7. Verify that the beam is still falling on the photoresistor.
8. Record the resistance, which corresponds to the light intensity.
9. Vary the angle of only one polarizer in 10° increments up to 180° and record the resistance values.
10. Record a few extreme points at, e.g., 35° and 190° to ensure that the minima and maxima are well-located.
11. Using your calibration function for the photoresistor, calculate the relative transmission corresponding to each resistance and complete the table below.

13.5.4 Three Polarizers

1. Set the first polarizer to 0° and the second to 90° . Place a third polarizer between the two polarizers you already have. Again, leave the transmission filter in place for 100% calibrated transmission.

^{iv}Even the so-called 100% transmission region results in some loss in intensity due to scattering by the plastic film. Leaving the filter in at 100% insures that your calibration above remains valid for absolute intensities.

Table 13.2: *Intensity versus relative angle between polarizers*

θ ($^\circ$)	R (k Ω)	Transmission
single pol.		
350		
0		
10		
...		
170		
180		
190		

2. Repeat the procedure above, varying the angle of only the central polarizer.

13.5.5 Diffraction Grating

You can now remove all optical components from the rail except for the laser.

1. Place the wooden block with graph paper at the far end of the rail. Center the laser beam on the center of the graph paper.
2. Insert the diffraction grating (with a magnetic mount) between the laser and graph paper. Adjust its horizontal position until you see a central laser spot and two diffracted spots on either side of it.
3. Align the grating and laser such that the incident beam is normal on the grating (i.e., $\theta_i = 0$). An easy way to do this is to check that the beam *reflected* from the grating hits the center of the laser.
4. Carefully measure the positions of the central spot and the two diffracted beams.
5. Carefully measure the distance between the grating and the graph paper.
6. Move the grating close enough to observe two more sets of diffracted spots (for a total of 5 spots, including the central beam). Measure the spot positions.
7. Switch to the other color laser and repeat the measurements above.
8. Qualitatively explore the patterns resulting when you project the laser beam through the various features on the diffraction optics wheel.

If there is time: Using the photoresistor, measure the intensity of the central and first two pairs of diffracted spots.

13.6 Data analysis

13.6.1 Two Polarizers

1. Plot the relative transmission versus angle, including error bars corresponding to the *propagated* error due to resistance fluctuations and your measurement of the polarizer angles.

2. On the same plot, include a calculated response from Malus' law, varying the parameters to determine the initial intensity and any angular offset until you achieve a satisfactory fit. Minimizing χ^2 is a good way to achieve this!

13.6.2 Three Polarizers

Repeat the analysis above, including a calculated response from Eq. 13.4.

13.6.3 Diffraction

From the positions of the spots and the distance to the grating, calculate the sine of the angle defined by the beam incident on the grating, the central beam on the graph paper, and the diffracted spots. If the optical elements are aligned for normal incidence of the beam on the grating, the n^{th} spots away from the central beam should have a position governed by

$$n\lambda = d \sin \theta \quad (13.7)$$

where n is the order of the spot ($n=1$ for the spots immediately following the central beam, λ is the incident wavelength, and d is the grating spacing (i.e., the inverse of the "lines per meter" of the grating). From your distance measurements, determine λ for each laser, including the sources and magnitude of the uncertainties.

13.7 Format of report

You may choose one of the following formats for your report:

1. two-page memo (excluding required plots)
2. group oral presentation
3. formal written lab report
4. research proposal

Further details and templates are provided for each of these formats. Keep in mind that over the course of all of the one-week experiments you must do each type of report (and two formal lab reports).

13.8 Discussion and topics for your report

In addition to the data analysis outline above, below are some questions and topics you may wish to address in your report.

1. How does the three-polarizer experiment relate to a liquid crystal display (LCD)? How can you prove that your cell phone display uses liquid crystal technology? Hint: try viewing it through a polarizer!

2. Why are polarized sunglasses useful?
3. Explain how polarization is used in some recent 3D movie technologies.
4. Does the order of the polarizers in the three polarizer experiment matter? Why?
5. How are different sorts of common polarizers constructed? What is the mechanism for producing polarized light?
6. How accurate is this technique of wavelength measurement? How could you improve it?
7. For what other experiments in this course will the diffraction measurement have relevance?
8. You can observe diffraction in a household setting with a simple laser pointer. A good “grating” is a CD or DVD disk. Why?
9. Discuss qualitatively the patterns resulting from the features on the optics wheel. Why do they appear the way they do?

Fine Structure in Atomic Spectra

14.1 Introduction

In this experiment, we will employ a plane diffraction grating, collimating optics, and a telescope to form a spectrometer, which we will use to measure a visible atomic emission lines of sodium (Na) and mercury (Hg). We will use the Hg lines to calibrate the spectrometer and subsequently determine the energy level diagram of the low lying levels of Na and the spin-orbit splitting of the $3p$ configuration.

14.1.1 Diffraction Grating

In this experiment, we will use a plane diffraction grating, the large N limit of diffraction by multiple parallel slits. Diffraction gratings can be made by ruling a large number of equidistant lines on a glass substrate. Most gratings today, however, are replicas (molds of ruled gratings). Parallel light rays incident on a diffraction grating produces a series of sharp spots at varying angles with respect to the incident beam. For parallel light incident on a diffraction grating with angle θ_i , the positions of the maxima are given by

$$p\lambda = d(\sin \theta_p + \sin \theta_i) \quad (14.1)$$

where d is the spacing between lines of the grating, θ_p the diffraction angle, θ_i the angle of incidence with respect to the grating normal, and p designates the order of the spectrum (see Fig. 14.1). It is clear from Eq. 14.1 that for fixed p , the angle θ_p will be a function of the wavelength of the light λ . Thus, if we illuminate the grating with parallel rays of light composed of several wavelengths, each wavelength will emerge with a different angle. Note, however, dispersion only occurs for $|p| \geq 1$. At zero order, $\theta_p = 0$, we will see all the wavelengths.

The diffraction grating is a particularly simple instrument to use for studying the components of light produced by an excited atom. When the electrons in an atom are excited, they return to lower states by emitting light of specific wavelengths known as spectral lines. The spectrum of an atom is one of its basic identifying signatures. The existence of many elements in astronomical studies is often established by measurements of their spectra. Atomic spectroscopy has also played a fundamental role in the development of quantum mechanics, and the great theoretical and experimental importance of atomic spectroscopy is well established.

14.1.2 Sodium Atom

The structure of Na is more complicated than that of H because of the presence of 10 additional electrons. Although these electrons are tightly bound in the $1s$, $2s$ and $2p$ levels, they produce a

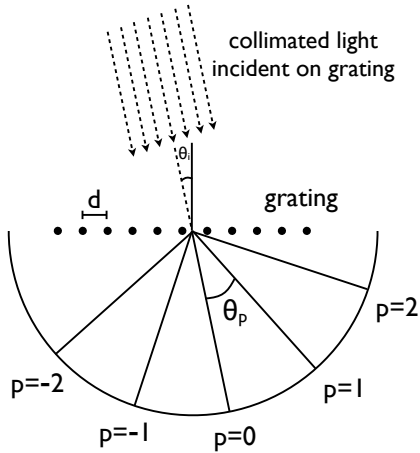


Figure 14.1: Schematic illustration of diffraction of light by a diffraction grating. Various orders of the spectrum are shown.

significant affect on the energy of the outer electron in the $3p$ level.

The well known bright doublet which is responsible for the bright yellow light from a sodium lamp may be used to demonstrate several of the influences which cause splitting of the emission lines of atomic spectra. The transition which gives rise to the doublet is from the $3p$ to the $3s$ level, levels which would be the same in the hydrogen atom. The fact that the $3s$ (orbital quantum number = 0) is lower than the $3p$ ($l=1$) is a good example of the dependence of atomic energy levels on angular momentum. The $3s$ electron penetrates the $1s$ shell more and is less effectively shielded than the $3p$ electron, so the $3s$ level is lower (more tightly bound). The fact that there is a doublet shows the smaller dependence of the atomic energy levels on the total angular momentum. The $3p$ level is split into states with total angular momentum $j=3/2$ and $j=1/2$ by the magnetic energy of the electron spin in the presence of the internal magnetic field caused by the orbital motion. This effect is called the spin-orbit effect. In the presence of an additional externally applied magnetic field, these levels are further split by the magnetic interaction, showing dependence of the energies on the z -component of the total angular momentum. This splitting gives the Zeeman effect for sodium.

The magnitude of the spin-orbit interaction has the form $\mu_z B = \mu_B S_z L_z$. That is, the spin-orbit interaction can be regarded as an internal magnetic field which raises or lowers the energy of electrons according to their spin. In the case of the sodium doublet, the difference in energy for the $3p^{3/2}$ and $3p^{1/2}$ comes from a change of 1 unit in the spin orientation with the orbital part presumed to be the same. The change in energy is of the form

$$\Delta E = \mu_B g B \quad (14.2)$$

where $\mu_B = 57.88 \mu\text{eV/T}$ is the Bohr magneton and $g \approx 2$ is the electron spin g -factor. This gives an estimate of the internal magnetic field needed to produce the observed splitting.

14.1.3 Mercury Atom

The structure and spectra of the Hg is very complicated, due to the large number of electrons ($Z = 80$). Mercury is added to fluorescent lights because the large number of different wavelength lines in the spectra yields an overall color close to white. We will use the most intense mercury lines to calibrate the spectrometer, and investigate its spectrum more qualitatively.

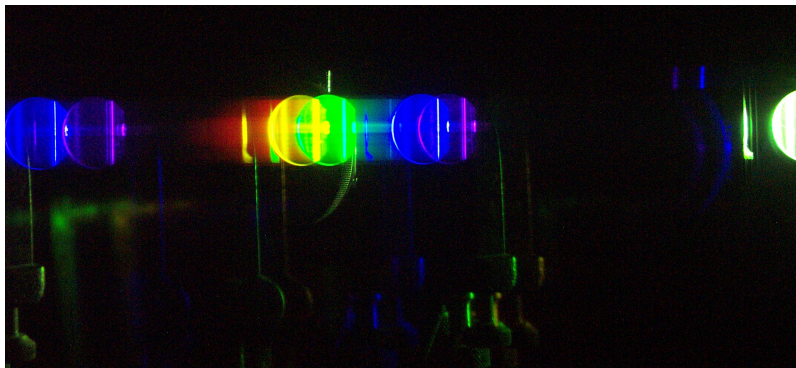


Figure 14.2: A digital picture of the Hg light source taken with a diffraction grating placed over the lens.

14.1.4 The Grating Spectrometer

The apparatus used in this experiment is schematically shown in Fig. 14.3. It consists of a plane transmission diffraction mounted on the rotating table of a spectrometer. The spectrometer also consists of a telescope and a collimator. The telescope can be rotated independently of the collimator about the axis of the table.

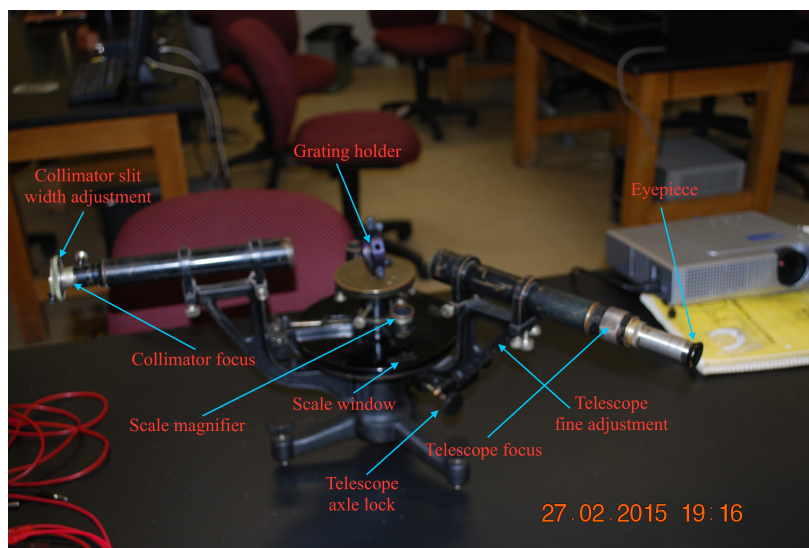


Figure 14.3: The experimental apparatus used to study atomic spectra. See the text for further description of the labeled parts.

1. Grating holder. Used for mounting the grating. Handle this part with extreme care. The table on which the grating holder sets can be leveled and rotated.
2. Eyepiece. At the end of the telescope, this eyepiece can be gently pulled or pushed to get a sharp focus on the cross-hairs without moving their position. This allows the apparatus to allow for different individual viewing preferences without up- setting the focus of the image being viewed.
3. Telescope focus. The chrome flange can be rotated to focus the telescope on the image. When properly focused, the image of an object of infinity falls in the plane of the cross hairs.
4. Collimator slit width adjustment. Allows adjustment in width and also orientation of the slit.
5. Collimator focus. Adjusts the length of the collimator so that the image of the slit appears to be at infinity. The telescope image of the slit therefore falls on the plane of the cross hairs.
6. Telescope arm clamping and tangent screws. The two screws on the side of the spectrometer just below the telescope each serve a distinct purpose. The screw projecting radially outward is the clamping screw (telescope axle lock) which should always be loosened before attempting to move the telescope. After the telescope is nearly in the desired position, the clamping screw should be gently tightened. The perpendicular tangent screw (telescope fine adjustment) then allows for very fine adjustment of the telescope position.
7. Angular position readout, with magnifier. The least division of the main scale is $1/3$ degree and there is a vernier to carry the measurement to $\frac{1}{3}$ minute of arc.

Below is a simplified schematic of the key optical elements of the spectrometer, showing the principle of operation.

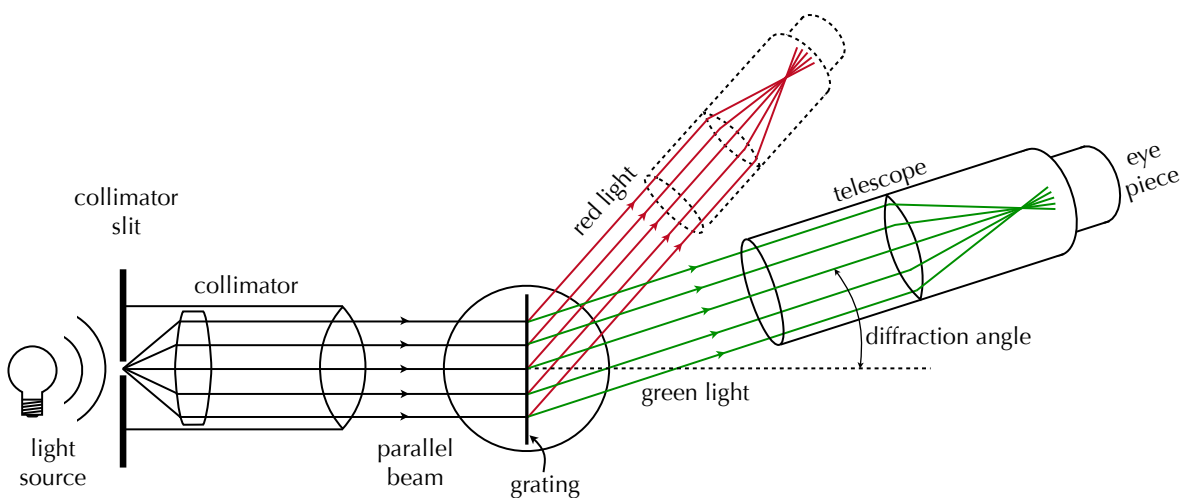


Figure 14.4: Principle of operation of the spectrometer.

14.1.5 Aligning the Spectrometer

First, having focused the eyepiece on the cross hairs, focus the telescope for parallel light (i.e., light from infinity) by focusing on an object approximately 100 meters away. Focusing through a window on a building across the street will do.

Second, put a light source close to the slit of the collimator. Using the telescope, view the slit. Make the slit as narrow as possible. While looking through the telescope, adjust the focus of the collimator so that the image of the slit falls on the plane of the cross-hairs and is sharp. Now the collimator is set to produce a collimated beam, i.e., a parallel beam of light.

Third, mount the grating on the table of the spectrometer. Make sure that the photographic film from which the grating is made is on the telescope side. Note the position of the telescope when it is in line with the collimator, i.e., when the slit image is aligned with the cross-hairs. Shine the light from your desk lamp or flashlight into the open side port of the telescope (near the eyepiece) to illuminate the cross-hairs. You might find it helpful to place a mirror just behind and flush with the grating to see the reflected image of the cross-hairs. Once you have located it, you can remove the mirror and continue with the alignment procedure. Rotate the grating so that the reflected image of the cross-hairs falls back on the cross-hairs. In this position, the grating is perpendicular to the optical axis of the telescope. The system is set up to approximately satisfy Eq. 14.1; the incident beam consists of parallel light falling nearly normally on the grating, i.e., $\theta_i = 0$. Lock the grating into place and record the position of the angular scale on the rotating table. Do not adjust the grating anymore. You will be able to see images of successive orders of the spectrum by rotating the telescope away from the zero order position.

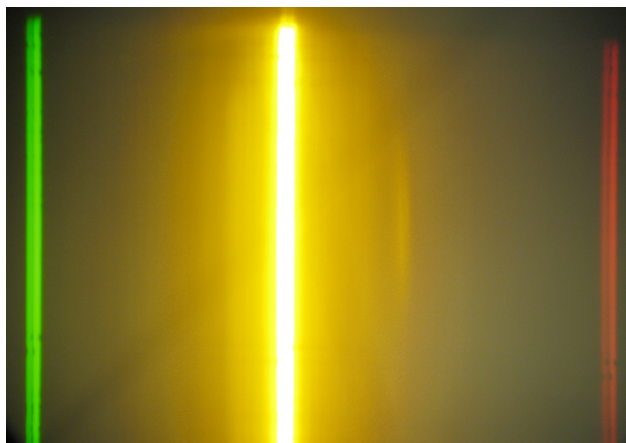


Figure 14.5: Green, yellow, and red Hg emission lines as viewed through the spectrometer. Note that all three lines are actually doublets.

14.2 Procedure for Taking Data

14.2.1 Making Measurements of Various Spectra

As mentioned earlier, the sources you will be observing are mercury and sodium. You will be able to see first order spectra for all of these light sources. With the spectrometer enclosed in the dark box, you will also be able to detect some of the lines in second order.

To make measurements, first record the value of the angular scale on the rotating table when the cross-hairs are aligned with the zero order spectrum (this corresponds to θ_o). Rotate the telescope to the right so that the cross-hairs align with a p^{th} order spectral line. Record the angular scale reading. Then, rotate the telescope so that the cross-hairs align with the corresponding spectral line on the left side of the zero order spectrum; record the angular scale reading. **You should measure the angle for each spectral line at least twice on each side**, giving multiple measurements of the angle, θ_p for the p^{th} order. For each color, lock the telescope in place where the cross hairs are nearly aligned with the spectral line and then use the fine adjustment to precisely center the cross hairs on the line. In the case of doublets, you should be able to measure both members of the doublet turning the fine adjustment knob only. Note, you might find it easier to measure all the lines on one side before measuring them on the other side.

The scale on the spectrometer is a vernier scale. Make sure you understand how to read a vernier scale. If you do not know how to read a vernier scale, ask your instructor for assistance. When making your measurements, always rotate the table in the same direction, cw or ccw, to minimize errors due to hysteresis.

14.2.2 Mercury Spectrum

You will use the Hg spectrum as your wavelength standards to calibrate your spectrometer. The wavelengths can be on the National Institute of Standards and Technology atomic database website: (<http://physics.nist.gov/PhysRefData/ASD/index.html>) or in the attached handout. You should measure the angles of the five lines (violet, blue, blue-green, green and yellow). Where the lines may belong to a doublet or a triplet, measure both lines in the case of a doublet but only the most intense line in the case of a triplet. In the case of a triplet, however, note that the line measured belongs to a triplet.

14.2.3 Sodium Spectrum

With the sodium lamp, you should measure the wavelengths of the green, yellow and red spectral lines in first order. Be sure to measure both members of the yellow doublet.

14.3 Analysis and Discussion

1. For each line you measured, calculate the diffraction angle by taking the difference between the measured angle for that line and the angle measured for the central maximum. Average diffraction angles measured for the same emission line and calculate the standard deviation of the mean. Tabulate the results.
2. From the known wavelengths and measured diffraction angles, you can use Eq. 14.1 to find the grating spacing d . Do this by fitting λ to $d \sin \theta$ and extracting a value for d from the fit parameters. Use the fitting method of linear least squares.
3. Calculate the wavelengths of the measured lines for Na and compare with reference values. For each line, note which transition is responsible.
4. Calculate the splitting of the yellow doublet in electron volts, and compare to the database values. What is the effective strength (in Tesla) of the spin-orbit interaction? Discuss the difference between the observed splitting and the database value in light of measurement uncertainties.

Appendix 1: Reference Spectral Data

Table 14.1: Hydrogen emission lines, UV-Vis-NIR[11]

λ (nm)	I (a.u.)	Assignment
383.5384	5	Balmer $9 \rightarrow 2$
388.9049	6	Balmer $8 \rightarrow 2$
397.0072	8	Balmer $7 \rightarrow 2$
410.174	15	Balmer $6 \rightarrow 2$
434.047	30	Balmer $5 \rightarrow 2$
486.133	80	Balmer $4 \rightarrow 2$
656.272	120	Balmer $3 \rightarrow 2^\dagger$
656.2852	180	Balmer $3 \rightarrow 2^\dagger$
954.597	5	Paschen $8 \rightarrow 3$
1004.94	7	Paschen $7 \rightarrow 3$

[†] Spin-orbit split lines, $2p_{1/2} \rightarrow 1s$ and $2p_{3/2} \rightarrow 1s$

Appendix 2: Angular Momentum Correction to Hydrogen Levels

The energies of the stationary states of hydrogen slightly depend on the orbital angular momentum quantum number l . An improved formula for the energy of the state of quantum numbers n and l for nonzero l is

Table 14.2: Selected Sodium emission lines, UV-Vis-NIR[11, 12]

λ (nm)	I (a.u.)	Assignment
466.5	19	$6^2D_{5/2,3/2} \rightarrow 3^2P_{1/2}$
466.9	19	$6^2D_{5/2,3/2} \rightarrow 3^2P_{3/2}$
474.8	4	$7^2S_{1/2} \rightarrow 3^2P_{1/2}$
475.2	4	$7^2S_{1/2} \rightarrow 3^2P_{3/2}$
497.9	104	$5^2D_{5/2,3/2} \rightarrow 3^2P_{1/2}$
498.3	104	$5^2D_{5/2,3/2} \rightarrow 3^2P_{3/2}$
514.8	23	$6^2S_{1/2} \rightarrow 3^2P_{1/2}$
515.4	23	$6^2S_{1/2} \rightarrow 3^2P_{3/2}$
568.3	130	$4^2D_{5/2,3/2} \rightarrow 3^2P_{1/2}$
568.8	130	$4^2D_{5/2,3/2} \rightarrow 3^2P_{3/2}$
589.0	139	$3^2P_{3/2} \rightarrow 3^2S_{1/2}$
589.6	139	$3^2P_{1/2} \rightarrow 3^2S_{1/2}$
615.4	67	$5^2S_{1/2} \rightarrow 3^2P_{1/2}$
616.1		$5^2S_{1/2} \rightarrow 3^2P_{3/2}$
818.3	90	$3^2D_{5/2,3/2} \rightarrow 3^2P_{1/2}$
819.48	129	$3^2D_{5/2,3/2} \rightarrow 3^2P_{3/2}$

Table 14.3: Selected Mercury emission lines, UV-Vis-NIR[11, 13]

λ (nm)	I (a.u.)	Assignment
365.02	2800	
365.48	300	
366.29	80	
404.66	1800	
407.78	140	
433.92	250	
434.75	400	
435.83	4000	
546.07	1100	
567.59	160	
576.68	240	
578.97	100	
579.07	280	
580.38	140	
614.95	1000	
671.64	160	
690.75	250	
708.19	250	
709.19	250	

$$E_{n,l} = \frac{m_e e^4}{2(4\pi\epsilon_0)^2 \hbar^2 n^2} \left[1 + \frac{e^4}{(4\pi\epsilon_0)^2 \hbar^2 c^2 n} \left(\frac{1}{1 + l + \frac{1}{2} \pm \frac{1}{2}} - \frac{3}{4n} \right) \right] \quad (14.3)$$

where the term $\pm\frac{1}{2}$ corresponds to the spin parallel and antiparallel, respectively, to the orbital angular momentum.

Appendix 3: Center-of-Mass Correction to the Hydrogen Energies

In a simple calculation of the energies of the stationary states of hydrogen, we pretend that the proton remains at rest. Actually, both the electron and proton orbit about their common center of mass. We can show that the energies of the stationary states, taking into account this motion of the proton, are given by

$$E_n = -\frac{me^4}{2(4\pi\epsilon_0)^2\hbar^2n^2} \quad (14.4)$$

where m is the reduced mass

$$m = \frac{m_em_p}{m_e + m_p} \quad (14.5)$$

There is a fairly good chance that you've not encountered the reduced mass in your previous mechanics classes, so we will go through it quickly. Essentially, the reduced mass is an "effective" inertial mass that can be used solve a two-body problem as if it is a single body problem.

Our system consists of a single proton and a single electron, separated by a distance r described by a vector \vec{r} . Classically, we expect both particles to orbit about their center of mass. The proton, being much heavier, will move very little in comparison to the lighter electron, and sit closer to the center of mass. This is depicted below.ⁱ

The center of mass for a simple two-body system like this essentially amounts to finding the point along their common axis at which their torques would balance.ⁱⁱ If the particles' positions are described by vectors \vec{r}_e and \vec{r}_p pointing from the center of mass toward the electron and proton, respectively, then this requires

$$m_e\vec{r}_e + m_p\vec{r}_p = 0 \quad \text{or} \quad \vec{r}_p = -\frac{m_e}{m_p}\vec{r}_e \quad (14.6)$$

Of course, we also know that the $\vec{r} = \vec{r}_e - \vec{r}_p$. Rearranging the first equation and substituting,

$$\vec{r} = \vec{r}_e - \vec{r}_p = \vec{r}_e \left(1 + \frac{m_e}{m_p}\right) = -\vec{r}_p \left(\frac{m_e}{m_p} + 1\right) \quad (14.7)$$

This leads us to expressions for the electron and proton orbital radii in terms of their separation and masses:

$$\vec{r}_e = \vec{r} \left(\frac{m_p}{m_p + m_e}\right) \quad \vec{r}_p = -\vec{r} \left(\frac{m_e}{m_p + m_e}\right) \quad (14.8)$$

ⁱSome neat animations can be found at <http://en.wikipedia.org/wiki/Barycenter>.

ⁱⁱPut more simply, if the two particles were on a see-saw, where should the center pivot be?

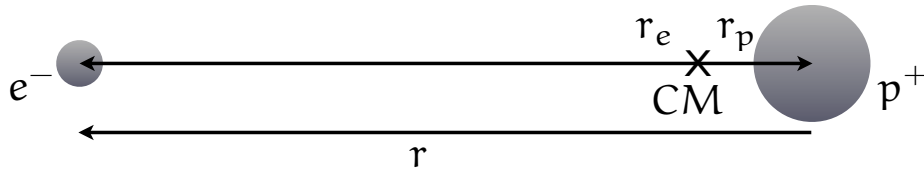


Figure 14.6: An electron and a proton, with a separation \vec{r} . The electron sits at a distance r_e from their center of mass, and the proton a distance r_p from the center of mass.

Here the negative sign for \vec{r}_p just indicates that the direction is opposite \vec{r} . Two other interesting things to note: first, only the relative mass m_e/m_p really matters; second, owing to the fact that $m_p \gg m_e$, the proton will orbit in a much smaller circle than the electron. So much smaller, in fact, that we are usually well-justified in taking the proton to be at rest. Now, let us write down the kinetic energy of the two-particle system. We'll have the kinetic energy of each particle, and their electrical potential energy :

$$E = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_p v_p^2 - \frac{k_e e^2}{r} = \frac{1}{2}m_e \left(\frac{dr_e}{dt}\right)^2 + \frac{1}{2}m_p \left(\frac{dr_p}{dt}\right)^2 - \frac{k_e e^2}{r} \quad (14.9)$$

Using the expressions above, we can write the individual particles' velocities in terms of the rate at which the separation changes with time:

$$v_e = \frac{dr_e}{dt} = \left(\frac{dr}{dt}\right) \left(\frac{m_p}{m_p + m_e}\right) \quad v_p = \frac{dr_p}{dt} = \left(\frac{dr}{dt}\right) \left(\frac{m_e}{m_p + m_e}\right) \quad (14.10)$$

But $d\vec{r}/dt$ is nothing more than the relative velocity of the proton and electron, which we'll simply call v :

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\vec{r}_e - \vec{r}_p) = \frac{d\vec{r}_e}{dt} - \frac{d\vec{r}_p}{dt} \quad (14.11)$$

With this definition, we may write the electron and proton velocities asⁱⁱⁱ

ⁱⁱⁱWe don't really need the vector notation any more, since we're just going to square them anyway.

$$v_e = v \left(\frac{m_p}{m_p + m_e} \right) \quad v_p = v \left(\frac{m_e}{m_p + m_e} \right) \quad (14.12)$$

Substituting these expressions into our energy equation,

$$E = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_p v_p^2 - \frac{k_e e^2}{r} = \frac{1}{2}m_e v^2 \frac{m_p^2}{(m_p + m_e)^2} + \frac{1}{2}m_p v^2 \frac{m_e^2}{(m_p + m_e)^2} - \frac{k_e e^2}{r} \quad (14.13)$$

$$E = \frac{m_e m_p}{2(m_e + m_p)^2} (m_p v^2 + m_e v^2) - \frac{k_e e^2}{r} = \frac{1}{2} \left(\frac{m_e m_p}{m_e + m_p} \right) v^2 - \frac{k_e e^2}{r} \quad (14.14)$$

If we define a quantity $m = m_e m_p / (m_e + m_p)$, known as the “reduced mass,” our energy equation is identical to that of a single particle of charge $-e$ in an electric potential $V = k_e e / r$:

$$E = \frac{1}{2} m v^2 - eV \quad (14.15)$$

We have successfully reduced a two-body problem to a single-body problem. How about the angular momentum? Taking the center of mass as our origin, the net angular momentum is mvr for each particle:

$$L = L_p + L_e = m_e \left(\frac{dr_e}{dt} \right) r_e + m_p \left(\frac{dr_p}{dt} \right) r_p \quad (14.16)$$

$$L = m_e \left(\frac{v m_p}{m_p + m_e} \right) \left(\frac{r m_p}{m_p + m_e} \right) + m_p \left(\frac{-m_e v}{m_e + m_p} \right) \left(\frac{-r m_e}{m_p + m_e} \right) \quad (14.17)$$

$$L = \frac{m_e m_p v r}{(m_e + m_p)^2} (m_e + m_p) = \left(\frac{m_e m_p}{m_e + m_p} \right) v r = mvr \quad (14.18)$$

Again, we have reduced a two-body problem to an equivalent single-body problem: the total angular momentum is simply the reduced mass times the relative velocity times the separation distance. We can now apply our quantization condition for the total angular momentum:

$$mvr = n\hbar \quad \text{or} \quad v = \frac{n\hbar}{mr} \quad (n \in \mathbb{N}_1 = \{1, 2, \dots\}) \quad (14.19)$$

Substituting this condition into our energy equation:

$$E = \frac{1}{2} m \left(\frac{n^2 \hbar^2}{m^2 r^2} \right) - \frac{k_e e^2}{r} = \frac{n^2 \hbar^2}{2mr^2} - \frac{k_e e^2}{r} \quad (14.20)$$

The definition of a stationary state is a state in which the energy has no time dependence. In this case, there is no explicit time dependence, but an equivalent condition is to require that the variation with respect to radius vanishes:

$$\frac{dE}{dt} = \frac{dE}{dr} \frac{dr}{dt} = 0 = \left(\frac{-n^2 \hbar^2}{mr^3} + \frac{ke^2}{r^2} \right) \frac{dr}{dt} \quad (14.21)$$

$$\implies \frac{n^2 \hbar^2}{mr} = ke^2 \quad \text{or} \quad r = \frac{n^2 \hbar^2}{kme^2} \quad (14.22)$$

Using this extremal radius in our energy equation, we find

$$E = \frac{n^2 \hbar^2}{2m} \left(\frac{k^2 m^2 e^4}{n^4 \hbar^4} \right) - \frac{k^2 m e^4}{n^2 \hbar^2} \quad (14.23)$$

$$E = \frac{-k^2 m e^4}{2n^2 \hbar^2} = \frac{-me^4}{2(4\pi\epsilon_0)^2 n^2 \hbar^2} \quad (14.24)$$

For the last line, we noted that $k_e = 1/4\pi\epsilon_0$. This implies that the hydrogen and deuterium^{iv} nuclei will have slightly different emission and absorption lines, owing to the fact that the deuterium nucleus is about twice as heavy as the hydrogen nucleus. The deuterium transitions should be at slightly higher energies (lower wavelengths). It is a tiny effect, about a 0.2 nm shift of the 656.3 nm H line, but quite measurable and in good agreement with this simple model.^v Similar effects are expected and observed for other hydrogen-like systems, e.g., He^+ and Li^{++} .

^{iv}Deuterium is hydrogen with a neutron in the nucleus in addition to the proton.

^vSee, for example, <http://hyperphysics.phy-astr.gsu.edu/Hbase/quantum/hydfin.html>.

Charge Quantization

15.1 Objective

Demonstrate that electric charge is quantized and determine the base unit of electric charge.

15.2 Supplies & Equipment

1. Lab notebook
2. Millikan oil drop apparatus and accessories (Pasco)
3. High-voltage power supply (Pasco)
4. Multimeter

15.3 Introduction

The electric charge carried by a particle may be calculated by measuring the force experienced by the particle in an electric field of known strength. Although it is relatively easy to produce a known electric field, the force exerted by such a field on a particle carrying only one or several excess electrons is very small. For example, a field of 10^5 V/m would exert a force of only 1.6×10^{-14} N on a particle bearing one excess electron. This is a force comparable to the gravitational force on a particle with a mass of 10^{-15} kg!ⁱ

The success of the Millikan Oil Drop experiment depends on the ability to measure forces this small. The behavior of small charged droplets of oil, having masses of only 10^{-15} kg or less, is observed in a gravitational and an electric field. Measuring the velocity of the drop falling in air enables, with the use of Stokes's law, the calculation of the mass of the drop. The observation of the velocity of the drop rising in an electric field then permits a calculation of the force on, and hence, the charge carried by the oil drop.

Although this experiment will allow one to measure the total charge on a drop, it is only through the careful analysis of the data obtained and a certain degree of experimental skill that the charge of a single electron can be determined. By selecting droplets which rise and fall slowly, one can be certain that the drop has a small number of excess electrons. A number of such drops should be observed and their respective charges calculated. If the charges on these drops are integral multiples of a certain smallest charge, then this is a good indication of the atomic nature of electricity. However, since a different droplet has been used for measuring each charge, there remains the question

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as to the effect of the drop itself on the charge. This uncertainty can be eliminated by changing the charge on a single drop while the drop is under observation. An ionization source placed near the drop will accomplish this. In fact, it is possible to change the charge on the drop several times. If the results of measurements on the same drop then yield charges which are integral multiples of some smallest charge, then this is proof of the atomic nature of electricity.

The measurement of the charge of the electron also permits the calculation of Avogadro's number. The amount of current required to electro-deposit one gram equivalent of an element on an electrode (the faraday) is equal to the charge of the electron multiplied by the number of molecules in a mole. Through electrolysis experiments, the faraday has been found to be 9.625×10^7 C per kg of equivalent weight. Dividing the faraday by the charge of the electron yields Avogadro's number N_A :

$$\frac{9.625 \times 10^7 \text{ C/kg equiv. weight}}{1.602 \times 10^{-19} \text{ C}} = 6.025 \times 10^{23} \text{ molecules/kg equiv. weight} = N_A \quad (15.1)$$

15.3.1 Equation for calculating the charge on a drop

An analysis of the forces acting on an oil droplet will yield the equation for the determination of the charge carried by the droplet.

First, let us consider the case of a spherical droplet of radius r and mass m falling without an electric field present. In this case, there are only two forces acting on the drop: the gravitational force F_g pulling it downward, and the frictional (drag) force F_d pushing upward. The drag force for very small spherical objectsⁱⁱ in a continuous viscous fluid can be accurately modeled by Stokes lawⁱⁱⁱ, which gives the drag force as^{iv}

$$F_d = 6\pi\eta r v \quad (15.2)$$

where F_d is the frictional (drag) force, η is the fluid's dynamic viscosity, r the radius of the particle, and v the particle's velocity. If the particle reaches terminal velocity, this means that the gravitational force is precisely balanced by the drag force. For a spherical drop of mass m , comprised of a substance of density ρ , the gravitational force is $mg = \frac{4}{3}\pi r^3 \rho g$, where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration of the drop. Figure 15.1a shows the forces acting on the drop when it is falling in air and has reached its terminal velocity. (Terminal velocity is reached in a few milliseconds for

ⁱⁱStrictly, for very small Reynolds numbers.

ⁱⁱⁱStoke's law is derived by solving the "Stokes flow" limit for small Reynolds numbers ($Re \ll 1$) of the Navier-Stokes equations.

^{iv}See Appendix 15.7 for a derivation.

the droplets used in this experiment.) At the terminal velocity v_f , the forces must balance:

$$F_g = F_d \quad (15.3)$$

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_f \quad (15.4)$$

$$v_f = \frac{2\rho g r^2}{9\eta} \quad (15.5)$$

Of course, we have in principle neglected one point: the buoyant force due to the differing density of the drop and the air. Since the density of the air is only about $\frac{1}{1000}$ that of oil, this force may be neglected.^v

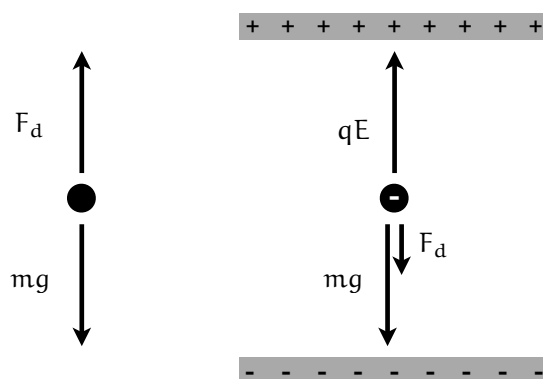


Figure 15.1: (a) This schematic shows the forces acting on the drop when it is falling in air and has reached its terminal velocity. (b) This schematic shows the forces acting on the drop when it is rising under the influence of an electric field.

If the drop in question also has a charge q , then it will experience an additional electric force in an electric field. Figure 15.1b shows the forces acting on the drop when it is *rising* under the influence of an electric field. Here E is magnitude of the electric field, q the charge on the drop, and v_r the velocity with which the particle rises. Adding the forces, we find

$$qE = \frac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_r \quad (15.6)$$

In principle the radius of the particle can be measured, but it is far easier to measure the terminal velocity. Based on Eq. 15.5, knowledge of the terminal velocity uniquely determines the particle radius for a given fluid. Eliminating r in favor of v_f , and solving for q :

$$qE = \frac{4}{3}\pi r^3 \rho g + 6\pi\eta r v_r = 6\pi\eta r (v_f + v_r) = 6\pi\eta \sqrt{\frac{9\eta v_f}{2g\rho}} (v_f + v_r) \quad (15.7)$$

$$q = \frac{6\pi}{E} \sqrt{\frac{9\eta^3}{2g\rho}} (v_f + v_r) \sqrt{v_f} \quad (15.8)$$

^vThe correction would involve replacing ρ in the equation above with $\rho - \rho_a$, where ρ_a is the density of air.

The electric field E is generated by applying a potential difference V between two parallel conducting plates, placed a distance d apart. If the spacing d is much smaller than the size of the plates in any lateral dimension (easily satisfied in the experiment), in the region between the plates the field is $E=V/d$. This lets us eliminate E in favor of the more easily measured V and a known distance.

$$q = \frac{6\pi d}{V} \sqrt{\frac{9\eta^3}{2g\rho}} (v_f + v_r) \sqrt{v_f} \quad (15.9)$$

Only one small detail remains: Stoke's law becomes inaccurate when the velocity of falling droplets is less than about 10^{-3} m/s. Droplets having this and smaller velocities have radii on the order of $2\ \mu\text{m}$, comparable to the mean free path of air molecules, a condition which violates one of the assumptions made in deriving Stokes's law. Since the velocities of the droplets used in this experiment will be in the range of $10^{-4} - 10^{-5}$ m/s, we must add a correction to Stoke's law. As it turns out, the correction is straightforward: we need only replace the viscosity by an effective value, which includes a correction factor (the Cunningham factor):

$$\eta \longrightarrow \eta_{eff} = \frac{\eta}{1 + \frac{b}{Pr}} \quad (15.10)$$

Here P is the atmospheric pressure, r is the radius of the drop, and b is a constant factor. This leads us to a more accurate form for the electric charge on a drop:

$$q = 6\pi d \sqrt{\frac{9\eta^3}{2g\rho \left(1 + \frac{b}{Pr}\right)^3}} \frac{(v_f + v_r) \sqrt{v_f}}{V} \quad (15.11)$$

We can regroup this in a somewhat more convenient manner:

$$q = \left[6\pi d \sqrt{\frac{9\eta^3}{2g\rho}} \right] \left[1 + \frac{b}{Pr} \right]^{-3/2} \left[\frac{(v_f + v_r) \sqrt{v_f}}{V} \right] \quad (15.12)$$

The first quantity in brackets is a function of the experimental apparatus and the composition of the drop alone, and need only be determined once for any particular apparatus. The second term in brackets is determined for each droplet, while the third term in brackets is calculated for each change of charge that the drop experiences. Finally, an alternate form you might find more convenient for your calculations is

$$q = \frac{4}{3} \pi \rho g d \left[\sqrt{\left(\frac{b}{2P}\right)^2 + \frac{9\eta v_f}{2g\rho}} - \frac{b}{2P} \right]^3 \left[\frac{v_f + v_r}{V v_f} \right] \quad (15.13)$$

Below, we summarize the definitions of symbols used, along with proper units and numerical values where applicable.

Symbol	Quantity	Units	Value
q	charge	coulombs [C]	
d	separation of plates in condenser	meters [m]	7.5×10^{-3} m
ρ	density of oil	[kg/m ³]	866 kg/m ³
g	acceleration of gravity	[m/s ²]	9.81 m/s ²
η	viscosity of air	Poise, or [Pa s]	see plot
b	constant	[Pa m]	8.23×10^{-3} Pa m
P	barometric pressure	[Pa]	101.325 kPa at 1 atm
r	radius of drop	[m]	
v_f	velocity of fall	[m/s]	
v_r	velocity of rise	[m/s]	
V	potential difference between plates	[V]	

15.4 Procedure

The following pages are taken from the PASCO write-up for this apparatus and are legally restricted by copyright law to use for our course only.

Begin legally restricted use material.

Equipment

Included equipment:

- Apparatus platform and plate charging switch (see detailed description below)
- 12 volt DC transformer for the halogen lamp
- Non-volatile oil (Squibb #5597 Mineral Oil, density = 886 kg/m^3)
- Atomizer

Note: We measured the density of the Squibb Mineral Oil and found it to be 886 kg/m^3 . However, the densities of different lots of mineral oil may vary slightly; therefore, for greatest precision, you should determine the density of the mineral oil you are using.

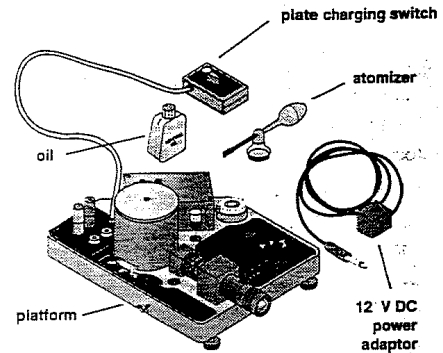


Figure 3. Included equipment

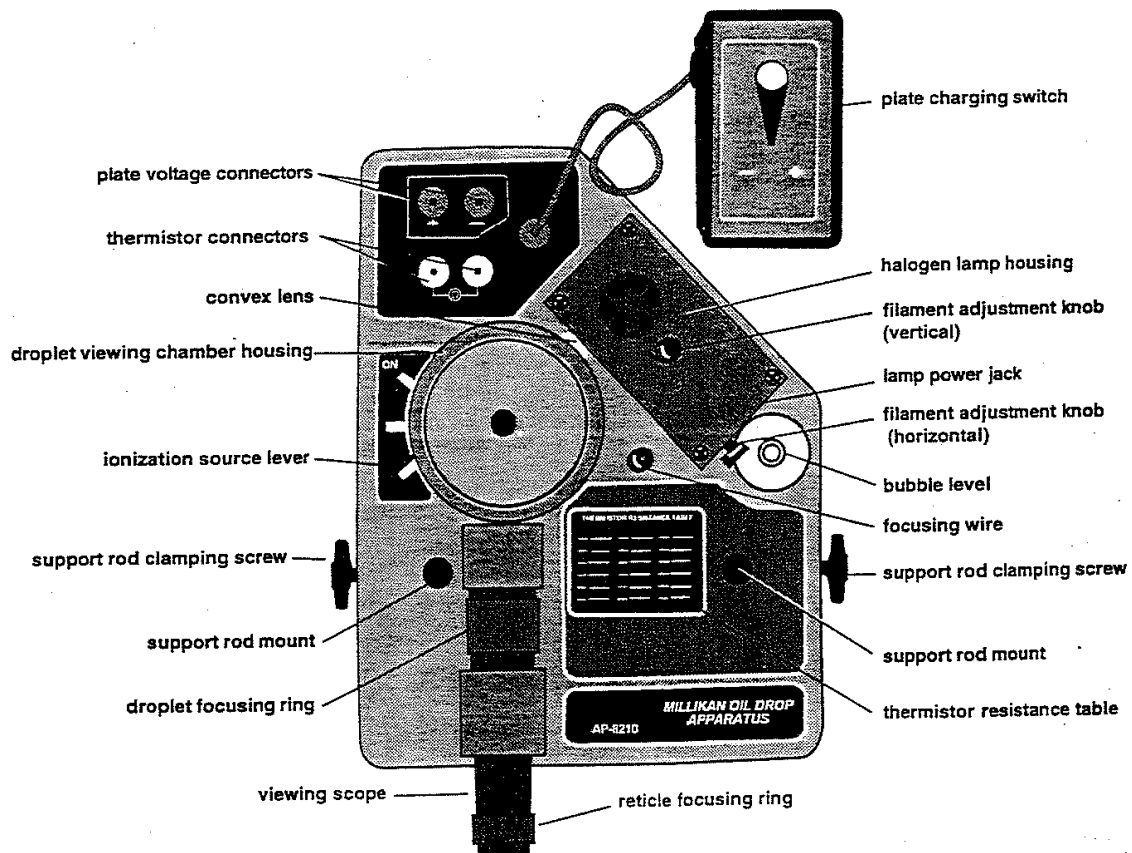


Figure 4. Apparatus platform

Components of Platform:

- Droplet viewing chamber (see details below)
 - Viewing scope (30X, bright-field, erect image) with reticle (line separation: 0.5 mm major divisions, 0.1 mm minor divisions), reticle focusing ring, and droplet focusing ring
 - Halogen lamp (12V, 5W halogen bulb and dichroic, infrared heat-absorbing window, horizontal and vertical filament adjustment knobs)
 - Focusing wire (for adjusting viewing scope)
 - Plate voltage connectors
 - Thermistor connectors (thermistor is mounted in the bottom plate)
- WARNING:** Do not apply voltage to the thermistor connectors.
- Thermistor table (resistance vs. temperature)
 - Ionization source lever (with three positions: Ionization ON, Ionization OFF, and Spray Droplet Position)
 - Bubble level
 - Support rod mounts and screws (to permit mounting of platform on a PASCO ME-8735 Large Rod Stand, so viewing scope can be raised to a comfortable eye level)
 - 3 leveling feet
 - Plate charging switch (on a 1 meter cord to prevent vibration of platform during a switching activity)

Components of Droplet Viewing Chamber (Figure 5)

- Lid
- Housing
- Droplet hole cover
- Upper capacitor plate (brass)
- Plastic spacer (approximately 7.6 mm thick)
- Lower capacitor plate (brass)
 - Thorium-232 alpha source (0.008 mcurie)
 - Electrical connection to upper capacitor plate
- Convex lens

Note: Thorium-232 is a naturally occurring, low level alpha-particle emitter with a half-life of 1.41×10^{10} years. It is not regulated in its use and poses no hazard to the user of the PASCO Millikan Oil Drop Apparatus.

It is recommended that you store the equipment in the original packing material. After unpacking, remove the foam insert from the droplet viewing chamber. Store the plate charging switch on the Velcro tabs located on the platform.

Required Equipment, Not Included:

- High voltage, well regulated power supply that delivers up to 500V DC, 10 mA minimum (for example, the PASCO SF-9585 High Voltage Power Supply)
- Digital multimeter (to measure voltage and resistance)(for example, the PASCO SB-9599A Universal Digital Multimeter)
- Patch cords with banana plug connectors (4) (for example, the PASCO SE-9415 Banana Plug Patch Cord)
- Stopwatch (for example, the PASCO SE-8702A Digital Stopwatch)

Additional Recommended Equipment:

- PASCO ME-8735 Large Rod Stand
- PASCO ME-8736 Steel Rods, 45 cm (2)

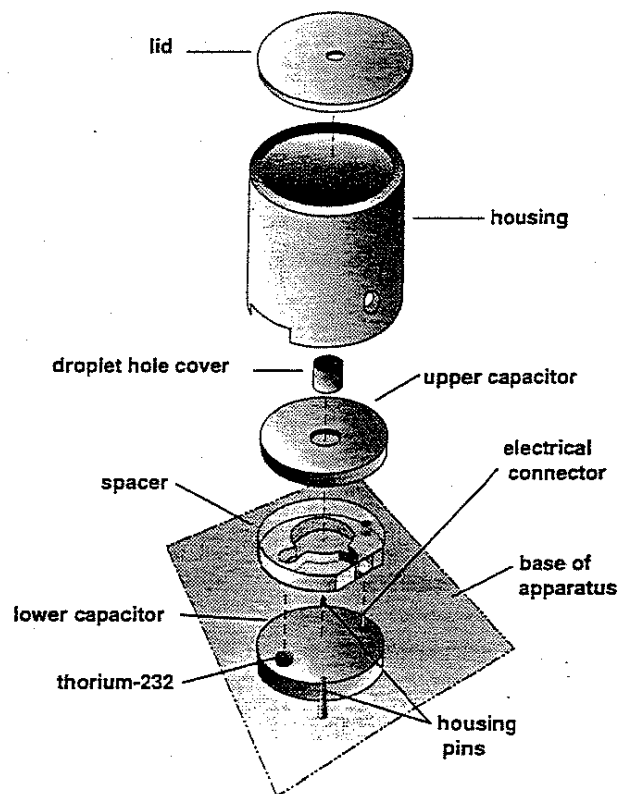


Figure 5. Droplet viewing chamber

Equipment Setup

Adjusting the Environment of the Experiment Room:

1. Make the room as dark as possible, while allowing for adequate light to read the multimeter and stopwatch, and to record data.
2. Insure that the background behind the apparatus is dark..
3. Select a location that is free of drafts and vibrations.

Adjusting the Height of the Platform and Leveling It:

1. Place the apparatus on a level, solid table with the viewing scope at a height which permits the experimenter to sit erect while observing the drops. If necessary to achieve the proper height, mount the apparatus on two support rods (ME-8736) on the large rod stand (ME-8735) (Figure 6).
2. Using attached bubble level as a reference, level the apparatus with the leveling screws on the rod stand or the leveling feet of the platform, as is appropriate for your setup.

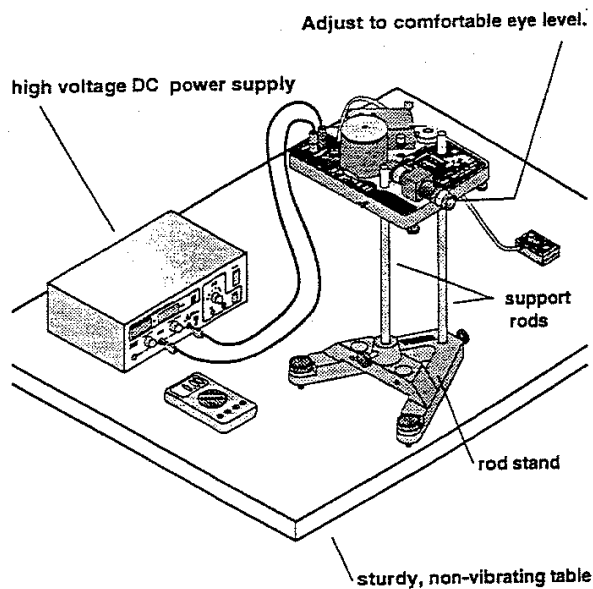


Figure 6. Equipment setup

Measuring Plate Separation

1. Disassemble the droplet viewing chamber by lifting the housing straight up and then removing the upper capacitor plate and spacer plate (see Figure 5). Measure the thickness of the plastic spacer (which is equal to the plate separation distance) with a micrometer. Be sure that you are not including the raised rim of the spacer in your measurement. The accuracy of this measurement is important to the degree of accuracy of your experimental results. Record the measurement

Use care when handling the brass plates and plastic spacer to avoid scratching them.

All surfaces involved in the measurement should be clean to prevent inaccurate readings.

Aligning the Optical System

Focusing the Viewing Scope

1. Reassemble the plastic spacer and the top capacitor plate onto the lower capacitor plate. Replace the housing, aligning the holes in its base with the housing pins (See Figure 5).

Note: The thorium source and the electrical connection on the lower capacitor plate fit into appropriately sized holes on the plastic spacer.

2. Unscrew the focusing wire from its storage place on the platform and carefully insert it into the hole in the center of the top capacitor plate (Figure 7).

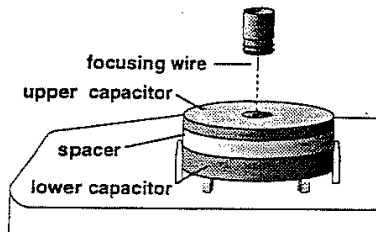


Figure 7. Insertion of the focusing wire into the top capacitor plate

3. Connect the 12V DC transformer to the lamp power jack in the halogen lamp housing and plug it into a wall socket.

Check to be sure that the transformer is the correct voltage: 100, 117, 220, or 240 V.

4. Bring the reticle into focus by turning the reticle focusing ring.
5. View the focusing wire through the viewing scope, and bring the wire into sharp focus by turning the droplet focusing ring.

Note: Viewing will be easier for experimenters who wear glasses if the viewing scope is focused without using the glasses.

Focusing the Halogen Filament:

1. Adjust the horizontal filament adjustment knob. The light is best focused when the

right edge of the wire is brightest (in highest contrast compared to the center of the wire).

2. While viewing the focusing wire through the viewing scope, turn the vertical filament adjustment knob until the light is brightest on the wire in the area of the reticle.
3. Return the focusing wire to its storage location on the platform.

Functions of Controls

Ionization Source Lever

1. When the lever is at the ionization OFF position, the ionization source is shielded on all sides by plastic, so that virtually no alpha particles enter the area of the drops.
2. At the ON position, the plastic shielding is removed and the drop area is exposed to the ionizing alpha particles emitted from the thorium-232.
3. At the Spray Droplet Position, the chamber is vented by a small air hole that allows air to escape when oil drops are being introduced to the chamber.

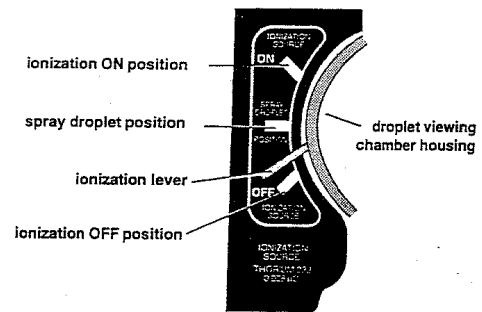


Figure 8. Ionization source lever settings

Plate Charging Switch

The plate charging switch has three positions:

1. TOP PLATE -: negative binding post is connected to the top plate.
2. TOP PLATE +: negative binding post is connected to the bottom plate.
3. PLATES GROUNDED: plates are disconnected from the high voltage supply and are electrically connected.

Adjusting and Measuring the Voltage

1. Connect the high voltage DC power supply to the plate voltage connectors using banana plug patch cords and adjust to deliver about 500 V.
2. Use the digital multimeter to measure the voltage delivered to the capacitor plates.

Measure the voltage at the plate voltage connectors, not across the capacitor plates. There is a 10 megohm resistor in series with each plate to prevent electric shock.

Determining the Temperature of the Droplet Viewing Chamber

1. Connect the multimeter to the thermistor connectors and measure the resistance of the thermistor. Refer to the Thermistor Resistance Table located on the platform to find the temperature of the lower brass plate. The measured temperature should correspond to the temperature within the droplet viewing chamber.

Although the dielectric window reflects much of the heat generated by the halogen bulb, the temperature inside the droplet viewing chamber should be determined periodically (about every 15 minutes).

Experimental Procedure

1. Complete the reassembly of the droplet viewing chamber by placing the droplet hole cover on the top capacitor plate and then placing the lid on the housing (see Figure 5).

Note: The droplet hole cover prevents additional droplets from entering the chamber once the experiment has started.

2. Measure and record the plate voltage and the thermistor resistance (temperature).

Introducing the Droplets into the Chamber

1. Put non-volatile oil of known density into the atomizer (for example, Squibb #5597 Mineral Oil, density: 886 kg/m^3).
2. Prepare the atomizer by rapidly squeezing the bulb until oil is spraying out. Insure that the tip of the atomizer is pointed down (90° to the shaft; see Figure 9)

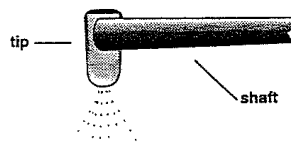


Figure 9. Correct position of the atomizer tip

3. Move the ionization source lever to the Spray Droplet Position to allow air to escape from the chamber during the introduction of droplets into the chamber.
4. Place the nozzle of the atomizer into the hole on the lid of the droplet viewing chamber.
5. While observing through viewing scope, squeeze the atomizer bulb with one quick squeeze. Then squeeze it slowly to force the droplets through the hole in the droplet hole cover, through the droplet entry hole in the top capacitor plate, and into the space between the two capacitor plates.
6. When you see a shower of drops through the viewing scope, move the ionization source lever to the OFF position.

If repeated “squirts” of the atomizer fail to produce any drops in the viewing area but produce a rather cloudy brightening of the field, the hole in the top plate or in the droplet hole cover may be clogged. Refer to the Maintenance section for cleaning instructions.

Note: The exact technique of introducing drops will need to be developed by the experimenter. The object is to get a small number of drops, not a large, bright cloud from which a single drop can be chosen. It is important to remember that the drops are being forced into the viewing area by the pressure of the atomizer. Therefore, excessive use of the atomizer can cause too many drops to be forced into the viewing area and, more important, into the area between the chamber wall and the focal point of the viewing scope. Drops in this area prevent observation of drops at the focal point of the scope.

Note: If the entire viewing area becomes filled with drops, so that no one drop can be selected, either wait three or four minutes until the drops settle out of view, or disassemble the droplet viewing chamber (after turning off the DC power supply), thus removing the drops. When the amount of oil on the parts in the droplet viewing chamber becomes excessive, clean them, as detailed in the Maintenance section. Remember, the less oil that is sprayed into the chamber, the fewer times the chamber must be cleaned.

Selection of the Drop

1. From the drops in view, select a droplet that both falls slowly (about 0.02 – 0.05 mm/s) when the plate charging switch is in the “Plates Grounded” position and can be driven up and down by turning on the voltage.

Hint: A drop that requires about 15 seconds to fall the distance between the major reticle lines (0.5 mm) will rise the same distance, under the influence of an electric field (1000 V/cm), in the following times with the following charges: 15 s, 1 excess electron; 7 s, 2 excess electrons; 3 s, 3 excess electrons. (Note: these ratios are only approximate)

Note: If too many droplets are in view, you can clear out many of them by connecting

power to the capacitor plates for several seconds.

Note: If you find that too few droplets have net charges to permit the selection of an appropriately sized and charged drop, move the ionization lever to the ON position for about 5 seconds.

2. When you find an appropriately sized and charged oil droplet, fine tune the focus on the viewing scope.

Note: The oil droplet is in best focus for accurate data collection when it appears as a pinpoint of bright light.

Collecting Data on the Rise and Fall of the Oil Droplet

1. Measure the rise (plates charged) and fall (plates not charged) velocities of the selected droplet about 10 – 20 times. Maneuver the droplet as needed using the plate voltage switch.

Note: The greatest accuracy of measurement is achieved if you time from the instant that the bright point of light passes behind the first major reticle line to the instant bright point of light passes behind the second major reticle line. (These lines are 0.5 mm apart.)

2. Calculate the charge on the droplet. If the result of this first determination for the charge on the drop is greater than for the charge on the drop is greater than 5 excess electrons, you should use slower moving droplets in subsequent determinations. You may find it convenient to set up a spreadsheet calculation so that the charge is immediately calculated for any given pair of rise and fall times.
3. Introduce more oil droplets into the chamber using the procedure previously described and select another droplet.
4. Measure the rise and fall velocities of the selected droplet about 10 – 20 times or until the charge changes spontaneously or the droplet moves out of view.

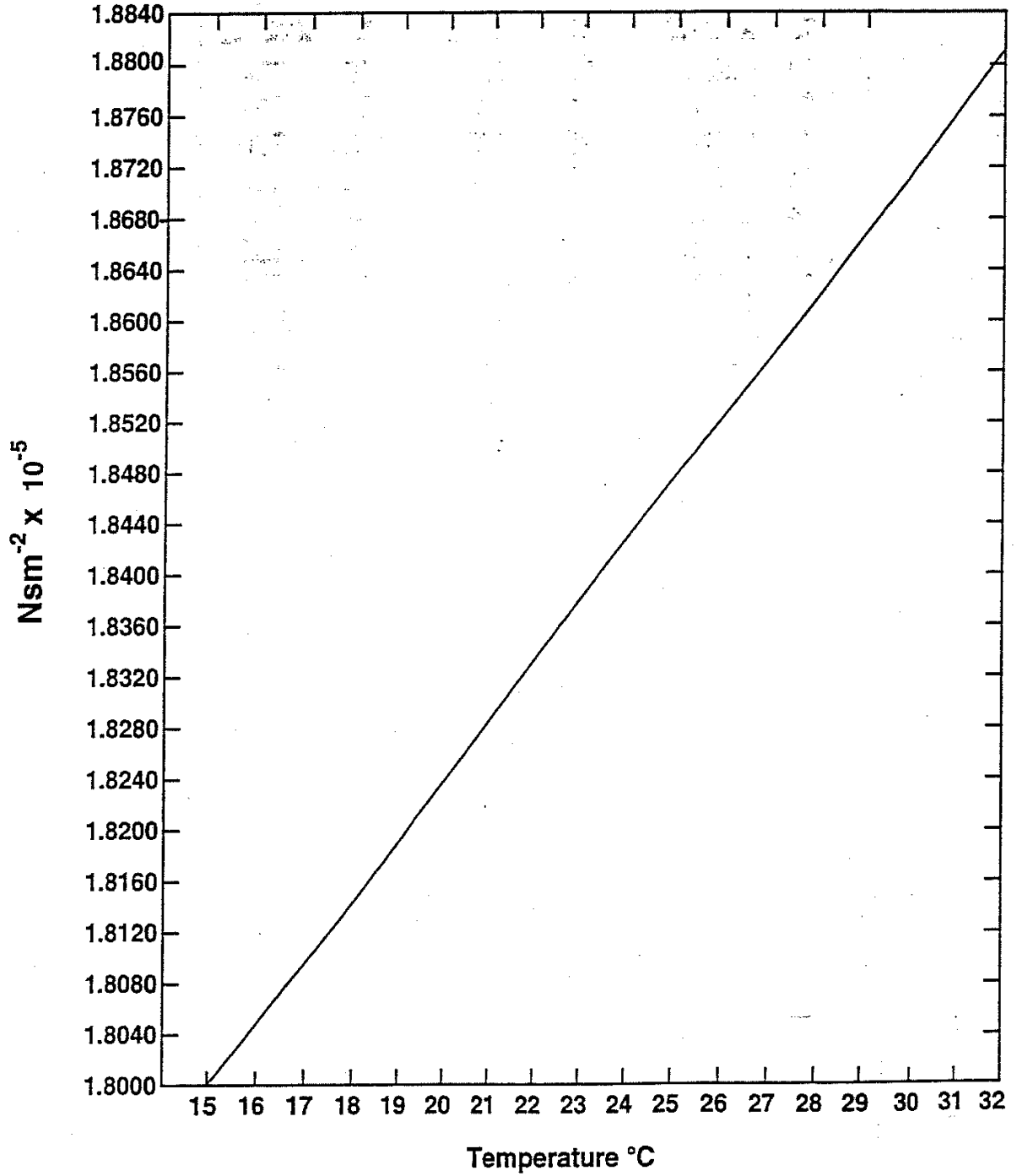
5. Bring the droplet to the top of the field of view and move the ionization lever to the ON position for a few seconds as the droplet falls.
 6. If the rising velocity of the droplet changes, make as many measurements of the new rising velocity as you can (10 – 20 measurements).
 7. If the droplet is still in view, attempt to change the charge on the droplet by introducing more alpha particles, as described previously, and measure the new rising velocity 10 – 20 times, if possible.
 8. Repeat step (7) as many times as you can.
 9. Record the plate potential, the oil density, the viscosity of air at the temperature of the droplet viewing chamber (see appendix A), and the barometric pressure for each set of velocity measurements.
2. **Make sure that you use the PLATES-GROUNDED setting in measuring the fall time, not the voltage polarity which drives the drops down.**

Notes:

1. It is desirable to observe as many different charges on a single drop as possible.

Appendix A: Viscosity of Dry Air as a Function of Temperature

Viscosity of Dry Air as a Function of Temperature



Appendix B: Millikan Oil Drop Apparatus Thermistor Resistance at Various Temperatures

THERMISTOR RESISTANCE TABLE

°C	X10⁶ Ω	°C	X10⁶ Ω	°C	X10⁶ Ω
10	3.239	20	2.300	30	1.774
11	3.118	21	2.233	31	1.736
12	3.004	22	2.169	32	1.700
13	3.897	23	2.110	33	1.666
14	2.795	24	2.053	34	1.634
15	2.700	25	2.000	35	1.603
16	2.610	26	1.950	36	1.574
17	2.526	27	1.902	37	1.547
18	2.446	28	1.857	38	1.521
19	2.371	29	1.815	39	1.496

End legally restricted use material.

In addition to recording measured rise and fall times, all measurements for temperature and pressure should also be recorded.

15.5 Analysis and Discussion

1. For each set of rise times and each set of fall times which are consistent within the set, calculate the average rise time and average fall time.
2. Create a table with one column being a drop label, the second column being the rise velocity, the third column the fall velocity, and the fourth column the calculated charge. The rise and fall velocities should be calculated from the average rise and fall times. There should be an entry in the table for each pair of rise and fall velocities. Show a sample calculation of the charge.
3. Take the difference between all possible pairs of charges. The number of possible pairs of charges for N charges is $\frac{1}{2}N(N - 1)$. Convert signed quantities to magnitudes. Tabulate the results.
4. Histogram the measured charges in one plot and all measured charge differences in a second plot.
5. Use whatever analysis method you like to discern any evidence for a fundamental electric charge. Comment on the evidence or lack of it for a fundamental unit of charge. In the case your data favor the existence of a fundamental charge, estimate its magnitude and assign an uncertainty to your estimate.

When you are finished

- Turn the voltage on the accelerating supply to zero, and turn the power off.
- Unplug the halogen light from the Millikan apparatus.
- Clean up your work area

15.6 Viscosity of dry air as a function of temperature

The viscosity of air can be computed using Sutherland's formula^{vi}

$$\eta(T) = \eta_o \left(\frac{T}{T_o} \right)^{3/2} \frac{T_o + S}{T + S} \quad (15.14)$$

Here η is the viscosity in poise (Ns/m²) at the input temperature T , μ_o is the reference viscosity in poise (Ns/m²) at a reference temperature T_o , T is the input temperature in Kelvin, T_o is a reference temperature in Kelvin, and S is an effective temperature called Sutherland's constant. For dry air, $\eta_o = 1.716 \times 10^{-5}$ Ns/m² at $T_o = 273$ K, $S = 111$ K, valid over a temperature range of 0–555 K. ^{vii}

^{vi}From <http://www.epa.gov/EPA-AIR/2005/July/Day-13/a11534d.htm> and <http://en.wikipedia.org/wiki/Viscosity>.

^{vii}Reference data: at 300 K, $\eta = 1.846 \times 10^{-5}$ Ns/m². Table A.4 in F.P. Incropera and D.P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 3rd ed, John Wiley, New York, NY, 1990. See also table B.4 in D.R. Poirier and G.H. Geiger, *Transport Phenomena in Materials Processing*, TMS, Warrendale, PA, 1994)

15.7 Derivation of Stoke's law and the Navier-Stokes Equation: A crash-course in fluids

Continuity Equation

First^{viii}, we need the continuity equation for a fluid. Qualitatively, a general continuity equation reads something like this:

$$(\text{rate of mass accumulation}) + (\text{rate of mass out}) - (\text{rate of mass in}) = 0 \quad (15.15)$$

We can be a bit more precise by applying our continuity equation to a specific volume of space V , which is defined by a bounding surface S . In this case, the net rate at which mass accumulates inside V depends on the net rate at which mass passes through S , either coming in or going out:

$$(\text{rate of mass accumulation in } V) + (\text{net rate of mass crossing } S) = 0 \quad (15.16)$$

This is basically just bookkeeping, or conservation of matter if you like. If the amount of mass in V is static, then it must be true that the amount of matter entering through S is the same as the amount of matter leaving through S . If the amount of mass in V is increasing, then there must be a net flow of matter in through S .

For a fluid, or any continuous substance, our continuity equation is just an expression of conservation of matter. Since we wish to deal with continuous substances like fluids, rather than particles, it is most convenient to put our equations in terms of the density of the substance ρ . Consider a tiny cube of our substance of dimensions $\Delta x \Delta y \Delta z$. The mass of this cube is simply $\rho \Delta x \Delta y \Delta z$. If we have a net flow of our substance through this cube, let's say in the x direction, how does the mass of the cube change with time? If the substance is incompressible, and the cube remains completely full, then the mass *doesn't* change, of course. However, in the general case, we just need to keep track of how much mass is in the cube at any moment, and how much mass enters and leaves.

We will presume that our cube is nicely aligned along the x , y , and z axes, and that there is a net flow of our substance with velocity \vec{v} , as shown in Fig. 15.2. We will assume that the density of our substance is constant. If we look first at the faces of the cube perpendicular to the x axis (i.e., the faces whose area normals are parallel to the x axis), the net flow through the cube along the x axis can be found by comparing the rate at which mass enters one side and leaves the other. The rate of mass flowing through the left side face at x is

$$\left. \frac{\partial m}{\partial t} \right|_x = \left. \frac{\partial}{\partial t} (\rho V) \right|_x = \left. \frac{\partial}{\partial t} (\rho \Delta x \Delta y \Delta z) \right|_x = \left. \Delta y \Delta z (\rho v_x) \right|_x \quad (15.17)$$

^{viii}Most of this section is based on Ch. 2 of D.R. Poirier and G.H. Geiger, *Transport Phenomena in Materials Processing*, TMS, Warrendale, PA, 1994) and Ch. 40-41 of the Feynman Lectures on Physics, vol. II

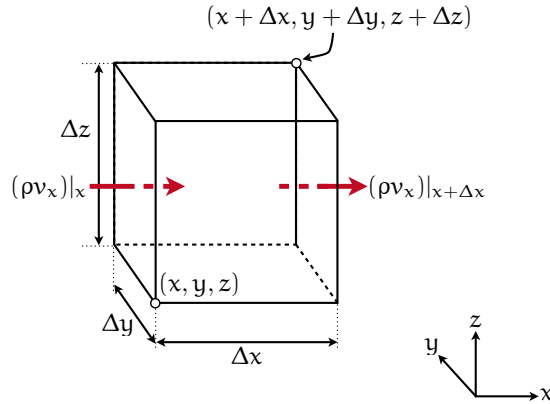


Figure 15.2: Volume element fixed in space with fluid flowing through it.

This is just the familiar result that the mass flow rate through a pipe is product of the velocity of the flow, the fluid density, and the pipe's cross-sectional area. In the same manner, we can find the flow rate through the right side face at $x + \Delta x$,

$$\left. \frac{\partial m}{\partial t} \right|_{x+\Delta x} = \Delta y \Delta z (\rho v_x) \Big|_{x+\Delta x} \quad (15.18)$$

We can proceed similarly for the other two pairs of faces perpendicular to the y and z axes, and then add up all the terms for fluid entering or leaving the cube to come up with a mass balance. If, when we add up the rates for all the sides, we have a non-zero result, then we must be either accumulating mass inside our cube, or it is experiencing a net loss in mass. Either way, the *accumulation* in mass inside our cube of constant volume can only reflect a change in density,

$$(\text{mass accumulation}) = \frac{\partial}{\partial t} \rho V = \Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} \quad (15.19)$$

Our mass balance is then simply relating this rate of mass accumulation to the net flow through the cube:

$$\begin{aligned} \Delta x \Delta y \Delta z \frac{\partial \rho}{\partial t} = & \Delta y \Delta z \left(\rho v_x \Big|_x - \rho v_x \Big|_{x+\Delta x} \right) + \Delta x \Delta z \left(\rho v_y \Big|_y + \rho v_y \Big|_{y+\Delta y} \right) \\ & + \Delta x \Delta y \left(\rho v_z \Big|_z - \rho v_z \Big|_{z+\Delta z} \right) \end{aligned} \quad (15.20)$$

Next, we can divide by $\Delta x \Delta y \Delta z$ and then take the limit of infinitesimal dimensions, and after recalling the definition of the derivative, we arrive at the continuity equation:

$$\frac{\partial \rho}{\partial t} = - \left(\frac{\partial}{\partial x} \rho v_x + \frac{\partial}{\partial y} \rho v_y + \frac{\partial}{\partial z} \rho v_z \right) = -\nabla \cdot \vec{v} \quad (15.21)$$

Hopefully, at least some of this material is familiar. For the present experiment, conducted in air at very low velocities, we may assume the air has approximately constant density (i.e., it is

incompressible), and the continuity equation simplifies to $\nabla \cdot \vec{v} = 0$.

Using the vector form of the continuity equation, we can reformulate it for different coordinate systems relevant to specific problems by expanding the divergence operator $\nabla \cdot$ appropriately. In spherical coordinates, we have

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 \partial v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} (\rho v_\varphi) = 0 \quad (15.22)$$

Our problem of interest is the slow flow of a fluid past a dense sphere. If the fluid flow is along the z axis, the problem is symmetric about the z axis, and we may neglect the φ components of velocity. In other words, the problem is essentially two-dimensional, thanks to the rotational symmetry about the z axis. In this special case,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (\rho r^2 \partial v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\rho v_\theta \sin \theta) = 0 \quad (15.23)$$

As an aside, the equivalent continuity equation in electromagnetism is conservation of charge, which you might have seen:

$$\partial \rho / \partial t + \nabla \cdot \vec{j} = 0 \quad (15.24)$$

where ρ is charge density and \vec{j} current density. The charge density in a region can only change if there is a net flow of charge, a current, into or out of that region.

Static fluids

Next, we will need the equation for the forces and momentum in the fluid. Let us consider a completely static volume of fluid, with no net flow in any direction. If we know the pressure at some point within the fluid (say, at its bottom surface) is P_o , then at any point a height h above that level, the pressure is just $P = P_o - \rho gh$ where g is the gravitational acceleration, and ρ the fluid density. Put another way, the pressure as a depth h differs from our reference level only by the weight of the fluid in a column of height h .

Of course, we can turn this equation around: if P_o is just an arbitrary, constant reference pressure, then this also implies that *anywhere* in the fluid $P + \rho gh = P_o$ must be constant! Actually, this is not so surprising either, if we multiply everything by a volume of interest:

$$P_o V = PV + \rho Vgh = PV + mgh \quad (15.25)$$

What we have is simply a result of the work-energy theorem: the work done in increasing the pressure on a given constant volume is $P(V - V_o)$, and this must be accounted for by the change in gravitational potential energy, mgh . Again, in dealing with continuous matter such as a fluid, it is more convenient to recast all of our equations in terms of *density* rather than mass and volume.

In this light, gh is just the gravitational potential *per unit mass*, so what we are really saying is that *pressure plus gravitational potential is a constant* for a fluid, or that pressure itself is a sort of volumetric potential. Thus, we define a gravitational potential per unit mass $\phi = mg$, giving us

$$P + \rho\phi = \text{const} \quad (15.26)$$

Now we have an energy balance for our static fluid, it is only a bit of mathematics to find a force balance. If we consider a one-dimensional fluid, we know that force is just the spatial derivative of the potential energy. The same will hold true of the potential energy per unit volume. If we consider only fluids of constant density, we can take the spatial derivative of both sides of Eq. 15.26:

$$\frac{\partial P}{\partial x} + \rho \frac{\partial \phi}{\partial x} = 0 \quad (15.27)$$

In three dimensions, we need only replace the spatial derivative with a gradient:

$$\nabla P + \rho \nabla \phi = 0 \quad (15.28)$$

This is nothing more than a Newton's law force balance for our stationary fluid, if we recognize that $\rho \nabla \phi$ is the force (per unit volume). In static equilibrium, the force per unit volume is precisely balanced by a gradient in pressure.

This equation is the complete description of hydrostatics, though it is quite a bit more complicated than it looks: there is no general solution. If the density of the fluid varies spatially ($\nabla \rho \neq 0$ somewhere), our continuity equation above tells us that there is no way that a *static* equilibrium can be maintained, we must have also have time-varying density. ^{ix} Only if ρ is constant in space do we have a simple solution for hydrostatics, viz., $P + \rho\phi = \text{const}$.

Equations of motion without viscosity (“Dry Water”)

What to do if the fluid is not static? We already know the continuity equation in general, but we still need to consider a more general force balance for our fluid. What we have derived above is the equilibrium condition for a static fluid, generalizing just means letting the pressure and potential gradient terms become unbalanced to yield a net acceleration. In the absence of viscous forces, this would simply be

$$\rho \times (\text{acceleration}) = -\nabla P - \rho \nabla \phi \quad (15.29)$$

The left side is the net force per unit volume, and the first two terms on the right are our pressure and potential gradients. Already, if these terms on the right are unbalanced (e.g., if we have a spatially-varying density) we will have a net acceleration of the fluid, and hence motion. What

^{ix}Strictly, for a fluid of constant density, a spatially-varying density in Eq. 15.21 implies that the velocity field must have zero divergence, or be zero everywhere to have a density which does not vary in time. Only the case $v=0$ corresponds to a truly static situation, and thus, if ρ has any spatial variation, a time variation is implied.

does the acceleration term look like?

What we really need to find is $\Delta\vec{v}/\Delta t$ for infinitesimal Δt , that is our acceleration. Just from the mathematics of partial derivatives, we can say quite a lot already. Say we know the velocity of a infinitesimal volume of fluid at some particular point in space and time, $\vec{v}(x, y, z, t)$. What is the velocity of the *same* bit of fluid at some later time $t + \Delta t$ when the bit of fluid is at a neighboring point $(x + \Delta x, y + \Delta y, z + \Delta z)$? From the definition of partial derivatives, for small changes in x , y , z , and t (i.e., to first order) we can write the change in velocity as

$$\Delta\vec{v} = \vec{v}(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t) - \vec{v}(x, y, z, t) \quad (15.30)$$

$$= \frac{\partial\vec{v}}{\partial x}\Delta x + \frac{\partial\vec{v}}{\partial y}\Delta y + \frac{\partial\vec{v}}{\partial z}\Delta z + \frac{\partial\vec{v}}{\partial t}\Delta t \quad (15.31)$$

This is not incredibly useful, as such, but we can multiply and divide every spatial derivative by Δt to put this in a more interesting form:

$$\Delta\vec{v} = \frac{\partial\vec{v}}{\partial x}\Delta x + \frac{\partial\vec{v}}{\partial y}\Delta y + \frac{\partial\vec{v}}{\partial z}\Delta z + \frac{\partial\vec{v}}{\partial t}\Delta t \quad (15.32)$$

$$= \frac{\partial\vec{v}}{\partial x}\frac{\Delta x}{\Delta t}\Delta t + \frac{\partial\vec{v}}{\partial y}\frac{\Delta y}{\Delta t}\Delta t + \frac{\partial\vec{v}}{\partial z}\frac{\Delta z}{\Delta t}\Delta t + \frac{\partial\vec{v}}{\partial t}\Delta t \quad (15.33)$$

$$= \frac{\partial\vec{v}}{\partial x}v_x\Delta t + \frac{\partial\vec{v}}{\partial y}v_y\Delta t + \frac{\partial\vec{v}}{\partial z}v_z\Delta t + \frac{\partial\vec{v}}{\partial t}\Delta t \quad (15.34)$$

$$= \left(\frac{\partial\vec{v}}{\partial x}v_x + \frac{\partial\vec{v}}{\partial y}v_y + \frac{\partial\vec{v}}{\partial z}v_z + \frac{\partial\vec{v}}{\partial t} \right) \Delta t \quad (15.35)$$

The acceleration $d\vec{v}/dt$ then becomes, in the limit $\Delta t \rightarrow 0$,

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial x}v_x + \frac{\partial\vec{v}}{\partial y}v_y + \frac{\partial\vec{v}}{\partial z}v_z + \frac{\partial\vec{v}}{\partial t} \quad (15.36)$$

This might not look like much, but if we look and rearrange it carefully we can recognize a nicer vector form:

$$\frac{d\vec{v}}{dt} = v_x \frac{\partial\vec{v}}{\partial x} + v_y \frac{\partial\vec{v}}{\partial y} + v_z \frac{\partial\vec{v}}{\partial z} + \frac{\partial\vec{v}}{\partial t} \quad (15.37)$$

$$= \left[(v_x \hat{x} + v_y \hat{y} + v_z \hat{z}) \cdot \left(\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \right) \right] \vec{v} + \frac{\partial\vec{v}}{\partial t} = (\vec{v} \cdot \nabla) \vec{v} + \frac{\partial\vec{v}}{\partial t} \quad (15.38)$$

Can you see why it must be $(\vec{v} \cdot \nabla) \vec{v}$ and not, e.g., $\vec{v} \cdot (\nabla \vec{v})$? (If for no other reason, the former is a vector while the latter is a scalar!)

Having found the acceleration, in the absence of viscous forces our equation of motion is complete:

$$\frac{d\vec{v}}{dt} = \rho (\vec{v} \cdot \nabla) \vec{v} + \rho \frac{\partial\vec{v}}{\partial t} = -\nabla P - \rho \nabla \phi \quad (\text{equation of motion, no viscosity}) \quad (15.39)$$

We can add a bit more physical content to our equation of motion by defining a new field from the curl of the velocity, $\vec{\Omega} = \nabla \times \vec{v}$. This quantity is called the *vorticity* of the fluid, and it characterizes the circulation of the fluid. If $\vec{\Omega} = 0$ everywhere, the fluid is said to be *irrotational*. By introducing the vorticity, we can separate the terms in our equation of motion to characterize two basic cases: fluids that swirl, and those that do not.^x If we are only interested in fluids that do not circulate, this will allow considerable simplification.

In order to achieve this separation, we can also make use of the following vector identity to introduce terms that contain $\nabla \times \vec{v}$:

$$(\vec{v} \cdot \nabla) \vec{v} = (\nabla \times \vec{v}) \times \vec{v} + \frac{1}{2} \nabla (\vec{v} \cdot \vec{v}) = \vec{\Omega} \times \vec{v} + \frac{1}{2} \nabla v^2 \quad (15.40)$$

This allows us to put our equation of motion in the following form:

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{\Omega} \times \vec{v} + \frac{1}{2} \rho \nabla v^2 = -\nabla P - \rho \nabla \phi \quad (15.41)$$

Now, taking advantage of this new form, we can consider only irrotational fluids for which $\vec{\Omega} = 0$ (as is the case in our experiment), in which case we have the simpler result

$$\rho \frac{\partial \vec{v}}{\partial t} + \frac{1}{2} \rho \nabla v^2 = -\nabla P - \rho \nabla \phi \quad (\text{equation of motion, no viscosity, irrotational}) \quad (15.42)$$

Finally, there is one more simplification we can make for many reasonable cases: the assumption of *steady flow*. This doesn't mean we have nothing happening, it is merely the condition that we have motion of the fluid at constant velocity, $\partial \vec{v} / \partial t = 0$. In this case,

$$\frac{1}{2} \rho \nabla v^2 = -\nabla P - \rho \nabla \phi \quad (\text{equation of motion, no viscosity, irrotational, steady flow}) \quad (15.43)$$

Since every term in this equation involves a gradient, we may simply integrate both sides to get rid of a gradient from every term, and once we remember to add in an integration constant, we have

$$\frac{1}{2} \rho v^2 + P + \rho \phi = (\text{const}) \quad (15.44)$$

This is Bernoulli's theorem, which is basically a statement of conservation of energy per unit volume for the fluid. Compare this to our starting point, $P + \rho \phi = (\text{const.})$, and you will see that the new term $\frac{1}{2} \rho v^2$ is nothing more than the kinetic energy of the moving fluid!

^xIt might help to recall the fundamental theorem of vector calculus, which roughly states that we can build any reasonable vector field out of the sum of an irrotational (zero curl) field and a solenoidal (zero divergence) field.

Viscosity in one dimension

Adding a viscous (drag) force to our equation of motion is not much of a problem, in *principle*. If we have a viscous force \vec{f}_v per unit volume, then Newton's law yields

$$\rho(\vec{v} \cdot \nabla) \vec{v} + \rho \frac{\partial \vec{v}}{\partial t} = -\nabla P - \rho \nabla \phi + \vec{f}_v \quad (15.45)$$

The problem is, what is the viscous force? The model of fluid flow above basically ignores the presence of any lateral shear forces, or forces perpendicular to the direction of the fluid flow. In other words, our first model assumes that the fluid will put up no resistance to being pushed around, which is clearly unrealistic. This is not even realistic for a solid: when we deform a solid, we know that it will produce a restoring force proportional to the strain it experiences, giving rise to Hooke's law macroscopically. Real fluids will also react to an applied force or pressure, but more important in this case than the *amount* of strain is the *rate* at which strain is produced. For example, in most fluids it is easier to move slowly than it is to move rapidly – think about swimming or stirring a jar of thick syrup.

Our previous model ignored any interactions between a moving fluid and a solid surface it encounters. In fact, it would have all but impossible to do so without some sort of empirical guidance or at least a hint at the answer. In this, we are lucky, however. One important experimental fact severely constrains models of viscous forces: *the velocity of a fluid is exactly zero at the boundary of a solid surface*. This is not an obvious fact, but one you can easily verify: how else would your fan blades have dust on them? Shouldn't it blow off?

With this fact, we can attempt a model of viscous forces. Image that we have two flat parallel plates of area A immersed in an initially stationary fluid, separated by a distance d . We hold one plate at rest in the fluid, and move the other plate at velocity v_o through the fluid. In a fluid without viscosity, the moving plate would not disturb the fluid at all, and the fluid velocity would be zero everywhere. However, if the relative velocity of fluid and plate must be zero at each plate's surface, that means that the fluid velocity varies from 0 to v_o moving from the stationary to the moving plate! At the surface of the moving plate, the fluid must have velocity v_o , and at the surface of the stationary plate, it must have $v=0$.

If you measure the force per unit area required to keep the top plate moving, it turns out to be proportional to the velocity of the plate divided by the spacing between the plates.

$$\frac{F}{A} = \eta \frac{v_o}{d} \quad (15.46)$$

The constant of proportionality η is known as the coefficient of viscosity, and to some extent it is a measure of how much force must be supplied to produce motion in a fluid. Noting that power is $\vec{F} \cdot \vec{v}$, you can see that the power required to maintain a speed v in a fluid scales as ηv^2 .

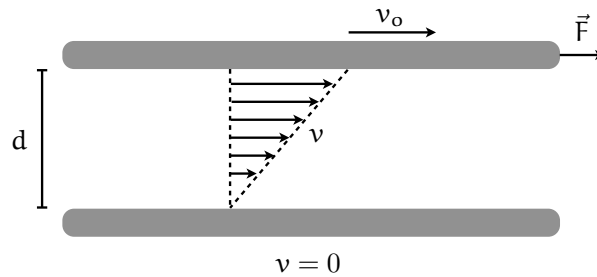


Figure 15.3: Viscous drag between two parallel plates in a fluid.

In moving beyond a single dimension, we have even more basic problems to consider: if we press on a fluid in one direction, it will move in *all* directions, not just along the direction of the applied force. This is in sharp contrast to our usual considerations of infinitesimal particles, or rigid objects. What do we do when the object can “squish?” In the example above, the moving plate will displace the fluid it is moving through, imparting velocity in the directions perpendicular to the motion of the plate. Evidently, what we lack is a way to relate displacement along one direction with force along another. The mathematical tool we are missing is the *tensor*, a generalization of vectors and scalars which turns out to be indispensable for many areas of physics.

The Stress Tensor

Tensors

So far as we need them, a tensor is a set of numbers (or a matrix) that when multiplied by a vector gives back a new vector. Of course, this much can be accomplished by vector or scalar multiplication, but what makes tensors special is that the two vectors need not be simply parallel or perpendicular. This is exactly what we need to understand stress and pressure in materials: relating a displacement or force in one direction to a resulting force along a different direction, particularly when materials are allowed to deform. A close analogy to the type of mathematical object we need is the rotation matrix: a multiplying a given vector by a rotation matrix gives a new vector of the same length, but pointing in a new direction. A tensor is a more general type of matrix, in which both the length and direction of the resulting vector are generally different.

As an example, let's say we want to consider the conductivity of a material, σ . Ohm's law states that the current density is proportional to the electric field via the conductivity:

$$\vec{j} = \sigma \vec{E} \tag{15.47}$$

In an isotropic, homogeneous material, σ is just a scalar, a plain number, that characterizes how much current density results from a specific electric field. As such, a scalar conductivity results in a current density which is always parallel to the electric field. In many materials, this is a perfectly reasonable assumption. However, this is clearly a simplification: what about crystals? In a perfect crystal, we have a symmetric arrangement of atoms which is clearly not isotropic, and it is unphysical to expect the conductivity to be the same along every direction in the crystal. If we have a simple cubic crystal, it would be reasonable to expect the conductivity to be the same along all three crystallographic directions, but we would certainly expect a different conductivity along other directions.

Consider a simple two-dimensional crystal, with a square grid of atoms along the x and y directions. If the spacing of atoms along x and y is the same, we expect that a given electric field applied along the x or y direction would lead to the same current density. However, if we applied the electric field along the line $y = x$, 45° with respect to the rows of atoms, we should expect a different current density. Thus, at the very least, our conductivity must be direction-dependent so long as the crystal is not isotropic! Moreover, this means that we can't even reasonably expect that the current density is along the same *direction* as the electric field. If the field is along the the line $y = x$, and we have different conductivities in the x and y directions, we should expect that the resulting current density has both x and y components, and they will not be the same. Even in an isotropic material we have to worry about this to an extent, current will spread out in all directions in a uniform conductor.

In general, the conductivity actually has *nine* components relating electric field to current density, since we have three directions for E combined with three components for j . The conductivity, then, is really a *matrix*:

$$j_i = \sum_j \sigma_{ij} E_j \quad \text{or} \quad \begin{bmatrix} j_x \\ j_y \\ j_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (15.48)$$

The nine components of σ make a *tensor*, relating \vec{E} along an arbitrary direction to a resulting \vec{j} along a different direction. The indices of σ signify which component of \vec{E} is being related to which component of \vec{j} : the first index is the component of \vec{E} , the second the component of \vec{j} . Incidentally, the fact that we require two indices to tabulate all of the components of σ makes it a “second rank” tensor.^{xi} Thus, for the x component of \vec{j} , we have

$$j_x = \sigma_{xx} E_x + \sigma_{xy} E_y + \sigma_{xz} E_z \quad (15.49)$$

On the other hand, if we apply an electric field along the x direction only, \vec{j} has components in all

^{xi}By the same logic, we can call vectors “first rank” tensors, needing only one index, and scalars “zero rank” tensors.

three directions:

$$j_x = \sigma_{xx}E_x \quad j_y = \sigma_{yx}E_x \quad j_z = \sigma_{zx}E_x \quad (15.50)$$

Usually, we don't need to deal with all nine components, and we can make use of symmetry to reduce the number of independent components. For instance, the conductivity tensor is symmetric, meaning that $\sigma_{ij} = \sigma_{ji}$. In fact, it is possible to simplify the conductivity even further. Our choice of axes along which to decompose the electric field and conductivity vectors, and thus the conductivity tensor, was completely arbitrary. In fact, it is always possible to choose axes such that the tensor is *diagonal*, e.g., such that only σ_{xx} , σ_{yy} , and σ_{zz} are non-zero:

$$\begin{bmatrix} j_x \\ j_y \\ j_z \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (15.51)$$

In a crystal, finding the diagonal representation of a tensor typically corresponds to choosing the natural crystallographic axes for decomposing the electric field and current density. Finally, in an isotropic material, in which the conductivity is independent of direction, $\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$, and we may treat the conductivity as a simple scalar.

Other examples of tensors

In fact, you've already encountered tensors many times, probably without knowing it. Generally speaking, if you need to relate two vectors, and they in general need not be strictly parallel or perpendicular, a tensor is probably involved. For instances, the moment of inertia is really a 2nd-rank tensor, since angular momentum and angular velocity are not in general parallel. Torque is also a 2nd-rank tensor, and *anti-symmetric* ($\tau_{ij} = -\tau_{ji}$), but happens to transform like a vector in three dimensions. For that reason, we usually just treat it as a vector (or pseudovector, really) since we can get away with it!

Stress

So what is stress? Essentially, it is nothing more than a generalization of pressure, a net force per unit area. Hydrostatic pressure we are used to dealing with is just a special type of stress, when the net force is normal to area of consideration. In a static fluid, the force on each side of an infinitesimal cube of fluid is the same in magnitude and always normal to the surfaces of the cube. In this case, the stress is just the hydrostatic pressure, and it is a simple scalar: $\vec{F} = PA\hat{n}$, where \hat{n} is a unit vector normal to the area A .

When we wish to deal with the internal forces in continuous objects, however, this need not be true. Inside a solid object or a fluid, we know there are internal forces between neighboring parts of the material holding it together. Consider first a cube of a nice squishy substance like gelatin, and cut it into two pieces. Clearly, before we cut the gelatin, there must have been a force holding the

two pieces together. Before the cut, each half exerted a force ΔF on the other to hold the block of gelatin together, so the stress in the material was simply this force divided by the area of the cut surface. However, the net force between the two pieces was not simply perpendicular to the cut surface. If that were true, any infinitesimal force along the cut plane would have separated the two pieces. Thus, there must be forces acting not just normal to any surface in the block, but also along the two tangential directions. In order to properly treat a patch of surface within a continuous object, we must deal with all three components of force acting on the surface. This is what stress is, a generalization of pressure to encompass forces acting on a surface in all three directions.

Let us go back to the example above, where we have a flat plate moving at velocity v_o through a fluid. In that case, we had two types of forces present. First, we had a force per unit area on the surfaces of the plate due to the hydrostatic pressure of the fluid, which acted equally in all directions and normal to each surface. This force can be described by a simple scalar, the pressure, and the area of the plate. Second, we had a force acting antiparallel to the velocity due to the viscous drag of the fluid. This is what we would call a shear force, being tangential to the surface of the plate. The force per unit area due to viscous drag is thus a *shear stress*, acting in the $-x$ direction, and it depends on a velocity in the x direction and an area in the xy plane. A complete description of such forces will require a tensor, the *stress tensor*. As another quick example, let's go back to our cube of gelatin. Say we press down on the upper face lying in the xy plane. This will clearly lead to a net force in the z direction on both faces in the xy plane, and a net shortening of the cube along the z direction. If the gelatin is incompressible, however, conservation of matter requires that the cube bulge out in the x and y direction, meaning there must be outward forces on the other four faces of the cube! Again, we will need a tensor to describe this situation, since we have an applied force in one direction leading to net forces in all three directions.

How can we figure out what the stress tensor looks like? Let's consider a volume of continuous incompressible material, of constant density ρ . Now, take a small slice of this material perpendicular to the x axis, making a little square of sides Δy and Δz with area normal \hat{x} . If we apply a force $\Delta \vec{F}_1$ to this surface along an arbitrary direction, we can break it up into components ΔF_{1x} normal to the surface and ΔF_{1y} and ΔF_{1z} tangential to the surface. The components of stress are just these forces divided by the area of our surface, labeled with two indices: the first labeling the direction of the force component, the second the area normal. For example, the force per unit area along the y direction is just

$$\tau_{yx} = \frac{\Delta F_{1y}}{\Delta y \Delta z} = \frac{\Delta F_{1y}}{\Delta a_x} \quad (15.52)$$

where Δa_x is just the area of our element of surface perpendicular to the x direction. Similarly, we have net forces per unit area in the x and z directions,

$$\tau_{zx} = \frac{\Delta F_{1z}}{\Delta y \Delta z} \quad \tau_{xx} = \frac{\Delta F_{1x}}{\Delta y \Delta z} \quad (15.53)$$

The stress τ_{xx} acts normal to our little area, just as a simply hydrostatic pressure would, while the stresses τ_{yx} and τ_{zx} act along the transverse directions. In total, just to describe the stress along a single axis, we need *three* components, which means that a full description of all the stresses on an object will require *nine* components. For example, if we now take a slice of material lying perpendicular to the y axis, lying in the xz plane, this area will have a net force $\Delta\vec{F}_2$ acting on it, and resolving it along the three axes leads to stresses τ_{xy} , τ_{yy} , and τ_{zy} . We can make a similar construction for a slice perpendicular to the z axis, and in total the stress on our object will be characterized by nine numbers, which we can conveniently express as a matrix:

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} \quad (15.54)$$

The diagonal components τ_{ii} are normal stresses, representing forces per unit area acting perpendicular to the area of a given face. These components are what we would usually just call pressure, the net force per unit area acting perpendicular to a given face. The off-diagonal components are the *shear stresses* acting along the two directions tangential to a given face, analogous to the tangential frictional force present when we slide two objects past one another. Our nine numbers τ_{ij} in total make up the *stress tensor*, where i indicates the direction along which the stress acts, and j indicates the surface normal of the relevant face. Thus, τ_{xy} represents a shear stress acting in the x direction on a face whose area normal points in the y direction.

At this point, it is probably useful to draw a little picture. Take a small cube of material, aligned along the x , y , and z axes. Looking down the z axis at one side of the cube, the components of the stress in the x and y directions acting on four of the faces looks like this:

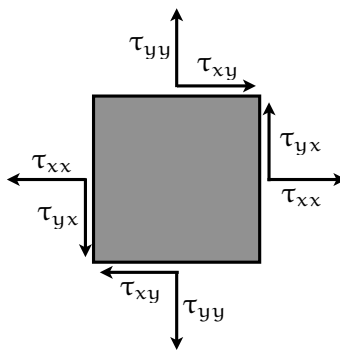


Figure 15.4: The forces in the x and y directions on the faces of an infinitesimal cube. The diagonal components of stress τ_{ii} act normal to each face, while the off-diagonal components τ_{ij} act tangentially to each face. Since the cube is very small, the stresses do not change appreciably across the cube.

As you can see, the forces on real continuous objects are rather complicated. From our initial

discussion of a static fluid, requiring only a simple scalar pressure, we now have a nine-component second-rank tensor. However, it is not as bad as it seems: the stress tensor turns out to be symmetric, and we don't need all nine components. If we consider an infinitesimally small cube of material, then we can imagine that the stresses do not change appreciable from one side to the other. As shown in the figure above, the forces on opposing sides of the cube must be equal and opposite in this case. This also implies that the *torque* about the center of the cube is zero – if it were not, the cube would start spinning, which would be unphysical for an infinitesimally small object. If our cube has sides of dimension Δa , then we can easily write down the torque about the center as $\Delta a(\tau_{yx} - \tau_{xy}) = 0$, which means $\tau_{xy} = \tau_{yx}$. We can apply the same logic looking at the other faces of the cube, and just by considering that the cube must be in rotational equilibrium we find that the stress tensor must be symmetric, $\tau_{ij} = \tau_{ji}$. In short, we have only have six unique components of stress, rather than nine.

Since our stress tensor is symmetric, it can be described by a symmetric matrix. If you have taken linear algebra, you might recall that this leads us to an even more important property of the stress tensor: since it is symmetric, it is always possible to find a choice of coordinate axes for which it is diagonal. That is, if we choose our coordinate axes carefully, it is always possible to find a special orientation for which our stress tensor has only the three components τ_{xx} , τ_{yy} , and τ_{zz} . In a perfect crystalline material, this special choice may correspond to the crystallographic axes, for instance. However, in general, the stress tensor varies from point to point in a material, meaning it is actually a tensor *field*. Just like we have a scalar field $T(x, y, z)$ describing the temperature everywhere in a room, or a vector field $\vec{E}(x, y, z)$ describing the electric field through all space, our stress tensor field describes the components of stress at all points in a material. At every point in space, the stress tensor gives us nine numbers – six unique numbers – describing the forces at that point, and thus a full description of the forces in a body require six functions of position.

Viscosity and stress in three dimensions

After a long detour, we can finally return to our parallel plates moving within a fluid from Sect. 15.7. Recall our setup:

To be a bit more concrete, let the x axis be in the direction of the applied force, the y direction upward, and the z direction out of the page. Using our new tensor machinery, we can write the force per unit area along the x direction required to keep the top plate moving as a stress:

$$\tau_{yx} = \frac{F_x}{\Delta y \Delta z} = \eta \frac{v_o}{d} \quad (15.55)$$

Thus, what we have previously considered is a shear stress acting tangential to the plates due to a viscous force along the x axis, and empirically it is found to be proportional to the speed of the upper plate through a scalar coefficient we call the viscosity.

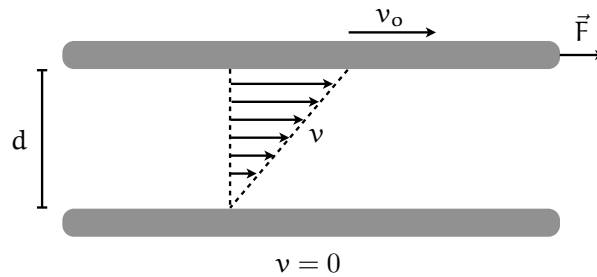


Figure 15.5: Viscous drag between two parallel plates in a fluid.

As a slightly more general case, we could forget about the plates, and only consider artificial surfaces within the fluid itself, moving at different velocities. In Fig. 15.6 below, we look at an extended region within a moving fluid, with its faces parallel to the fluid flow. In general, the velocity of the fluid will vary along the vertical direction (y), such that at the top of our cell the velocity is $v + \Delta v$, and at the bottom it is just v . Based on our simpler model above, the net shear force acting on the cell in the x direction will be the difference between the forces on the top and bottom of the cell, divided by the area of the plate ΔA . By analogy with the situation with two plates, the shear stress is then proportional to the difference in velocity between the top and bottom of the cell divided by the vertical extent of the cell:

$$\frac{\Delta F_x}{\Delta A} = \eta \frac{\Delta v}{\Delta y} = \eta \frac{\partial v_x}{\partial y} = \tau_{yx} \quad (\text{fluid velocity along } x \text{ only}) \quad (15.56)$$

This net shear stress acts to the right on the top face, and would tend to either deform our cell into a parallelogram or lead to a rotation of our cell in the clockwise direction. The only way that the fluid will be irrotational is if $\partial v_x / \partial y = 0$, that is, the velocity is constant along the vertical direction and there is no net force at all. Otherwise, a variation in fluid velocity along the *vertical* direction leads to a *horizontal* shear stress.

What if the velocity of the fluid isn't strictly parallel to the faces of our cell? Let's say the vertical component of the velocity varies across the top and bottom faces, with the velocity being higher on the left side of the cell. This situation would also tend to cause a clockwise rotation of our cell, meaning that there must also be stress components along the x direction due to the variation of fluid velocity in the x direction (as well as normal components along the y direction). For a general fluid velocity, the horizontal shear stress must then have two components:

$$\tau_{yx} = \eta \frac{\partial v_y}{\partial x} + \eta \frac{\partial v_x}{\partial y} \quad (15.57)$$

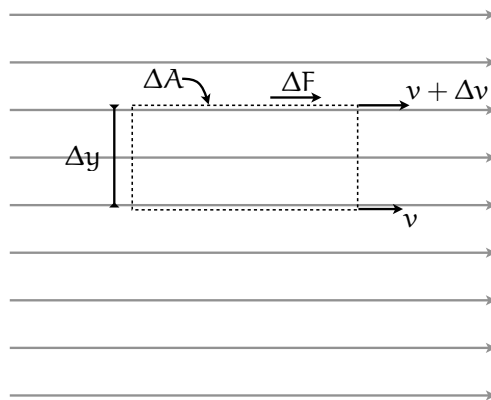


Figure 15.6: A small volume of fluid within a flow.

We could find the other shear components τ_{yz} and τ_{zx} similarly. Note that this equation immediately satisfies our symmetry requirement $\tau_{xy} = \tau_{yx}$. We can also see from this general expression that there are only three cases in which there is *no* shear stress in the fluid: either the fluid is static ($\vec{v} = 0$, the fluid's velocity varies only along the out-of-plane z direction ($\partial v_x / \partial y = \partial v_y / \partial x = 0$), or the fluid is uniformly rotating ($\partial v_x / \partial y = -\partial v_y / \partial x$). Of course, there are also no shear forces in a fluid with zero viscosity, but such things are incredibly rare.^{xii}

Along these same lines, we can also find the normal stresses, those acting perpendicularly to the faces of our cell. For example, if there is a variation in velocity along the vertical direction, then there will also be a net force on the cell in the vertical direction, along with the shear component:

$$\tau_{yy} = 2\eta \frac{\partial v_x}{\partial x} \quad (15.58)$$

The normal components of the stress, τ_{ii} , are what we would simply call pressure if the fluid were static. In the case of a moving fluid, the total normal force per unit area would be the static pressure P plus the normal stress due to the fluid motion.

In general, *so long as the fluid is incompressible*, based on our description above you should be able to convince yourself that the stress components are given by

$$\tau_{ij} = \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (15.59)$$

^{xii}Liquid helium is a so-called “superfluid” with zero viscosity, a macroscopic quantum-mechanical effect that can be observed only at very low temperatures.

Viscous forces in three dimensions

Now that we have the shear stresses in the fluid in the presence of viscosity, we can complete our equation of motion. All we need to do is work backwards to determine the forces on an arbitrary cell within a moving fluid from the stress components. Imagine we have again a small cube of fluid, Fig. 15.7, whose faces are aligned with our coordinate axes, with sides of length Δx , Δy , and Δz . In addition to any hydrostatic pressure, our cube will have a force on each of its six sides due to the stress in the moving fluid fluid, whose velocity we will assume to vary in magnitude and direction.

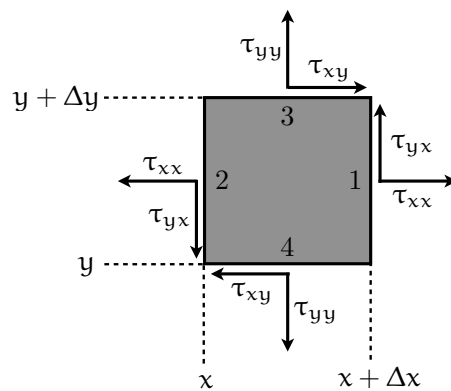


Figure 15.7: The stresses in the x and y directions on the faces of a cube of fluid.

First, we can tabulate all of the forces along a given axis, starting with x . All six faces of the cube will have a stress component in the x direction: four shear forces, and two normal forces. On face 1, we have a normal stress component τ_{xx} acting over an area $\Delta y \Delta z$, and net x component of the force on face 1 will be the product of stress and area. However, we must be careful: the stress is really a tensor *field*, and it varies with position, so the value of τ_{xx} is different for face 1 and face 2, for instance. Thus, we should explicitly note at which position we are evaluating the stress components. With that in mind,

$$F_{x1} = \tau_{xx} \Big|_{x+\Delta x} \Delta y \Delta z \tag{15.60}$$

The x component force on face two will be similar and opposite in sign, the only substantial difference is that we are evaluating the stress tensor at a different position:

$$F_{x2} = -\tau_{xx} \Big|_{\Delta x} \Delta y \Delta z \tag{15.61}$$

Faces 3 and 4 will also have forces in the x direction through the shear stress τ_{xy} . Face 3 has area $\Delta x \Delta z$ and it is located at vertical position $y + \Delta y$. Face 4 has the same area, but the force is in

the opposite direction and τ_{xy} should be evaluated at a vertical position y .

$$F_{x3} = \tau_{xy} \Big|_{y+\Delta y} \Delta x \Delta z \quad (15.62)$$

$$F_{x4} = -\tau_{xy} \Big|_y \Delta x \Delta z \quad (15.63)$$

$$(15.64)$$

Finally, faces 5 and 6 (on the front and back of the cube in Fig. 15.7) also have forces along the x direction through the shear stress τ_{xz} :

$$F_{x5} = \tau_{xz} \Big|_{z+\Delta z} \Delta x \Delta y \quad (15.65)$$

$$F_{x6} = -\tau_{xz} \Big|_z \Delta x \Delta y \quad (15.66)$$

$$(15.67)$$

All that is required now is to tabulate the net force along the x direction for the whole cube:

$$F_x = F_{x1} + F_{x2} + F_{x3} + F_{x4} + F_{x5} + F_{x6} \quad (15.68)$$

$$= \left(\tau_{xx} \Big|_{x+\Delta x} - \tau_{xx} \Big|_x \right) \Delta y \Delta z + \left(\tau_{xy} \Big|_{y+\Delta y} - \tau_{xy} \Big|_y \right) \Delta x \Delta z + \left(\tau_{xz} \Big|_{z+\Delta z} - \tau_{xz} \Big|_z \right) \Delta x \Delta y \quad (15.69)$$

$$= \Delta \tau_{xx} \Delta x \Delta z + \Delta \tau_{xy} \Delta x \Delta z + \Delta \tau_{xz} \Delta x \Delta y \quad (15.70)$$

As with our equation of motion without viscosity, it is more useful to consider the force per unit volume along x , which we'll call f_x :

$$f_x = \frac{F_x}{\Delta x \Delta y \Delta z} = \frac{\Delta \tau_{xx}}{\Delta x} + \frac{\Delta \tau_{xy}}{\Delta y} + \frac{\Delta \tau_{xz}}{\Delta z} \quad (15.71)$$

If we take the limit that the dimensions of our cube become infinitesimally small, what we have is the definition of a partial derivative:

$$f_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \quad (15.72)$$

We can repeat the analysis for the forces along the other directions, and our general expression for an incompressible fluid is

$$f_i = \sum_{j=1}^3 \frac{\partial \tau_{ij}}{\partial x_j} \quad \text{with} \quad \tau_{ij} = \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (15.73)$$

The stresses in a fluid depend on the velocity gradients in the fluid (or, equivalently, the rate of

change of shear strain), while the *forces* depend on the *stress gradients*. There will only be a net force on a volume of fluid if there is a net spatial variation of stress, and there will only be stress if there is a net spatial variation in velocity. Combining the two relationships above, we can cut out the middleman and relate the viscous force per unit volume directly to the velocity distribution:

$$f_i = \eta \sum_{j=1}^3 \frac{\partial}{\partial x_j} \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] \quad (15.74)$$

If we write out all three components of the force and rearrange the terms, we can recover a much more compact vector equation. Let's rearrange the sum and see what comes out:

$$f_i = \eta \sum_{j=1}^3 \frac{\partial}{\partial x_j} \left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right] = \eta \sum_{j=1}^3 \frac{\partial^2 v_i}{\partial x_j^2} + \eta \frac{\partial}{\partial x_i} \sum_{j=1}^3 \frac{\partial v_j}{\partial x_j} = \eta \nabla^2 v_i + \eta \frac{\partial}{\partial x_i} \nabla \cdot \vec{v} \quad (15.75)$$

Considering all three components of the force per unit volume, we have a nice vector equation in the end:

$$\vec{f} = \eta \nabla^2 \vec{v} + \eta \nabla (\nabla \cdot \vec{v}) \quad (15.76)$$

Here we have used the vector Laplacian $\nabla^2 \vec{v}$, which is just $\nabla^2 v_x \hat{x} + \nabla^2 v_y \hat{y} + \nabla^2 v_z \hat{z}$. We can make this still simpler, however, by remembering that for an incompressible fluid, the continuity equation reads $\nabla \cdot \vec{v} = 0$. With that in mind, after much pain we ultimately have a simple form for the viscous force in an incompressible fluid

$$\vec{f}_v = \eta \nabla^2 \vec{v} \quad (\text{viscous force, incompressible fluid}) \quad (15.77)$$

Our pervasive assumption of an incompressible fluid does come at a price. For instance, we will not be able to treat density variations in the fluid, such as sound waves. However, a wide variety of interesting fluids are essentially incompressible, and the form of the viscous force per unit volume above is sufficient. This assumption serves quite well for, e.g., air flowing at low speeds compared to the speed of sound.

The equation of motion with viscosity for incompressible fluids

The general equation of motion we developed was

$$\rho (\vec{v} \cdot \nabla) \vec{v} + \rho \frac{\partial \vec{v}}{\partial t} = -\nabla P - \rho \nabla \phi + \vec{f}_v \quad (15.78)$$

where f_v was our yet-to-be-determined viscous force. For an incompressible fluid, using the form of the viscous force from Eq. 15.77, our equation of motion reads

$$\rho (\vec{v} \cdot \nabla) \vec{v} + \rho \frac{\partial \vec{v}}{\partial t} = -\nabla P - \rho \nabla \phi + \eta \nabla^2 \vec{v} \quad (15.79)$$

This non-linear partial differential equation is the Navier-Stokes equation for flow of Newtonian

incompressible fluids. The Navier-Stokes equation is not straightforward to interpret qualitatively, and famously difficult to solve in even the simplest cases. Another more common form substitutes the potential per unit mass $\nabla\phi = -\vec{g}$:

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla P + \rho \vec{g} + \eta \nabla^2 \vec{v} \quad (15.80)$$

This form is more easily interpreted as a statement of Newton's law: mass (ρ) times acceleration equals the sum of forces, namely pressure ($-\nabla P$), viscous forces ($\eta \nabla^2 \vec{v}$), and gravity ($\rho \vec{g}$). Since we are assuming constant density (incompressible fluid), the continuity equation is simply $\nabla \cdot \vec{v} = 0$, which is a statement of conservation of fluid volume.

Incidentally, we can also reintroduce our vorticity $\vec{\Omega} = \nabla \times \vec{v}$:

$$\rho \left(\frac{\partial \rho}{\partial t} + \vec{\Omega} \times \vec{v} + \frac{1}{2} \nabla v^2 \right) = -\nabla P + \rho \vec{g} + \eta \nabla^2 \vec{v} \quad (15.81)$$

That means that for an irrotational fluid ($\vec{\Omega} = 0$), we have

$$\rho \left(\frac{\partial \rho}{\partial t} + \frac{1}{2} \nabla v^2 \right) = -\nabla P + \rho \vec{g} + \eta \nabla^2 \vec{v} \quad (15.82)$$

The steady-state ($\partial \rho / \partial t = 0$) equation for an irrotational fluid reads

$$\frac{1}{2} \rho \nabla v^2 = -\nabla P + \rho \vec{g} + \eta \nabla^2 \vec{v} \quad (15.83)$$

Finally, if we are interested in the steady-state behavior of fluids at ve

Stoke's flow around a solid sphere

If we are only interested in slow and steady flow (small Reynolds' numbers) of an incompressible fluid, we may neglect the "acceleration" term in the Navier-Stokes equation $\frac{1}{2} \rho \nabla v^2$:

$$-\nabla P + \eta \nabla^2 \vec{v} + \rho \vec{g} = 0 \quad (15.84)$$

This is just one step up from our equation of state for a completely static fluid, we now retain only the viscous force $\eta \nabla^2 \vec{v}$. Further simplification is possible due to the symmetry of the problem. Let the fluid flow be along the z axis with constant velocity V_∞ , with the solid sphere of radius R at the origin. In this case, by symmetry the fluid momentum is clearly independent of φ (the angle in the xy plane). In spherical coordinates the Navier-Stokes and continuity equations read

$$-\frac{\partial P}{\partial r} + \eta \left[\nabla^2 v_r - \frac{2v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - \frac{2}{r^2} v_\theta \cot \theta \right] + \rho g_r = 0 \quad (15.85)$$

$$-\frac{1}{r} \frac{\partial P}{\partial \theta} + \eta \left[\nabla^2 v_\theta + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2 \sin^2 \theta} \right] + \rho g_\theta = 0 \quad (15.86)$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) = 0 \quad (15.87)$$

Here we have expanded the θ and r portions of the gradient operators in spherical coordinates. Perhaps surprisingly, the stress distribution, pressure distribution, and velocity components can be found analytically:

$$\tau_{r\theta} = \frac{3\eta V_\infty}{2R} \left(\frac{R}{r} \right)^4 \sin \theta \quad (15.88)$$

$$P = P_o - \rho g z - \frac{3\eta V_\infty}{2R} \left(\frac{R}{r} \right)^2 \cos \theta \quad (15.89)$$

$$v_r = V_\infty \left(1 - \frac{3}{2} \left(\frac{R}{r} \right) + \frac{1}{2} \left(\frac{R}{r} \right)^3 \right) \cos \theta \quad (15.90)$$

$$v_\theta = -V_\infty \left(1 - \frac{3}{4} \left(\frac{R}{r} \right) - \frac{1}{4} \left(\frac{R}{r} \right)^3 \right) \sin \theta \quad (15.91)$$

Note the following boundary conditions: at the sphere's boundary $r = R$, $v_r = v_\theta = 0$, and at $r = \infty$, $v_z = V_\infty$. Equation 15.89 is readily parseable: P_o is the pressure in the plane $z=0$ far from the sphere, $-\rho g z$ is the hydrostatic pressure effect, and the term with V_∞ results from fluid flow around the sphere. These equations are valid for Reynolds numbers less than approximately one. In Fig. 15.8, we show fluid flow around a sphere under these conditions.

What we are interested in now is the force on the sphere due to this flow. The normal force (along the z axis) acting on the solid sphere is due to the pressure given by Eq. 15.89 with $r = R$ and $z = R \cos \theta$:

$$P(r = R) = P_o - \rho g R \cos \theta - \frac{3\eta V_\infty}{2R} \cos \theta \quad (15.92)$$

The net upward force in the z direction due to the pressure difference on the 'top' and 'bottom' portions of the sphere is found by multiplying this pressure times the infinitesimal bit of surface area over which it acts, $R^2 \sin \theta d\theta d\varphi$ and integrating over the surface of the sphere:

$$F_n = \int_0^{2\pi} \int_0^\pi \left[P_o - \rho g R \cos \theta - \frac{3\eta V_\infty}{2R} \cos \theta \right] R^2 \sin \theta d\theta d\varphi \quad (15.93)$$

$$F_n = \frac{4}{3} \pi R^3 \rho g + 2\pi \eta R V_\infty \quad (15.94)$$

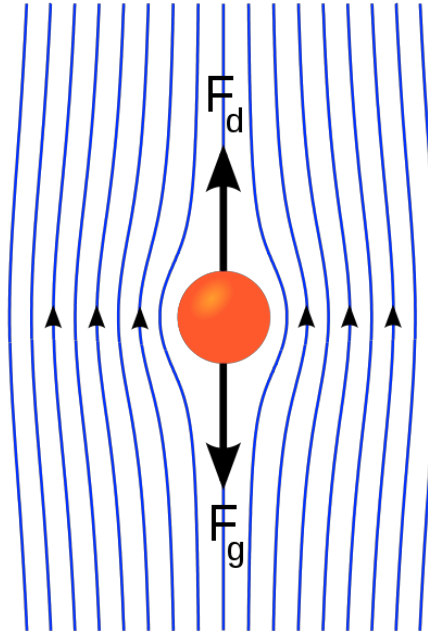


Figure 15.8: Forces on and streamlines around a sphere in Stokes flow. From http://en.wikipedia.org/wiki/File:Stokes_sphere.svg.

We recover two terms for the normal force: the first is the *buoyant force* and the second the *form drag*. At each point on the surface, there is also a shear stress acting tangentially, $-\tau_{r\theta}$. This tangential force, since we are dealing with a curved surface, has both $x - y$ and z components. Over the whole sphere, the former will vanish by symmetry, but the latter will give rise to a net force for any non-zero fluid velocity. On any infinitesimal patch of surface, the z -component of this tangential force is $(-\tau_{r\theta})(-\sin\theta)R^2\sin\theta d\theta d\varphi$, and once again integrating over the sphere's surface we find

$$F_t = \int_0^{2\pi} \int_0^\pi (\tau_{r\theta}|_{r=R} \sin\theta) R^2 \sin\theta d\theta d\varphi \quad (15.95)$$

From Eq 15.88,

$$\tau_{r\theta} \Big|_{r=R} = \frac{3}{2} \frac{\eta V_\infty}{R} \sin\theta \quad (15.96)$$

which results in a net *frictional drag* from the tangential flow of

$$F_t = 4\pi\eta R V_\infty \quad (15.97)$$

Thus, the total force on our sphere in the flowing fluid is

$$F = \frac{4}{3}\pi R^3 \rho g + 6\pi\eta R V_\infty \quad (15.98)$$

The force has two terms, as expected: the first due to gravity (the weight of the fluid), exerted even if the fluid is stationary, and the second associated with fluid motion, sometimes called the

“drag force.” Both forces act in the same direction, opposing the direction of fluid flow.

Equation 15.98 is known as Stoke's law, and from it we may determine the terminal velocity of a falling sphere. Consider a sphere falling in a stagnant fluid of density ρ_s . In this case V_∞ is the *relative* velocity of the fluid with respect to the sphere, which in this case is just the velocity of the falling sphere since the fluid is stationary. The static and drag Stoke's forces act opposite the direction that the sphere falls, and at the terminal velocity V_t , precisely balance the sphere's weight. If the sphere has density ρ_s , this means

$$\frac{4}{3}\pi R^3 \rho g + 6\pi\eta R V_t = \frac{4}{3}\pi R^3 \rho_s g \quad (15.99)$$

This leads to a terminal velocity of

$$V_t = \frac{2gR^2(\rho_s - \rho)}{9\eta} \quad (15.100)$$

As expected, it is the relative density of fluid and solid that determine the behavior of the sphere: if the sphere is more dense than the fluid, it sinks, and if it is less dense than the fluid, it rises. In the present experiment, the fluid density (air) is negligible compared to the ‘solid’ density (oil), and we arrive at Eq. 15.5.

Millikan's correction to Stoke's flow for low velocities

Under construction....

Gamma ray spectroscopy

Note: It is assumed that you have read the introduction to the previous experiment, gamma ray attenuation. See Sec. 6.

16.1 Introduction

The purpose of this lab is to become familiar with the response of a NaI detector to gamma rays and to identify unknown gamma sources. NaI, doped with thallium, is an inorganic scintillator detector, that is, it produces scintillation light when excited by the ionizing radiation of charged particles. Gamma sources are unstable nuclei which emit photons when they decay. The combination of gamma rays emitted by an unstable nucleus is unique. Besides gamma rays emitted due to nuclear de-excitations, annihilation radiation may be produced if the decay products include a positron: the positron annihilates with an atomic electron to produce two 511 keV photons.”ⁱ

16.1.1 Scintillation Counters

When radiation interacts with certain substances, a small flash of visible light (a scintillation) is produced. This flash of light can be detected by a special kind of photocell (called a photomultiplier tube) which is capable of multiplying the resulting signal by a very large factor. The combination of a scintillator mounted on a photomultiplier tube is referred to as a scintillation counter.

Depending on the type of scintillator, these counters are capable of detecting as well as determining the energy of alpha, beta, gamma, and x-rays. To detect alphas, zinc sulphide and plastic phosphors may be used, while betas are usually best detected by anthracene or plastics. The best scintillator for gammas (the type of radiation studied in this experiment) is a large crystal of sodium iodide containing a trace of thallium as an activant. This is referred to as a NaI(Tl) crystal. The crystal used in this experiment is roughly 4 cm in diameter and 2.5 cm thick, and it is mounted in an aluminum can which is optically coupled to the end of the photomultiplier tube. The entire detector assembly is contained within the aluminum can to shield it from external light.

The scintillation counter has several advantages over the Geiger-Müller counter for the detection of gamma rays. These advantages include:

- greater detection efficiency, due to the use of a solid (rather than a gaseous) detector
- capability for handling higher counting rates (due to shorter resolving times)
- capability for energy analysis (due to the fact that the amplitude of the output pulse is proportional to the energy absorbed in the scintillator)

ⁱMuch of this procedure is reproduced from ref. [4]. This version is only for use in PH255 at the University of Alabama.

16.1.2 Compton Scattering

The energy of gamma rays is transferred to the NaI(Tl) scintillator chiefly by two processes: the photoelectric effect and the Compton effect. At higher energies (> 1.022 MeV), pair production also plays a role, and will be discussed further below. In the photoelectric effect, the entire energy of the gamma ray is transferred to the phosphor, resulting in what is known as a photopeak or full absorption peak. Put another way, the photons in the incident radiation are of high enough energy to release electrons from atoms and molecules in the detector crystal. Photons in the beam are thus annihilated, leading to a corresponding reduction in intensity. This photoeffect will lead to a peak in the γ energy spectrum, known as the photopeak or full absorption peak.

In a real experiment, however, you will notice more structure in your spectrum than just photopeaks corresponding to gamma emission. Specifically, we must also consider the scattering of gamma photons by electrons in the detector, i.e., the Compton effect. This will lead to two additional features: a “Compton edge” and a “backscatter peak.” Recall that the Compton effect is a “collision” of a photon with an electron in which relativistic energy and momentum must be conserved. After the collision, the photon imparts some of its energy and momentum to an electron, and thus the photon exits the collision with a slightly lower energy and a new direction. This process is shown schematically in Fig. 16.1.

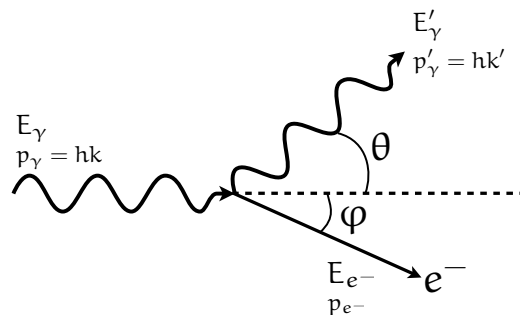


Figure 16.1: Illustration of Compton scattering. An incident photon of energy E_γ and momentum hk scatters off of an electron at rest. The photon emerges at angle θ with reduced energy E'_γ and momentum hk' . The electron is ejected with energy E_{e^-} and momentum p_{e^-} .

Analyzing the collision and conserving relativistic energy and momentum, one can readily determine the exiting photon’s energy as a function of its energy and ejection angle:

$$E'_\gamma = \frac{mc^2}{(1 - \cos \theta) + mc^2/E_\gamma} \quad (16.1)$$

Here mc^2 is the electron rest mass, approximately 511 keV. This formula might appear more familiar in terms of the incident and exiting photon wavelengths:

$$\lambda' = \lambda + \frac{h}{mc} (1 - \cos \theta) \quad (16.2)$$

In short, a gamma enters the detector crystal and Compton scatters off of an electron which is then detected by scintillation. The Compton-scattered gamma leaves the detector, and the amount of detected energy is thus the kinetic energy given to the electron, or the difference between the incident and exiting photon energies. Compton scattering necessarily results in a small energy transfer than the photoelectric process, where the entire incident photon energy is transferred to an electron. The maximum possible energy transfer defines the so-called ‘‘Compton edge,’’ and occurs when the gamma is scattered backwards at $\theta = 180^\circ$. This maximum energy transfer is thus:

$$E_{ce} = E_\gamma - E'_\gamma(\theta = 180^\circ) = \frac{2E_\gamma^2}{mc^2 + 2E_\gamma} \quad (16.3)$$

The Compton edge represents the *maximum* energy transfer to an electron, or the maximum energy *loss* by the photon. Of course, the collision may be considerably more gentle and result in a smaller energy transfer. Considering all possible collisions, one arrives at a broad distribution of events at energies less than the Compton edge, and a cutoff at the edge itself. For a 662 keV ^{137}Cs gamma, the Compton edge occurs at approximately 478 keV.

Still one more effect of Compton scattering must be considered. Gammas may also interact with electrons outside the detection region, and these electrons may be Compton-scattered back into the detector and be detected by the photoeffect. Only a few angles near 180° will result in electrons entering the detector, which results in a peak in the energy spectrum at an energy corresponding to that of the exiting photon:

$$E_{bsc} = E_\gamma - E_{ce} = E'_\gamma(\theta = 180^\circ) = \frac{E_\gamma}{1 + 2E_\gamma/mc^2} \quad (16.4)$$

Essentially, the backscatter peak arises from absorption in the detector of scattered gammas from Compton interactions in the material surrounding the scintillator, mainly in the glass layer between the crystal and the cathode of the photomultiplier. For a 662 keV ^{137}Cs gamma, the backscatter peak should occur at approximately 184 keV. In Fig. 16.2 we show what the gamma ray spectrum from ^{137}Cs might look like as measured with an *ideal* detector (in which backscattering does not occur). At the lowest energies, an x-ray emission from the ^{137}Ba daughter is visible, followed by the Compton spectra. The Compton spectrum increases monotonically until the Compton edge is reached, after which the photopeak is observed. The real ^{137}Cs spectrum, however, shows several other features, most notably the backscattering peak.

In the analysis of complex scintillation spectra, care must be taken to identify backscatter peaks and Compton electron distributions to avoid confusing this with photoelectron peaks. It is also notable that the photopeak obtained with a real detector is broadened compared to the ideal case. The amount of broadening depends on the properties of the particular scintillator and photomultiplier

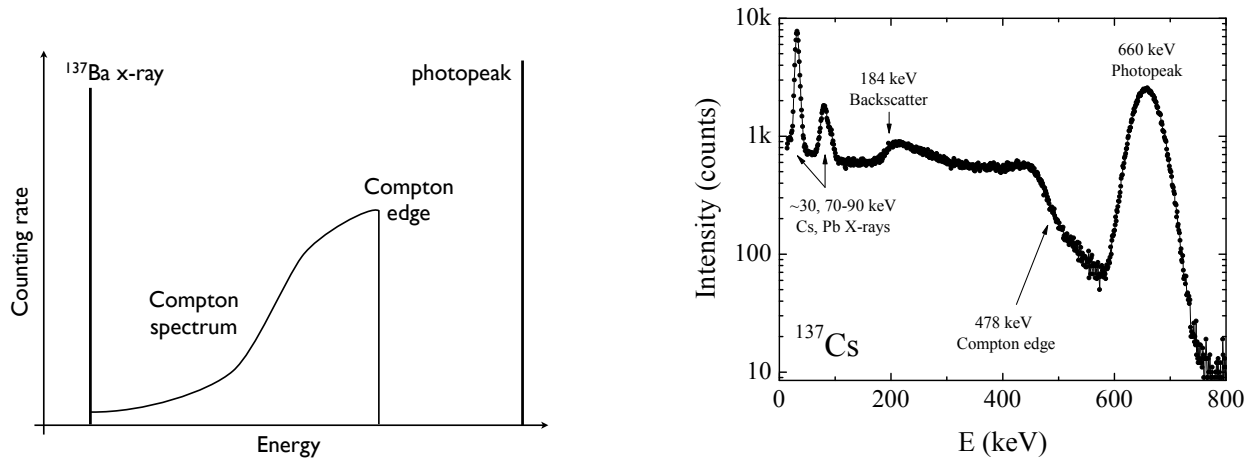


Figure 16.2: Left: Gamma spectrum of ^{137}Cs for an ideal detector and no backscattering. Right: Measured gamma spectrum of ^{137}Cs . Aside from the main photopeak at ≈ 660 keV, clear Compton edge and backscatter peak are visible, as well as X-ray absorption edges. Note the logarithmic scale on the vertical axis.[3]

used. A quantitative measure of the broadening is the resolution, which is calculated as the ratio of the width of the photopeak at one half its maximum intensity (the full width at half-maximum, or FWHM) divided by the pulse height (or energy where the maximum occurs, expressed as a percentage).

16.1.3 Pair Production

The antiparticle of the electron (e^- or β^-) is the positron (e^+ or β^+). When a positron and electron collide, they annihilate each other and produce two photons, each of energy corresponding to their rest energy of 511 keV and traveling in opposite directions to conserve linear momentum. Some radioactive isotopes, particularly those with proton-rich nuclei, decay by positron emission. This reduces their nuclear charge and atomic number by one. The 511 keV photons (gammas) from the subsequent annihilation of the positron can be observed in gamma ray spectroscopy, clear evidence of nuclear annihilation.

A positron can also be produced along with an electron as a pair by a gamma ray with energy greater than two electron masses (1.022 MeV). That is, the photon is annihilated into an electron-positron pair. This process conserves charge as well as linear and angular momentum, and is usually referred to as pair production. Any excess energy above 1.022 MeV is found as kinetic energies of the pair. After pair production, the positron will be annihilated by another electron, producing two 511 keV gammas. This is shown schematically in Fig. 16.3.

Pair production and positron emission have a number of effects on gamma ray spectra. Annihilation will produce two 511 keV gammas moving in opposite directions, only one of which may possibly enter your detector if the annihilation occurs outside the detector. Positrons can only travel very

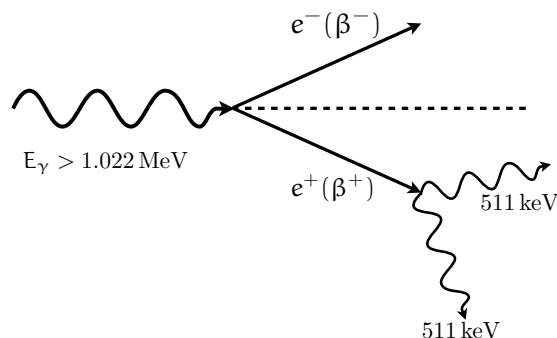


Figure 16.3: Illustration of pair production. A gamma enters the detector from the left with energy $E_\gamma > 1.022 \text{ MeV}$, producing an electron-positron pair in the detector. The positron is annihilated producing two 511 keV gammas.

short distance before being annihilated, meaning annihilation almost always occurs in the source material. The result is a 511 keV photopeak and Compton distribution in the gamma spectrum, as shown in Fig. 16.4.

Gammas with energies greater than 1.022 MeV can enter your detector and interact by pair production, producing a positron inside the detector, as shown in Fig. 16.3. Annihilation occurs so rapidly that the light produced by the two 511 keV gammas combines with the light produced from the total kinetic energies of the electron and positron to produce a pulse that represents the energy of the original gamma. This appears as a photopeak pulse. Sometimes one or both of the 511 keV gammas escape the detector, and this creates photopeaks that are missing 511 keV of energy (first escape peak) or 1.022 MeV (second escape peak).

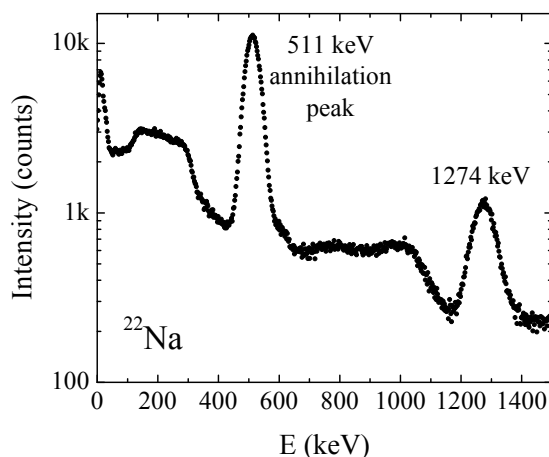


Figure 16.4: Gamma spectra of ^{22}Na , showing a 511 keV annihilation peak.

Two other mechanisms are worth noting, though they do not change the essential physics of the problem.ⁱⁱ A fourth mechanism, Rayleigh scattering, corresponds to the elastic (photon-energy-

ⁱⁱOf course, there are still other mechanisms, such as nuclear-resonance scattering and Delbrück scattering, but

conserving) scattering of light, which simply changes the direction of incident photons. Since no energy is lost, this process will merely reduce the measured intensity of radiation (because some photons will be deflected out of the detection region) and will not alter the spectrum.

Finally, in principle must also consider direct absorption of incident photons by the nucleus itself, photonuclear absorption. This process usually results in the ejection of one or more neutrons and/or protons. This interaction can contribute $\sim 5-10\%$ to the total photon interaction, though within a fairly narrow energy region usually occurring somewhere between 5 MeV and 40 MeV. At the energies of the present experiment (< 2 MeV), this effect is negligible.

16.2 Objective

The objective of this experiment is to apply understand the interaction of high-energy gamma-ray photons with matter. You will perform spectroscopic measurements on known materials to identify and explain the key features in the spectra, and apply your knowledge to the determination of an unknown material by analysis of its gamma spectrum. You will also be able to determine the rest mass-energy of electrons.

Hypothesis: Gamma ray spectra are characteristic of particular nuclei, and may be used as an analytical tool. The spectra can be explained by considering the photoelectric effect, Compton scattering, pair production, and characteristic x-ray emissions.

16.3 Preparatory Questions

You should touch on these questions in your report.

1. How does a photomultiplier tube work?
2. How does a scintillation detector work?

16.3.1 Relevant Reading

Taylor[1], Ch.

Pfeffer & Nir[2], Ch.

http://en.wikipedia.org/wiki/Gamma_ray

See [5-7] for relevant nuclear data and [8] for example gamma spectra.

16.4 Supplies

1. Spectech UCS30 spectrometer

they are negligible from our point of view.

2. NaI(Tl) detector
3. Multi-channel analyzer (MCA) / control electronics
4. Pb cylinder for shielding
5. radioactive sources
6. lab notebook and USB drive for saving spectra

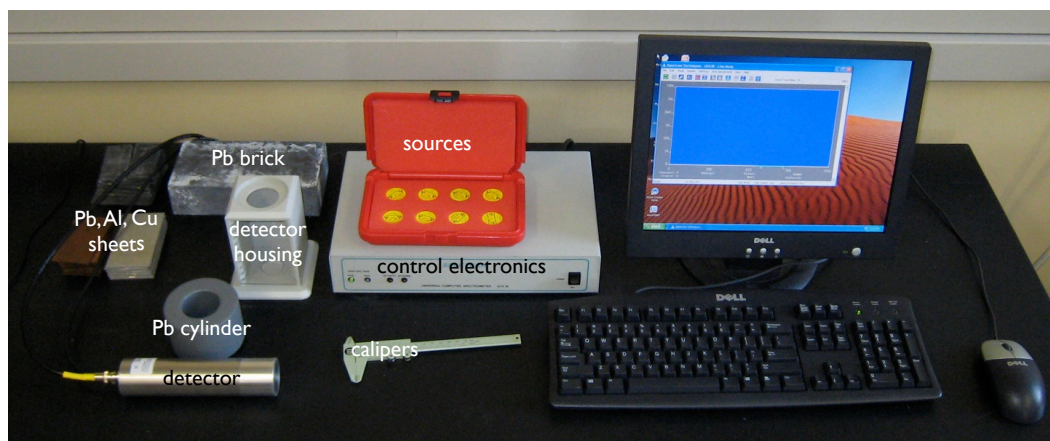


Figure 16.5: The Spectech UCS30 spectrometer and accessories.

16.5 Suggested procedure

16.5.1 Startup

1. Turn on the power to the Spectech UCS30
2. run the program “USX USB” from the shortcut on the desktop
3. place the ^{60}Co source on top of the table, and place the lead cylinder around it
4. place the cylindrical NaI detector inside the lead cylinder
5. Turn the high voltage on and set to 650 V.
6. Expose the detector to the ^{60}Co source. Set the gain in UCS30, by adjusting the coarse gain and fine gain, such that two peaks (the 1173 and 1333 keV photopeaks) are visible in the collected spectrum on the right hand (high energy) side. If these peaks are too far to the left, you will have insufficient dynamic range for your measurements.

16.5.2 Running the experiment

1. Once you have set the detector to a reasonable gain, giving you sufficient dynamic range to detector the gamma radiation of all sources, perform an energy calibration. This is done by accumulating gamma ray spectra with a sufficient number of sources. Press the green diamond to start collecting data and the red octagon to stop, and use the “X” to clear the spectrum. Before clearing a spectrum to take the next measurement, save the data to the

computer's hard disk (to be transferred to your thumb drive) as a Comma Separated File so you can get back to the data and analyze it in Excel on your own computer. Collect data for at least 5 minutes for each source. To analyze your data highlight the characteristic peaks that become visible. These correspond to the full absorption of the radiation emitted in the decay of the excited nuclear states. When you hold the pointer over the highlighted areas the computer will display the number of accounts within the highlighted area, the average ADC count (sometimes called *peak centroid*, and the FWHM (Full Width at Half Maximum) value. FWHM is another measure for the standard deviation, σ , of a Gaussian. They relate to each other as $FWHM = 2 \cdot \sqrt{2 \cdot \ln(2)} \approx 2.36 \cdot \sigma$. Further record the counting time of each source measurement. Record all these data in an Excel spread sheet. You will use Excel for your data analysis.

Using Excel plot the measured gamma ray peak centroid (in ADC counts) versus their tabulated energy values. You will have to look up the known energies of the gamma lines on the web. The instructor can help you figure out how. Tip: the US national labs are wonderful sources of online data. The data points you collected should more or less line up on a straight line. The slope of this line gives you the energy calibration. If you are adventurous try to describe the data with a parabola. Does this give a better fit to the experimental data? The tabulated data and the energy calibration plot with fit need to be part of your written report. In your Excel spread sheet convert the FWHM values into σ . Plot σ as a function of their corresponding peak centroids. Is there a trend? Can you fit the data with some appropriate mathematical function? Explain your data and choice of fit function in your report.

- Put the unknown source on the detector and collect data for 5 minutes. Using your newly found energy calibration identify the radio nuclide(s) contained in this radioation source. Your report needs to explain how you identified the unkonwn material.

16.6 Data analysis

For each source, measure the photopeak, the Compton edge, and the backscatter peak energies. The Compton edge is at an energy corresponding to about half the change in the count rate from the trough between the photopeak and edge to the immediate maximum in the edge. The photopeak should be easily identified, and the backscatter peak should be more triangular and asymmetric in shape, with the peak flat or slightly rounded. The energy of the backscatter peak corresponds to the lowest energy of the top of the peak. Test your energy measurements by verifying that

$$E_{\gamma} = E_{ce} + E'_{\gamma} = E_{bsc} + E_{ce} \quad (16.5)$$

Compute the position of the Compton edges and backscatter peaks for each gamma and indicate their ideal locations on the spectra.

Solve Eq. 16.3 for mc^2 as a function of E_{γ} and E_{ce} . Using the photopeak and Compton edge en-

ergies, calculate the mass energy of the electron and the measurement error in that mass. Average your results to obtain one value for the electron's rest mass energy and the standard deviation in that value.

Compute the percent resolution for each peak.

The resolution should improve (get smaller) as the energy increases. Is that what you observe? Further, if the distribution of channel numbers for a given energy deposit truly follows gaussian statistics, the resolution is proportional to $E^{-1/2}$. How does the expectation of gaussian statistics compare with your observations?

For each of the unknown sources:

1. Estimate the energies of all prominent peaks. (If one of the peaks has an energy consistent with 511 keV, you may assume that it is the annihilation peak and thus that the parent emits positrons.)
2. Identify the parent. Show your reasoning. You may use the on-line table of gamma energies, in <http://nucleardata.nuclear.lu.se/toi/radSearch.asp>. (Use your uncertainty in the measurement of the peak energy as a guide in setting the width of the window of gamma ray energies in which you search.) You may assume that the half-life of the unknown source is at least 1 month.
3. If there are several candidates, look at the list of gamma energies and corresponding branching ratios for each candidate. Are there any other gammas with a comparable or larger branching ratio than the ones you see? (The branching ratio is labeled as $I_g(\%)$ in the on-line tables.) The branching ratio indicates the fraction of decays in which the gamma is emitted, smaller is less likely.) If so, are peaks present in the spectrum corresponding to these gammas? If not, the candidate is ruled out.
4. Identify the source of each peak, e.g. gamma from nuclear decay, sum of two gammas from nuclear decay, sum of a gamma from nuclear decay and annihilation gamma, etc.

16.7 Format of report

You may choose one of the following formats for your report:

1. two-page memo (excluding required plots)
2. group oral presentation
3. formal written lab report
4. research proposal

Further details and templates are provided for each of these formats. Keep in mind that over the course of all of the one-week experiments you must do each type of report (and two formal lab reports).

16.8 Discussion and topics for your report

The backscatter peak in each spectrum should change in intensity with the addition of a lead sheet. Could any other material work as well? It is the product of the electron density and the material thickness that correlates directly with the intensity of the backscatter peak.

Gamma sources available

Table 16.1: Disk gamma source set

Isotope	Activity (μCi)	Half-life	Peaks (keV)
^{133}Ba	1	10.8 yrs	81, 276, 303, 356, 384
^{109}Cd	1	462 days	88
^{137}Cs	1	30.2 yrs	662
^{57}Co	1	272 days	122, 136
^{60}Co	1	5.27 yrs	1173, 1333
^{54}Mn	1	313 days	835
^{22}Na	1	2.6 yrs	511, 1275
^{207}Bi		31.6 yrs	569.7, 894, 1063.66, 1770.2
unknown	1.5	?	?

Sources from Spectrum Techniques, Oak Ridge, TN (www.spectrumtechniques.com). USNRC and state license exempt quantities.

Decay of Radon Daughters from Air

In this experiment you will study natural radioactivity and the radioactive decay law. The investigation of a complex decay sequence will allow you to utilize coupled differential equations in the description of the data. Decay data of unstable radon daughters, sampled from the air, will be used to determine the concentration of the radioactive noble gas ^{222}Rn in the air and therefore demonstrate the presence of *natural radioactivity* in your environment. To do that radon (^{222}Rn) decay products are collected by sucking a known volume of air through a filter. Their subsequent decay will be measured by counting the filter with a NaI scintillation detector. This detector will measure the gamma radiation emitted in the decay of Rn daughters. The collected data will be used to determine the activities of the Rn daughters by means of statistical analysis of the gamma-ray spectra. You will then infer the specific radon activity of the air.

17.1 Objectives

Determine the specific ^{222}Rn activity of air. Determine the half lives of the radon daughters ^{214}Pb and ^{214}Bi (optional).

17.2 Tools

- A calibrated blower and filter are used to catch radon-daughter loaded aerosols from the air.
- A NaI scintillation detector is used to measure the gamma radiation emitted by the filter.
- Statistical data analysis is performed to determine the time dependent activity content of the filter, allowing to infer the specific Rn-activity of the air and the half lives of the radon daughters.

17.3 Tasks

1. Read and understand the experiment description.
2. Solve the pre-lab problems given in this write-up.
3. Collect Radon decay daughters from the air on a filter.
4. Count the filter in order to quantify the activities of the unstable Radon daughters as a function of time.

5. Determine the half lives and activities of ^{214}Pb (and ^{214}Bi) by means of a statistical analysis. In your write-up you have to show graph of the ^{214}Pb (and ^{214}Bi) activity versus time since end of filtering.
6. Determine the Rn activity per unit air volume from your data.

17.4 Introduction

Radioactivity is the spontaneous disintegration of an unstable atomic nucleus and the emission of particles, sometimes together with very energetic electromagnetic radiation. Ionizing radiation is emitted during radioactive decays. This is what is measured. Radioactivity is categorized into α or β -decay or spontaneous fission.

The earth is radioactive and has been since it formed 4.54×10^9 *y* ago. Many radio nuclides can be found in the environment. They are generally categorized as:

1. Natural or primordial - from before the formation of the earth
2. Cosmogenic - formed as a result of cosmic ray interactions
3. Human produced or anthropogenic - enhanced or formed due to human actions (typically minor amounts compared to natural, except after *incidences*)

Radio-nuclides are found in the soil, air, and water. They are even found in us, being that we are products of our environment. Every day, we ingest and inhale radio-nuclides in our air, food, and the water. Natural radioactivity is common in the rocks and soil that makes up our planet, in water and oceans, and in our building materials and homes. There is nowhere on earth that you can not find natural radioactivity. The radio-nuclides ^{40}K , ^{232}Th , and ^{238}U (along with their decay products) are called *natural radioactivity*. Their half lives (the time taken for the radioactivity of a specified isotope to fall to half its original value) are with 1.28×10^9 *y*, 1.41×10^{10} *y*, and 4.49×10^9 *y* comparable to the age of the earth and they have thus not yet decayed away.

17.4.1 Units of Radioactivity

There is a confusing multitude of units measuring different things. The nuclear decay rate is measured in units of Becquerel (*Bq*). 1 *Bq* is equivalent to one decay per second. This is the SI unit for activity. Sometimes you will encounter an outdated unit for activity the Curie (*Ci*). The Curie is a historical unit, defined as the activity of 1 g of the radionuclide ^{226}Ra . This definition is not very practical for standard keeping and comparison. $1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$. The activity *A* is proportional to the amount of radionuclide present in the sample. If the sample contains *N* atoms of a radionuclide with a mean life time of $\tau = \frac{T_{1/2}}{\ln(2)}$ then:

$$A = \frac{N}{\tau}. \quad (17.1)$$

The *rad* (radiation absorbed dose) is a unit used to measure a quantity called absorbed dose. This relates to the amount of energy actually absorbed in some material, and is used for any type of radiation and any material. It measures the effect of the radiation on matter. One *rad* is defined as the absorption of 100 *ergs* ($1 \text{ erg} = 10^{-7} \text{ J}$) per *gram* of material. The unit *rad* does not describe the biological effects of the different radiations. The modern SI unit for radiation exposure is the *Gray* (Gy), defined as an energy deposit of 1 J/kg. *rad* and *Gy* measure the same effect, their relation is: $100 \text{ rad} = 1 \text{ Gy}$.

The unit *röntgen* (R), adopted in 1928, measures the exposure of *air* to ionizing radiation in units of the free charge induced per volume of air. It is defined as the charge of one *esu* per cm^3 of dry air at standard temperature and pressure. It is now stated in terms $1 \text{ R} = 2.58 \cdot 10^{-4} \text{ C/kg}$. Because this unit is a measure of the exposure of air only it is not used as often as *rad*.

Sometime you will find the unit *rem* (roentgen equivalent man). The *rem* is a unit used to derive a quantity called equivalent dose. This relates the absorbed dose in human tissue to the effective biological damage of the radiation. Not all radiation has the same biological effect, even for the same amount of absorbed dose. Equivalent dose is often expressed in terms of thousandths of a *rem*, or *mrem*. The unit *rem* has been replaced by the modern SI unit *Sievert* (Sv), with the definition $1 \text{ rem} = 0.01 \text{ Sv}$. To determine the equivalent dose (*rem*), you multiply absorbed dose (*rad*) by a quality factor (Q) that is unique to the type of incident radiation. The following quality factors are applied:

- Photons, all energies : $Q = 1$
- Electrons and muons, all energies : $Q = 1$
- Protons and charged pions : $Q = 2$
- Neutrons : Q is a continuous function of neutron energy
- Alpha particles and other atomic nuclei : $Q = 20$

The natural background effective dose rate varies considerably from place to place, but typically is around 2.4 mSv/year or 240 mRem/y.

In this lab we will determine activities in *Bq*. We will not work with absorbed dose or equivalent dose. These units have only been stated for your orientation.

17.4.2 Natural Radioactivity

As already stated in section 17.4 the radio-nuclides ^{40}K , ^{232}Th , and ^{238}U are called natural radioactivity. ^{40}K undergoes beta decay into ^{40}Ca which is stable. The time (t) dependence of the number of K-atoms, $N(t)$, is governed by the well known radioactive (exponential) decay law. The exponential decay law is a consequence of the requirement that the decay probability of each atom is equal and independent of its age. Let $\frac{dN}{dt}$ be the decay rate. As the decay probability per unit

time is equal at any time and for any atom the decay rate has to be proportional to the number of unstable atoms, thus N . This proportionality can be converted into an equality by introducing a constant of proportionality. To match the units on both sides the constant of proportionality needs to have dimension $time^{-1}$, we use the inverse mean life time $1/\tau$:

$$\begin{aligned} \frac{dN}{dt} &= -\frac{N}{\tau} \\ \Rightarrow \frac{dN}{N} &= -\frac{dt}{\tau} \\ \Rightarrow \int_{N(0)}^{N(t)} \frac{dN}{N} &= -\frac{1}{\tau} \cdot \int_0^t dt' \\ \Rightarrow [\ln N]_{N(0)}^{N(t)} &= -\frac{t}{\tau} \\ \Rightarrow \ln N(t) - \ln N(0) &= \ln \frac{N(t)}{N(0)} = -\frac{t}{\tau} \\ \Rightarrow N(t) &= N(0) \cdot e^{-\frac{t}{\tau}} \end{aligned} \quad (17.2)$$

$N(0)$ denotes the number of atoms present at time zero $t = 0$. So the assumption of equal decay probability and simple integration of the resulting first order linear differential equation yields the well known radioactive (exponential) decay law.

For ^{232}Th , and ^{238}U the situation is more complicated: something unstable decays into something

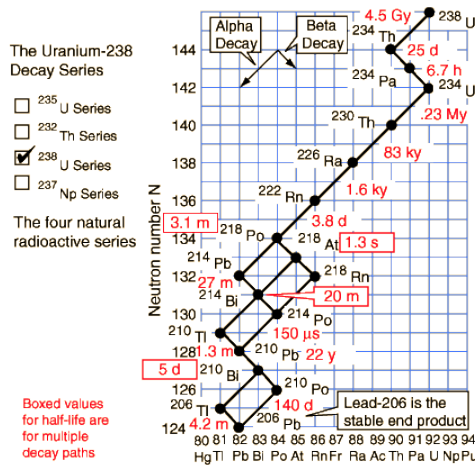


Figure 17.1: ^{238}U decay series.

that is again unstable and so forth. This situation is illustrated for ^{238}U in figure 17.1. ^{238}U decays into ^{206}Pb but via many unstable intermediate radio-nuclides.

The radioactive noble gas ^{222}Rn ($T_{1/2}=3.8$ d) is a member of the ^{238}U decay series. It can be found in trace concentrations everywhere, as it is gaseous and relatively long lived. It is released by minerals into the air where it exists in some equilibrium concentration. There is another species of Rn present in the air: the ^{232}Th decay series, also naturally present in our environment, has ^{220}Rn

($T_{1/2}=55.6$ s) as a member. Due its short half live its equilibrium concentration is much lower than that of ^{222}Rn . We can hence neglect its effect in this experiment. Airborne Rn-gas is one of

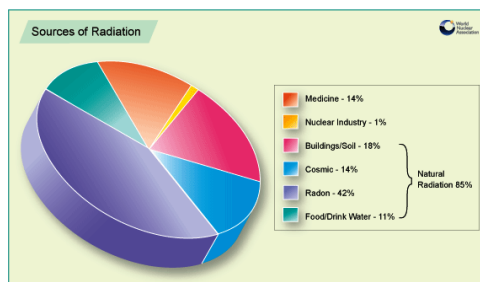


Figure 17.2: Fractional contribution of various radiation sources to the dose rate received by humans.

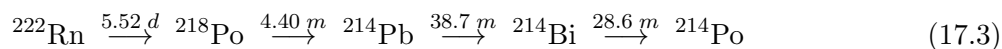
the principal components of our natural radiation exposure, as shown in figure 17.2. Depending on geographical location about half of your radiation exposure is due to radon gas!

Problem 1:

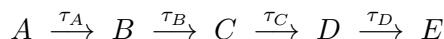
Show that the activity $A = -\frac{dN}{dt}$, or number of decays per unit time, of a radioactive species is given as $A = \frac{N}{\tau}$, where τ denotes the mean life time.

17.5 Radon Decay

Rn decays into unstable elements having short half lives. Equation 17.3 shows part of the ^{222}Rn decay sequence. This sequence offers a convenient way to study radioactive decays in the laboratory without the need to produce costly isotopes at a particle accelerator or by neutron activation at a nuclear reactor. Nature delivers them to us gratis! Here part of the decay sequence (mean life times are given too):



or



^{222}Rn and ^{218}Po undergo α -decay, ^{214}Pb and ^{214}Bi beta decay. During the beta decays of ^{214}Pb and ^{214}Bi energetic gamma radiation is emitted. This is not so for the alpha decays. The NaI detector used in our experiment is only sensitive to gamma radiation and thus cannot measure ^{222}Rn decays directly. It is only sensitive to the radon decay products ^{214}Pb and ^{214}Bi . You will be able to measure activities C and D with the NaI detector. Two practical questions arise:

1. How do we determine the ^{222}Rn activity from measured ^{214}Pb and ^{214}Bi activities? Is there a reason why they should be the same?

2. What time dependence do we expect to observe for the ^{214}Pb and ^{214}Bi decay rates?

17.5.1 Sequential Radioactive Decays

As explained above the radon decay products are filtered out of the air. Radon itself is a noble gas and will thus not be collected by a filter. How are the radon decay products captured?

The ^{222}Rn decay product, in this case ^{218}Po , is originating from an alpha-decay and left in a doubly charged (ionized) atomic state after its birth. The charged Po atoms tend to adhere to small dust particles (aerosols) suspended in the air. In order to collect the aerosols we will suck a calibrated volume of air through a filter. A suitable calibrated blower is available. Our filter then catches the aerosols and together with them the Po. The ^{218}Po (from here on called activity B) α -decays quickly within a few minutes. You are then left with ^{214}Pb (activity C) which β -decays into ^{214}Bi (activity D), which is β -unstable and decays into ^{214}Po . What we want to know is the ^{222}Rn activity (A). The collected ^{218}Po is unstable. So some of it will already decay while still running the blower. This poses a third question:

- 3 How do we quantify the fraction of the radon decay products that were collected but decayed before we started counting?

When starting the blower no radon decay products are present on the filter. ^{218}Po is collected and decays into ^{214}Pb and ^{214}Bi , the nuclides that are actually being measured. We thus need to determine how to relate the observable quantities (^{214}Pb and ^{214}Bi activities) to amount of filtered gas, filtering time, counting time.

17.5.2 Mathematical Treatment of Sequential Decays

17.5.2.1 Collection Phase

Let's examine the math describing both build-up and decay of the activities during the filtering. When the blower is *on*, activities B, C and D are building up on the filter. At the same time some of it already decays because these elements are unstable. There is no buildup of activity A, the gaseous radon, on the filter. We thus don't need to set up an equation for radionuclide A. The build up of species B, C, and D is described by the following differential equations:

$$\frac{dN_B}{dt_f} = k \cdot n_B - \frac{N_B}{\tau_B} \quad (17.4)$$

$$\frac{dN_C}{dt_f} = k \cdot n_C + \frac{N_B}{\tau_B} - \frac{N_C}{\tau_C} \quad (17.5)$$

$$\frac{dN_D}{dt_f} = k \cdot n_D + \frac{N_C}{\tau_C} - \frac{N_D}{\tau_D} \quad (17.6)$$

Here N_B , N_C , and N_D are the numbers of atoms of species B, C and D on the filter (unitless). n_A , n_B , n_C , and n_D are the numbers of atoms of species A, B, C and D per unit volume of air (unit $[l^{-1}]$). k is the volume of air passing through the filter per unit filtering time t_f (unit is $[l/s]$). We

are assuming that the collection efficiency for nuclide A is 0% (gas doesn't get stuck in a filter), and that of nuclide B to be 100%. You also note that the positive terms ($k \cdot n$) used in all three equations assumes equal efficiency for catching all three decay products of Rn on the filter. This is in fact not a very good assumption but we will stick with it to simplify the math.

How do we come up with these equations? Let's examine equation 17.5. $\frac{dN_C}{dt}$ describes the change of the number of nuclei of species C . It is hence a measure of the *decay rate* (unit is $[Bq]$ equivalent to $[s^{-1}]$). This is what we can directly measure with our radiation detector. The parent concentrations have then to be *inferred* from this data using an appropriate set of assumptions. We assume:

- The rate with which we capture species C from the air is proportional to the blower speed, $k=0.85 \text{ m}^3/\text{min}$, and the concentration, n_C , of species C in the air. The term $k \cdot n_C$ is positive (it is a gain) and measures this collection.
- While filtering is still in progress the sample of species C is already decaying. As in all radioactive decays the decay rate is proportional to the number of atoms on the filter. The shorter the live time, τ_C , the more "unstable" the sample is. Hence the decay rate has to be inversely proportional to τ_C . This term is negative as it represents a loss.
- Species B is collected on the filter in parallel. It is decaying as well while filtering is still in progress. The decay of species B leaves species C behind. The decay rate of B , as given in equation 17.4 has hence to be booked as a gain (with "+" sign) for B . This couples the equations.

This set of assumptions allows us now to determine the elemental concentrations n_A , n_B , and n_C from the measured decay rates. Clearly, differential equations are needed to describe this situation.

Above equations 17.4, 17.5, 17.6 are solved by the following functions (remember that t_f measures the flow of time during filtering):

$$N_B(t_f) = k \cdot n_B \cdot \tau_B \cdot (1 - e^{-t_f/\tau_B}) \quad (17.7)$$

$$N_C(t_f) = k \cdot \tau_C \cdot (n_B + n_C) \cdot (1 - e^{-t_f/\tau_C}) + \frac{k \cdot n_B \cdot \tau_B \cdot \tau_C}{\tau_B - \tau_C} \cdot (e^{-t_f/\tau_C} - e^{-t_f/\tau_B}) \quad (17.8)$$

$$\begin{aligned} N_D(t_f) = & k \cdot \tau_D \cdot (n_B + n_C + n_D) \cdot (1 - e^{-t_f/\tau_D}) + \\ & \frac{k \cdot \left(\frac{n_C}{\tau_C} - \frac{n_B}{\tau_B} - \frac{n_C}{\tau_B}\right) \cdot \tau_B \cdot \tau_C^2 \cdot \tau_D}{(\tau_B - \tau_C) \cdot (\tau_C - \tau_D)} (e^{-t_f/\tau_D} - e^{-t_f/\tau_C}) + \\ & \frac{k \cdot n_B \cdot \tau_B^2 \cdot \tau_D}{(\tau_B - \tau_C) \cdot (\tau_B - \tau_D)} \cdot (e^{-t_f/\tau_D} - e^{-t_f/\tau_B}) \end{aligned} \quad (17.9)$$

The appendix shows how these solutions are obtained.

Problem 2:

Show that function 17.8 is a solution of equation 17.5. You can do this by either substitution of the solution into the equation or by solving the equation using the method outlined in the appendix.

The time dependent activities A_X are obtained by dividing the number of atoms N_X by the respective life time τ_X . The activities of species B, C, and D at the end of filtering is thus obtained by dividing by the respective mean life times. These equations can be further simplified using some numerical values. We will use a filtering time of $t_f = 30 - 60$ m. The mean life times are given in equation 17.3. Thus $t_f \gg \tau_B$. The exponential containing the ratio of these two values can be neglected. We further assume that the activities of species A, B, and C in the atmosphere are in *secular equilibrium*, meaning are equal at all times. This approximation is well justified and explained in the appendix. It means the decay rate is determined by the long lived mother (in our case ^{222}Rn) of the decay series and is *equal* for all members of the series:

$$\frac{n_A}{\tau_A} = \frac{n_B}{\tau_B} = \frac{n_C}{\tau_C} \quad (17.10)$$

The activities at the end of filtering are given (to good approximation):

$$A_B(t_f) = k \cdot n_B \quad (17.11)$$

$$A_C(t_f) = k \cdot (n_B + n_C) \cdot (1 - e^{-t_f/\tau_C}) + k \cdot n_B \cdot \frac{\tau_B}{\tau_B - \tau_C} \cdot e^{-t_f/\tau_C} \quad (17.12)$$

$$A_D(t_f) = k \cdot (n_B + n_C + n_D) \cdot (1 - e^{-t_f/\tau_D}) - \frac{k \cdot n_C \cdot \tau_C^2}{(\tau_B - \tau_C) \cdot (\tau_C - \tau_D)} (e^{-t_f/\tau_D} - e^{-t_f/\tau_C}) + \frac{k \cdot n_B \cdot \tau_B^2}{(\tau_B - \tau_C) \cdot (\tau_B - \tau_D)} \cdot e^{-t_f/\tau_D} \quad (17.13)$$

We define the activities $A_C(t_f)$ and $A_D(t_f)$ as those present on the filter *at the end of filtering*.

For the data analysis it is useful to make use of the chain equilibrium:

$$\begin{aligned} \frac{n_B}{\tau_B} &= \frac{n_C}{\tau_C} \\ \Rightarrow n_C &= \frac{\tau_C}{\tau_B} \cdot n_B \end{aligned}$$

Substitute this into above equation for $A_C(t_f)$:

$$A_C(t_f) = k \cdot n_B \cdot \underbrace{\left[\left(1 + \frac{\tau_C}{\tau_B}\right) \cdot (1 - e^{-t_f/\tau_C}) + \frac{\tau_B}{\tau_B - \tau_C} \cdot e^{-t_f/\tau_C} \right]}_{\equiv f(t_f)} \quad (17.14)$$

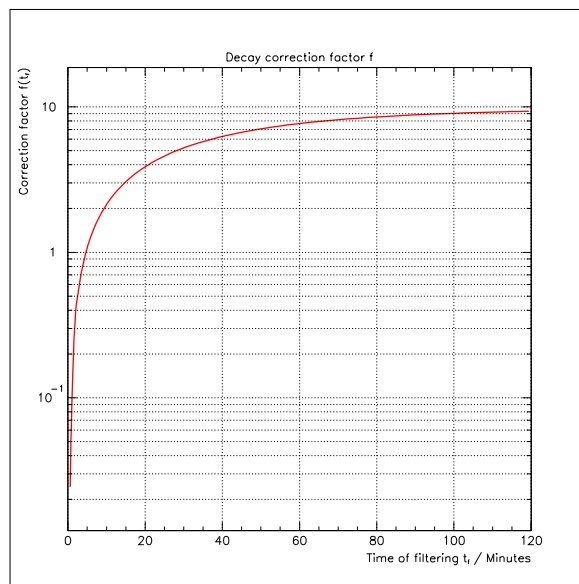


Figure 17.3: Graphical representation of the correction factor f as a function of the filtering time t_f .

The initial activity hence depends on some decay factor $f(t_f)$ that can be calculated using tabulated life times. You will use an Excel spread sheet to keep track of these activities during the experiment and to calculate $f(t_f)$ for your particular filtering time. Figure 17.3 shows how the f-factor depends on the length of time the filtering goes on. As you can see from figure 17.3 the correction factor can be quite large. Neglecting this correction would thus lead to a substantial error in the ^{222}Rn specific activity determined from a measured ^{214}Pb activity.

The equilibrium condition thus allows us to calculate the Radon decay rate n_A/τ_A from a derived concentration of activity B obtained from the measured activity C (^{214}Pb) on the filter at the end of filtering.

$$\begin{aligned} \frac{n_A}{\tau_A} &= \frac{n_B}{\tau_B} \\ &= \frac{A_C(t_f)}{k \cdot f(t_f) \cdot \tau_B} \end{aligned} \quad (17.15)$$

This is how we will determine the Radon activity of the air: calculate the correction factor $f(t_f)$ using equation 17.14. You have to keep track of the flow of time to get this right. You will measure the ^{214}Pb activity present on the filter at t_f , the end of filtering. Plug these two numbers into equation 17.15 to obtain the final result.

17.5.2.2 Counting Phase

What follows is a description of how to model the ^{214}Pb activity A_C as a function of counting time t_d . This activity is not *constant* in time, as seen with the calibration sources, but goes down as time goes on and relatively quickly. This obviously needs to be described properly in order to get a correct result.

In order to count the radon decay products, collected during the time t_f , the filter needs to be removed from the blower and placed in front of the NaI detector. As soon as the filtering process is stopped the activities are described by a different system of differential equations. The term k , describing the gain of activities, vanishes because the air flow is given turned off. We introduce a new decay time t_d , starting at zero at start of counting. We have:

$$\frac{dN_B}{dt_d} = -\frac{N_B}{\tau_B} \quad (17.16)$$

$$\frac{dN_C}{dt_d} = \frac{N_B}{\tau_B} - \frac{N_C}{\tau_C} \quad (17.17)$$

$$\frac{dN_D}{dt_d} = \frac{N_C}{\tau_C} - \frac{N_D}{\tau_D} \quad (17.18)$$

These equations are solved by the following functions (see appendix):

$$N_B(t_d) = N_B(t_f) \cdot e^{-t_d/\tau_B}$$

$$N_C(t_d) = N_C(t_f) \cdot e^{-t_d/\tau_C} - N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} \cdot \left(e^{-t_d/\tau_C} - e^{-t_d/\tau_B} \right).$$

$$\begin{aligned} N_D(t_d) = & N_D(t_f) \cdot e^{-t_d/\tau_D} - N_C(t_f) \frac{\tau_D}{\tau_C - \tau_D} \cdot \left(e^{-t_d/\tau_D} - e^{-t_d/\tau_C} \right) + \\ & N_B(t_f) \frac{\tau_D}{\tau_B - \tau_C} \cdot \left(\tau_C \frac{e^{-t_d/\tau_D} - e^{-t_d/\tau_C}}{\tau_C - \tau_D} - \tau_B \frac{e^{-t_d/\tau_D} - e^{-t_d/\tau_B}}{\tau_B - \tau_D} \right) \end{aligned}$$

As before the time dependent activities can be obtained by dividing the number of atoms by its corresponding life time. It is again assumed that $t_d \gg \tau_B$ or $e^{-t_d/\tau_B} \approx 0$.

$$A_B(t_d) = A_B(t_f) \cdot \underbrace{e^{-t_d/\tau_B}}_{\approx 0}$$

$$A_B(t_d) \approx 0$$

$$A_C(t_d) \approx A_C(t_f) \cdot e^{-t_d/\tau_C} \quad (17.19)$$

$$A_D(t_d) \approx A_D(t_f) \cdot e^{-t_d/\tau_D} - A_C(t_f) \frac{\tau_C}{\tau_C - \tau_D} \cdot \left(e^{-t_d/\tau_D} - e^{-t_d/\tau_C} \right) \quad (17.20)$$

This is true to good approximation. You note that the decay of species B does not play an important role here which simplifies the equations considerably. This is because B has a mean life time of only 4.4 min. It hence decays quickly. If you wait a few half lives after end of filtering before you start counting, then B will effectively have died out. In this limit equations 17.19 and 17.20 describe the data well. From a practical point of view: $A_B(t_d)$ is unmeasurable in our setup because it (a) doesn't live long enough and (b) it undergoes an α -decay which, in this case, doesn't result in emission of γ -radiation; and that's what we measure with our NaI detector.

In a practical sense you will determine $A_C(t_f)$ from a time fit to the ^{214}Pb data and then use it, together with the calculated $f(t_f)$ value to determine the specific Radon activity of the air.

You see from this discussion that the "exponential" decay is not really a law of nature. It is derived from the assumption that the decay probability only depends on the number of decaying nuclei and not their age. If you have a balance of production and decay the time development of the observed activity (or equivalently the of the emitted photon flux) may well be described by a more complicated function, which, however, always contains exponentials. This follows naturally from the same assumption, as shown in this experiment.

It is essential that you keep track of time in this experiments. Note the time you start and stop the blower, and the time you start counting the filter. Try to keep the delay between end of filtering and beginning of counting to a minimum, but keep in mind that the equation require a delay of at least 10 min to be valid.

17.6 Procedure

It is assumed that you are familiar with the basic data taking and analysis techniques used in the "Gamma Spectroscopy" experiment. If have trouble remembering consult the write-up of that experiment.

17.6.1 Data Collection

Put a new red filter into the blower. The lab supervisor will help you with this. Switch the fan *on* and read the air flow as measured with the flow meter on the back of the fan. If the flow meter doesn't work (sometimes the little ball gets stuck) use a flow of $0.85 \text{ m}^3/\text{min}$, which I determined earlier. Note the time you started the fan into your lab book. Collect aerosols for a period of 30-60 min. Switch the blower off and retrieve the filter. Note the time of end of filtering. Bring the loaded filter to the lab and put it in front of the NaI detector. Put it as close as possible. You should start counting the filter no more than 10 min after turning the blower off. Don't waste time! During the time you are collecting aerosols with the blower you should quickly re-check the energy calibration of the NaI detector. Use the ^{57}Co , ^{137}Cs and ^{60}Co sources for that. Apply the techniques and procedures described in the write-up of the experiment on "Gamma Spectroscopy". After the filtering is over you will measure the γ -ray flux emitted by the filter as a function of time. This will allow you to determine the wanted half lives. Collect 10 successive spectra using a measuring time of 10 minutes for each run. You will have to save each spectrum to disk and write down the starting time into your log book. Collect as many data files as possible, a minimum of 6.

17.6.2 Data Analysis

Use the *UCS* software for the data analysis. After you're done with the data analysis you will have to load your data files back into the software in order to determine the number of counts in the 351 keV peak (for ^{214}Pb) and the 609 keV peak for (^{214}Bi OPTIONAL). You will do this for each of your data files. Before embarking on this find on the web which peaks are to be expected in the ^{214}Pb and ^{214}Bi decays. This is just a sanity check. Note the number of counts in the respective peaks for each file into an EXCEL spread sheet. For every run note the number of events under the peaks of interest (N) and their respective errors (\sqrt{N}) into an Excel spread sheet. Divide the number of events and their errors by the counting time to arrive at an estimate for the peak counting rates $r(t_d)$ as a function of time. To do so you need to keep track of time and you know when you started data taking. To convert the measured counting rate $r(t_d)$ into an activity $a(t_d)$ you will need to make correction for the detector efficiency ε and the finite photon emission probability (called branching ratio) b . You will use the counting efficiency determined in the "Gamma Spectroscopy" experiment. Figure 17.4 shows an example efficiency curve with fit. This curve yields $\varepsilon = 0.034 \text{ counts}/\gamma$ for the 351 keV peak and $\varepsilon = 0.012 \text{ counts}/\gamma$ for the 609 keV line.

Calculate the time dependent activity of ^{214}Pb (and optionally ^{214}Bi) contained in the filter. Let $c(E_\gamma, t_d)$ be the integral of the peak you are going to analyze. This integral was measured at a time t_d for a peak energy E_γ . The measured activity of the decaying nuclide, $A(t_d)$ is given as:

$$A(t_d) = \frac{c(E_\gamma, t_d)}{\varepsilon \cdot b \cdot t}, \quad (17.21)$$

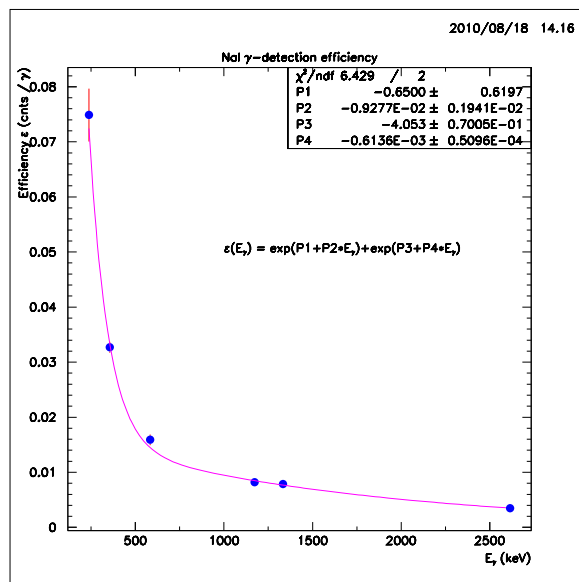


Figure 17.4: Detection efficiency of one of the lab NaI detectors (the one near the window).

with t denoting the duration of the data taking in [s]; yet another time you need to keep track of! Energy dependent values of ϵ are given in figure 17.4. Assume that the ϵ 's are free of error, for simplicity.

In this experiment you will only use your measurement of the peak energy to identify the appropriate peak and associated nuclear decay. The quantity of interest is the area under the peak. This is a measure of the strength of the gamma emitter. As pointed out before systematic corrections need to be made to get the activity from the peak integrals. You need to (a) convert into rates, (b) account for the fact that the characteristic gamma is only emitted in a fraction of the decays of interest (described by the *branching ratio*), and (c) that only some of the gammas emitted by the filter will result in a “count” under the full absorption peak (an energy dependent detection efficiency, containing both the geometrical factor and the finite interaction probability is given in figure 17.4). All this is summarized in equation 17.21.

As you go along in your data analysis assign a time in minutes since end of filtering to each run. A run has a certain length which means it corresponds to a time interval. In your analysis you need to assign a certain “point” in time to each run. How will you chose the time?

Problem 3:

What is the average time, $\langle t \rangle$, for a decay? To find the answer remember that an average over any function $f(t)$ is defined as:

$$\langle t \rangle = \frac{\int_0^\infty t \cdot f(t) dt}{\int_0^\infty f(t) dt} = \frac{\int_0^\infty t \cdot e^{-t/\tau} dt}{\int_0^\infty e^{-t/\tau} dt}$$

In our case the mid point between start and stop is a good choice.

How do we get the Radon activity quantitatively? Use:

$$\frac{n_A}{\tau_A} = \frac{A_C(t_f)}{k \cdot f(t_f) \cdot \tau_B}. \quad (17.22)$$

You have measured $A_C(t_f)$ by analyzing the time development of the 351 keV peak. You will need to calculate $f(t_f)$ for the specific filtering time t_f you have chosen ($\tau_B=4.4$ m).

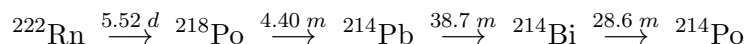
Enter the run times, peak counting rates and their errors into EXCEL. Use an exponential fit in EXCEL to describe the ^{214}Pb line derived activity. Finally determine the specific ^{222}Rn activity in [Bq/l] from your data using equation 17.22. It is imperative to use consistent units or you will get a wrong result.

Problem 4:

The nuclei you are counting were not all collected at the same time; they have a range of ages. This means you don't really know "time zero". How can you determine the half live of ^{214}Pb and ^{214}Bi in the presence of this "uncertainty"?

17.7 Appendix A

The decay sequence we wish to describe has the form:



or



During the filtering phase the number of atoms on the filter is growing. The growth is described by the system of linear first order coupled differential equations given before:

$$\frac{dN_B}{dt_f} = k \cdot n_B - \frac{N_B}{\tau_B} \quad (17.4)$$

$$\frac{dN_C}{dt_f} = k \cdot n_C + \frac{N_B}{\tau_B} - \frac{N_C}{\tau_C} \quad (17.5)$$

$$\frac{dN_D}{dt_f} = k \cdot n_D + \frac{N_C}{\tau_C} - \frac{N_D}{\tau_D} \quad (17.6)$$

How do we solve this problem? We first solve equation 17.4 to obtain the time dependent function $N_B(t_f)$. We substitute this solution into equation 17.5 and solve for $N_C(t_f)$ and so on. This is an iterative process. Unfortunately it becomes more and more cumbersome the longer it goes on!

The problem is that equation 17.4 has an additive, constant term ($k \cdot n_B$), we can thus not obtain a solution by separation of variables (all N's on one side all t's on the other) and integration. We need to use a different method to solve this equation.

After the blower has been turned off the air flow goes abruptly to zero, $k = 0$. The decays are going on. The problem is now governed by a modified set of differential equations:

$$\frac{dN_B}{dt_d} = -\frac{N_B}{\tau_B} \quad (17.12)$$

$$\frac{dN_C}{dt_d} = \frac{N_B}{\tau_B} - \frac{N_C}{\tau_C} \quad (17.13)$$

$$\frac{dN_D}{dt_d} = \frac{N_C}{\tau_C} - \frac{N_D}{\tau_D} \quad (17.14)$$

17.7.1 Mathematical Tools

Equation 17.4 has the general form:

$$\frac{dy(x)}{dx} + P(x) \cdot y(x) = Q(x), \quad (17.20)$$

P and Q may be constants or functions of x . They may not be explicit functions of y . This equation is called the Leibniz equation. Such equation can be solved by multiplication with an *integrating*

factor. To see how this works multiply both sides of equation 17.20 with $\exp(\int P dx)$ to get:

$$\exp\left(\int P dx\right) \cdot \frac{dy}{dx} + y \cdot \exp\left(\int P dx\right) \cdot P = Q \cdot \exp\left(\int P dx\right) \quad (17.21)$$

Since $\frac{d}{dx} \int P dx = P$ we can write the left hand side of equation 17.21 as a product differentiation:

$$\begin{aligned} \frac{d}{dx} [y \cdot \exp(\int P dx)] &= \frac{dy}{dx} \cdot \exp(\int P dx) + y \cdot \frac{d}{dx} \exp(\int P dx) \\ &= \exp(\int P dx) \cdot \frac{dy}{dx} + y \cdot \exp(\int P dx) \cdot P \end{aligned}$$

We substitute this result back into equation 17.21 to get:

$$\frac{d}{dx} [y \cdot \exp(\int P dx)] = Q \cdot \exp(\int P dx) \quad (17.22)$$

We can solve equation 17.22 by separation of variables:

$$\begin{aligned} d [y \cdot \exp(\int P dx)] &= Q \cdot \exp(\int P dx) dx \\ \int d (y \cdot \exp(\int P dx)) &= \int Q \cdot \exp(\int P dx) dx + C \\ y \cdot \exp(\int P dx) &= \int Q \cdot \exp(\int P dx) dx + C \end{aligned}$$

This type of first order differential equation thus has an analytic solution.

$$y(x) = \exp\left(-\int P(x) dx\right) \cdot \left[\int \left\{Q(x) \cdot \exp\left(\int P(x) dx\right)\right\} dx + C\right] \quad (17.23)$$

Here an integration constant can be used to accommodate the boundary conditions (knowledge of the value of the solution at a particular point). In the discussion of exponential decay, given above, definite integrals, not requiring an integration constant, have been used. Both methods are equivalent. This type of first order differential equation and its analytical solution has application in defining orthogonal trajectories, electronic circuits containing resistance and inductance (capacitance), mixing problems and so on.

17.7.2 Growth Phase

We can now construct the solution of equation 17.4 using formula 17.23. As boundary condition we assume that at time zero ($t_f = 0$ the flow of time during the filtering phase), none of the radon decay products is present on the filter: $N_B(0) = N_C(0) = N_D(0) = 0$. We make the following

choices for the variables and functions:

$$\begin{aligned}
 \frac{dN_B(t_f)}{dt_f} &= k \cdot n_B - \frac{N_B(t_f)}{\tau_B} \\
 \frac{dy(x)}{dx} + P(x) \cdot y(x) &= Q(x) \\
 \Rightarrow x &\equiv t_f \\
 \Rightarrow y &\equiv N_B \\
 \Rightarrow P &\equiv \frac{1}{\tau_B} \\
 \Rightarrow Q &\equiv k \cdot n_B \\
 N_B(t_f) &= \exp\left(-\int \frac{1}{\tau_B} dt_f\right) \cdot \left[\int k \cdot n_B \cdot \left[\exp\left(\int \frac{1}{\tau_B} dt_f\right)\right] dt_f + C\right] \\
 &= \exp\left(-\frac{t_f}{\tau_B}\right) \cdot \left[k \cdot n_B \cdot \int \exp\left(\frac{t_f}{\tau_B}\right) dt_f + C\right] \\
 &= \exp\left(-\frac{t_f}{\tau_B}\right) \cdot \left[k \cdot n_B \cdot \tau_B \cdot \exp\left(\frac{t_f}{\tau_B}\right) + C\right] \\
 N_B(t_f) &= k \cdot n_B \cdot \tau_B + C \cdot \exp\left(-\frac{t_f}{\tau_B}\right)
 \end{aligned}$$

We now make use of the boundary condition to determine C : we demand that $N_B(t_f = 0) = 0$.

$$\begin{aligned}
 0 &= k \cdot n_B \cdot \tau_B + C \\
 \Rightarrow C &= -k \cdot n_B \cdot \tau_B
 \end{aligned}$$

Substitute this choice back into the solution yields:

$$\begin{aligned}
 N_B(t_f) &= k \cdot n_B \cdot \tau_B + (-k \cdot n_B \cdot \tau_B) \cdot \exp\left(-\frac{t_f}{\tau_B}\right) \\
 N_B(t_f) &= k \cdot n_B \cdot \tau_B \cdot \left[1 - \exp\left(-\frac{t_f}{\tau_B}\right)\right] \mid \cdot \frac{1}{\tau_B} \\
 A_B(t_f) &= k \cdot n_B \cdot \left[1 - \exp\left(-\frac{t_f}{\tau_B}\right)\right]
 \end{aligned}$$

We have derived the solution listed in equation 17.7 above. The activity A_B is obtained by dividing the number of atoms by the mean life time.

17.7.3 The Decay Phase

The abrupt change of the conditions at the end of filtering results in a kink in the time dependent number of atoms. For the sake of time counting during the decay phase we reset the clock to zero and introduce a new time variable t_d . We can not simply set the flow rate to zero ($k = 0$) in the solutions to describe the decay phase as the growth is proportional to k and the result would be

identical to zero.

The description of activity B is straight forward, it is a simple exponential decay. I will not go through the details of this again. At time zero ($t_d = 0$) the number of atoms present is whatever it was at the end of filtering: $N_B(t_f)$, thus:

$$N_B(t_d) = N_B(t_f) \cdot \exp\left(-\frac{t_d}{\tau_B}\right) \quad | \cdot \frac{1}{\tau_B}$$

$$\Rightarrow A_B(t_d) = A_B(t_f) \cdot \exp\left(-\frac{t_d}{\tau_B}\right) \quad (17.24)$$

$$(17.25)$$

As a further example let's solve equation 17.17 with the help of the general solution 17.23.

$$\frac{dN_C(t_d)}{dt_d} = \frac{N_B(t_d)}{\tau_B} - \frac{N_C(t_d)}{\tau_C}$$

$$\frac{dy(x)}{dx} + P(x) \cdot y(x) = Q(x)$$

$$x \equiv t_d$$

$$y \equiv N_C$$

$$P \equiv \frac{1}{\tau_C}$$

$$Q \equiv \frac{N_B(t_f)}{\tau_B} \cdot \exp\left(-\frac{t_d}{\tau_B}\right)$$

In the previous example we have seen that the integrating factor is:

$$\exp\left(\int P(x) dx\right) \equiv \exp\left(\int \frac{1}{\tau_C} dt_d\right)$$

$$= \exp\left(\frac{t_d}{\tau_C}\right)$$

We note that the number of atoms of species B, present after the filtering phase is over, namely $N_B(t_f)$, does not depend on the decay time t_d . It can thus be treated as a constant as far as the integral is concerned. The solution is now obtained as:

$$N_C(t_d) = \exp\left(-\frac{t_d}{\tau_C}\right) \cdot \left[\int \left\{ \frac{N_B(t_f)}{\tau_B} \cdot \exp\left(-\frac{t_d}{\tau_B}\right) \cdot \exp\left(\frac{t_d}{\tau_C}\right) \right\} dt_d + C \right]$$

$$= \exp\left(-\frac{t_d}{\tau_C}\right) \cdot \left[\frac{N_B(t_f)}{\tau_B} \cdot \int \exp\left(\frac{t_d \cdot (\tau_B - \tau_C)}{\tau_C \cdot \tau_B}\right) dt_d + C \right]$$

$$= \exp\left(-\frac{t_d}{\tau_C}\right) \cdot \left[\frac{N_B(t_f)}{\tau_B} \cdot \frac{\tau_C \cdot \tau_B}{\tau_B - \tau_C} \cdot \exp\left(\frac{t_d \cdot (\tau_B - \tau_C)}{\tau_C \cdot \tau_B}\right) + C \right]$$

$$= N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} \cdot \exp\left(\frac{t_d \cdot (\tau_B - \tau_C)}{\tau_C \cdot \tau_B} - \frac{t_d}{\tau_C}\right) + \exp\left(-\frac{t_d}{\tau_C}\right) \cdot C$$

$$= N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} \cdot \exp\left(-\frac{t_d}{\tau_B}\right) + \exp\left(-\frac{t_d}{\tau_C}\right) \cdot C$$

The constant of integration is determined by demanding that at decay time zero ($t_d = 0$) the number of atoms present equals those left over after filtering was over:

$$\begin{aligned} N_C(t_d = 0) &= N_C(t_f) \\ N_C(t_f) &= N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} + C \\ \Rightarrow C &= N_C(t_f) - N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} \end{aligned}$$

Given the starting conditions the final time function is:

$$\begin{aligned} N_C(t_d) &= N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} \cdot \exp\left(-\frac{t_d}{\tau_B}\right) + \exp\left(-\frac{t_d}{\tau_C}\right) \cdot \left[N_C(t_f) - N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} \right] \\ &= N_C(t_f) \cdot \exp\left(-\frac{t_d}{\tau_C}\right) - N_B(t_f) \cdot \frac{\tau_C}{\tau_B - \tau_C} \cdot \left[\exp\left(-\frac{t_d}{\tau_C}\right) - \exp\left(-\frac{t_d}{\tau_B}\right) \right] \end{aligned}$$

The time dependent activity of activity C is again obtained by dividing the the number of atoms by the mean life time τ_C :

$$A_C(t_d) = A_C(t_f) \cdot \exp\left(-\frac{t_d}{\tau_C}\right) - A_B(t_f) \cdot \frac{\tau_B}{\tau_B - \tau_C} \cdot \left[\exp\left(-\frac{t_d}{\tau_C}\right) - \exp\left(-\frac{t_d}{\tau_B}\right) \right] \quad (17.26)$$

Taking into account that typical counting times are long compared to the mean life time of activity B: $t_d \gg \tau_B$ and thus $A_B(t_d) \approx 0$:

$$A_C(t_d) = A_C(t_f) \cdot \exp\left(-\frac{t_d}{\tau_C}\right)$$

17.7.4 Secular Equilibrium

In the analysis of the data we make the assumption of secular equilibrium: $\frac{n_B}{\tau_B} = \frac{n_C}{\tau_C} = \frac{n_D}{\tau_D}$. What does this mean and why is it a good assumption?

Calculate the ratio of the decay rate of two sequential decays $X \rightarrow Y \rightarrow Z$. We calculated the decay rates in the previous section, with the time dependent activities given in equations 17.24 and 17.26. Further assume the first decay to have a much longer life time than the second $\tau_X \gg \tau_Y$. Further assume that at time zero there was none of species Y present. All of it comes from the decay of X. $A_Y(t = 0) = 0$. Using above solutions we get:

$$\begin{aligned} A_X(t) &= A_X(0) \cdot \exp\left(-\frac{t}{\tau_X}\right) \\ A_Y(t) &= A_X(0) \cdot \frac{\tau_X}{\tau_X - \tau_Y} \cdot \left[\exp\left(-\frac{t}{\tau_X}\right) - \exp\left(-\frac{t}{\tau_Y}\right) \right] \end{aligned}$$

Lets get the ratio of activities (decay rates) for X and Y:

$$\frac{A_X(t)}{A_Y(t)} = \frac{A_X(0) \cdot \exp\left(-\frac{t}{\tau_X}\right)}{A_X(0) \cdot \frac{\tau_X}{\tau_X - \tau_Y} \cdot \left[\exp\left(-\frac{t}{\tau_X}\right) - \exp\left(-\frac{t}{\tau_Y}\right)\right]}$$

For observing time long compared to the life time of species Y ($t \gg \tau_Y$) one of the exponential is negligible: $\exp\left(-\frac{t}{\tau_Y}\right) \approx 0$. We thus get:

$$\begin{aligned} \frac{A_X(t)}{A_Y(t)} &= \frac{A_X(0) \cdot \exp\left(-\frac{t}{\tau_X}\right)}{A_X(0) \cdot \frac{\tau_X}{\tau_X - \tau_Y} \cdot \exp\left(-\frac{t}{\tau_X}\right)} \\ &= \frac{\tau_X - \tau_Y}{\tau_X} \end{aligned}$$

τ_Y is the short life time. This scaling of rates is called *transient equilibrium*. If we further demand $\tau_X \gg \tau_Y$ and thus $\tau_X - \tau_Y \approx \tau_X$ then the ratio of activities becomes 1. This situation is called *secular equilibrium*. You can understand this as a case were you have a sequence of events, all of them except the first very fast. Although all member of this chain react promptly they can only take what trickles down from above. The rate of the entire chain is thus determined by its slowest link. Given the large difference between the ^{222}Rn life time and those of its decay products the assumption of equal decay rate of all member of the decay sequence is well justified, giving a good approximation. In this situation one needs to measure only one decay rate to know them all.

Reference Data

17.8 Periodic Table of the Elements

On the following page, a freely-distributable periodic table from <http://www.nist.gov/physlab/data/periodic.cfm> [19].

PERIODIC TABLE

Atomic Properties of the Elements

Frequently used fundamental physical constants

For the most accurate values of these and other constants, visit physics.nist.gov/constants

1 second = $9\ 192\ 631\ 770$ periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs

speed of light in vacuum c $299\ 792\ 458\ \text{m s}^{-1}$ (exact)

Planck constant h $6.626\ 070\ 15 \times 10^{-34}\ \text{J s}$ (exact)

elementary charge e $1.602\ 2 \times 10^{-19}\ \text{C}$ (exact)

electron mass m_e $9.109\ 4 \times 10^{-31}\ \text{kg}$

proton mass m_p $1.672\ 6 \times 10^{-27}\ \text{kg}$

fine-structure constant α $1/137.036$

Rydberg constant R_∞ $10\ 973\ 732\ \text{m}^{-1}$

$R_\infty c$ $3.289\ 842 \times 10^{15}\ \text{Hz}$

$R_\infty hc$ $13.6057\ \text{eV}$

Boltzmann constant k $1.380\ 7 \times 10^{-23}\ \text{J K}^{-1}$

- Solids
- Liquids
- Gases
- Artificially Prepared

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1 1H	2 2He	3 3Li	4 4Be	5 5B	6 6C	7 7N	8 8O	9 9F	10 10Ne	11 11Na	12 12Mg	13 13Al	14 14Si	15 15P	16 16S	17 17Cl	18 18Ar	19 19K	20 20Ca	21 21Sc	22 22Ti	23 23V	24 24Cr	25 25Mn	26 26Fe	27 27Co	28 28Ni	29 29Cu	30 30Zn	31 31Ga	32 32Ge	33 33As	34 34Se	35 35Br	36 36Kr	37 37Rb	38 38Sr	39 39Y	40 40Zr	41 41Nb	42 42Mo	43 43Tc	44 44Ru	45 45Rh	46 46Pd	47 47Ag	48 48Cd	49 49In	50 50Sn	51 51Sb	52 52Te	53 53I	54 54Xe	55 55Cs	56 56Ba	57 57La	58 58Ce	59 59Pr	60 60Nd	61 61Pm	62 62Sm	63 63Eu	64 64Gd	65 65Tb	66 66Dy	67 67Ho	68 68Er	69 69Tm	70 70Yb	71 71Lu	72 72Hf	73 73Ta	74 74W	75 75Re	76 76Os	77 77Ir	78 78Pt	79 79Au	80 80Hg	81 81Tl	82 82Pb	83 83Bi	84 84Po	85 85At	86 86Rn	87 87Fr	88 88Ra	89 89Ac	90 90Th	91 91Pa	92 92U	93 93Np	94 94Pu	95 95Am	96 96Cm	97 97Bk	98 98Cf	99 99Es	100 100Fm	101 101Md	102 102No	103 103Lr	104 104Rf	105 105Db	106 106Sg	107 107Bh	108 108Hs	109 109Mt	110 110Un	111 111Uun	112 112Uub	113 113Uuh	114 114Uuq	115 115Uup	116 116Uuq	117 117Uus	118 118Uuo	119 119Uuu	120 120Uuq	121 121Uub	122 122Ubc	123 123Ubn	124 124Ubu	125 125Ubu	126 126Ubu	127 127Ubu	128 128Ubu	129 129Ubu	130 130Ubu	131 131Ubu	132 132Ubu	133 133Ubu	134 134Ubu	135 135Ubu	136 136Ubu	137 137Ubu	138 138Ubu	139 139Ubu	140 140Ubu	141 141Ubu	142 142Ubu	143 143Ubu	144 144Ubu	145 145Ubu	146 146Ubu	147 147Ubu	148 148Ubu	149 149Ubu	150 150Ubu	151 151Ubu	152 152Ubu	153 153Ubu	154 154Ubu	155 155Ubu	156 156Ubu	157 157Ubu	158 158Ubu	159 159Ubu	160 160Ubu	161 161Ubu	162 162Ubu	163 163Ubu	164 164Ubu	165 165Ubu	166 166Ubu	167 167Ubu	168 168Ubu	169 169Ubu	170 170Ubu	171 171Ubu	172 172Ubu	173 173Ubu	174 174Ubu	175 175Ubu	176 176Ubu	177 177Ubu	178 178Ubu	179 179Ubu	180 180Ubu	181 181Ubu	182 182Ubu	183 183Ubu	184 184Ubu	185 185Ubu	186 186Ubu	187 187Ubu	188 188Ubu	189 189Ubu	190 190Ubu	191 191Ubu	192 192Ubu	193 193Ubu	194 194Ubu	195 195Ubu	196 196Ubu	197 197Ubu	198 198Ubu	199 199Ubu	200 200Ubu	201 201Ubu	202 202Ubu	203 203Ubu	204 204Ubu	205 205Ubu	206 206Ubu	207 207Ubu	208 208Ubu	209 209Ubu	210 210Ubu	211 211Ubu	212 212Ubu	213 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Bibliography

- [1] J. R. Taylor, *An Introduction to Error Analysis*. University Science Books (Sausalito, Ca), 2nd ed., 1997.
- [2] J. I. Pfeffer and S. Nir, *Modern Physics*. Imperial College Press (London), 1st ed., 2000.
- [3] P. LeClair, 2009. unpublished work.
- [4] R. S. Peterson, “Experimental γ ray spectroscopy and investigations of environmental radioactivity,” 1996. Hard copy available in the laboratory; soft copy available on spectrometer computer.
- [5] J. Hubbel and S. Seltzer, “Tables of x-ray mass attenuation coefficients and mass energy-absorption coefficients.” NISTIR 5632. Available online at <http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html>.
- [6] G. Audi and A. H. Wapstra, “Masses, Q-values and nucleon separation energies: The 1995 update to the atomic mass evaluation,” *Nuclear Physics*, vol. A595, pp. 409–480, 1995. <http://nucldata.nuclear.lu.se/nucldata/toi/>.
- [7] “National nuclear data center.” <http://www.nndc.bnl.gov/index.jsp>.
- [8] Example gamma ray spectrum and decay schematics for isotopes. http://www.radiochemistry.org/periodictable/gamma_spectra/.
- [9] M. E. M. da Piedade and M. N. Berberan-Santos, “Atomic emission spectra using a UV-Vis spectrophotometer and an optical fiber guided light source,” *J. Chem. Ed.*, vol. 75, pp. 1013–1017, 1998.
- [10] O. Optics. Operating instructions manual for USB-650 and USB-4000 spectrometers and <http://www.oceanoptics.com>.
- [11] CRC Handbook of Chemistry and Physics, online edition. <http://www.hbcpnetbase.com>.
- [12] H. G. Kuhn, *Atomic Spectra*. Longmans (London), 1964.
- [13] V. Kondratyev, *The Structure of Atoms and Molecules*. Dover (New York), 1965.

- [14] R. J. W. H.-G. B. Jeff Rylander, Tom Jordan and R. Gran, “Quarknet cosmic ray detectors users manual.” <http://quarknet.fnal.gov/toolkits/ati/det-user.pdf>.
- [15] M. Planck, “Über das Gesetz der Energieverteilung im Normalspektrum,” *Ann. Phys.*, vol. 309, p. 533, 1901. English translation: <http://dbhs.wvusd.k12.ca.us/webdocs/Chem-History/Planck-1901/Planck-1901.html>.
- [16] A. Einstein, “Über einen die Erzeugung and Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt,” *Ann. d. Physik*, vol. 17, p. 132, 1905.
- [17] R. Millikan, “A direct photoelectric determination of Planck’s ‘h’,” *Phys. Rev.*, vol. 7, p. 355, 1916.
- [18] P. Mohr, B. Taylor, and D. Newell, “CODATA recommended values of the fundamental physical constants: 2006,” *Rev. Mod. Phys.*, vol. 80, p. 633, 2008. See also the NIST database: <http://physics.nist.gov/cgi-bin/cuu/Value?h>.
- [19] R. Dragoset, A. Musgrove, C. Clark, and W. Martin. Work of the US government excluded from copyright, <http://www.nist.gov/physlab/data/periodic.cfm>.