Measurement of e/k_B

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January 25, 2010

Contents

1   Abstract

By measuring the relationship between the applied voltage and the net current flow across a transistor, we were able to find the ratio between e, the elementary charge, and k_B, the Boltzmann constant. This relationship is found in the exponential factor of the Boltzmann distribution, used to statistically approximate the distribution of thermal energies of the electrons. By plotting the natural log of the net current generated against the voltage applied, we obtained a linear graph with a slope of $e/k_B T$. We then multiplied by the temperature of the transistor to calculate the e/k_B ratio. We were able to extract an experimental value for k_B by using the accepted value of e. We determined Boltzmann's constant to be

$$k_B = (1.61 \pm 0.04) * 10^{-23} \text{J/K}$$

2   Introduction

The e/k_B ratio is the fundamental relationship between **e**, the elementary charge, and **k_B**, the Boltzmann constant. By measuring the current generated across a transistor as a function of the potential difference applied between the emitter and collector, we were able to determine this ratio from the Boltzmann distribution of statistical physics.

3   Extracting e/k_B from the Boltzmann Distribution

3.1   Introduction

In this experiment, we manipulated the Boltzmann distribution to extract the e/k_B term from its exponential. The model used to design the experiment depends on the current flow across semiconductors of different types, and on the independence of the reverse current from the diffusion current in the presence of a forward bias.

3.1.1   The Junction Field

P-type and n-type semiconductors differ in their so-called majority carriers, i.e. the conductors of electrical current. N-type semiconductors are materials that have been doped with an element with Z+1 valence electrons. Therefore, despite being net neutral, the doped semiconductor has an excess of free electrons that can be easily promoted via thermal excitation into the conduction energy band. The p-type semiconductor is the opposite; having been doped with an element with Z-1 valence electrons, it has an abundance of "holes" in its valence band, which flow in exactly the same manner as would a group of positively charged particles. At the junction of the two semiconductors, conduction electrons from the n-type flow across and combine with holes in the p-type. These charges remain stationary, establishing an electric field known as the junction field, as shown below.

1 The Junction of a p-n diode. From http://en.wikipedia.org/wiki/P-n_junction
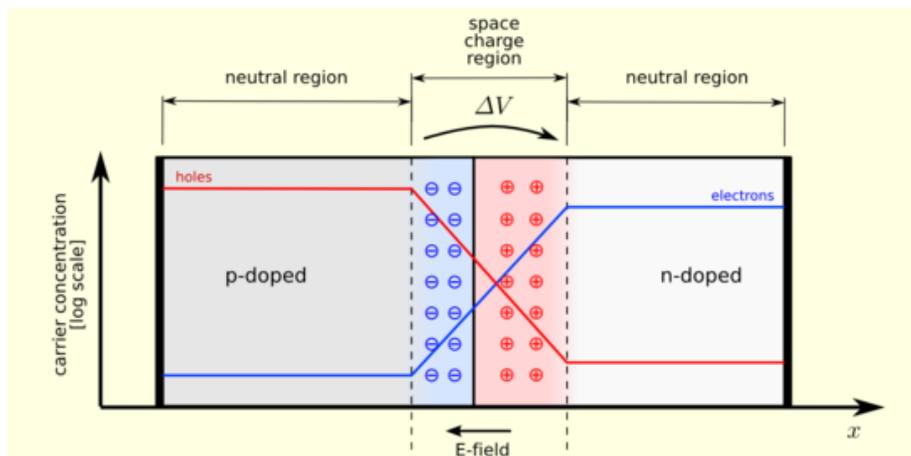
3.1.2 Current across the Potential Barrier

When no external potential difference is applied, very few electrons have the energy required to cross the junction field. The potential barrier is equal to the charge of an electron times the potential difference, or

$$E = eV_o \tag{1}$$

Only electrons with energy equal to the height of the potential barrier will be able to diffuse across. The number of electrons with that energy is given by the Boltzmann distribution.

$$F_B = Ae^{-eV_o/kT} \tag{2}$$

Therefore, there must be a diffusion current across the potential barrier against the junction field such that

$$I_o = Ae^{-eV_o/kT} \tag{3}$$

Because there can be no net current flow across a diode across which no external potential difference has been applied, there must then be an independent return current equal in magnitude but opposite in direction. Thus,

$$I_R = I_o = Ae^{-eV_o/kT} \tag{4}$$

If, however, an external potential difference is applied, we see a lowering of the potential barrier. In the presence of the potential difference V, the potential is lowered to

$$E = e(V_o - V)$$

This leads to a corresponding increase in the number of electrons energetic enough to cross the barrier, reflected in the alteration of the Boltzmann exponential. The diffusion current term becomes

$$I_o = Ae^{-e(V_o-V)/kT} \tag{5}$$

However, because it is independent of the diffusion current, the return current remains constant. Therefore, there is a net current flow

$$I_{net} = I_o - I_R = I_o(e^{\frac{eV}{kT}} - 1)$$

Because of the order of magnitude of eV/kT, net current flow becomes, to exceedingly good approximation,

$$I_{net} = I_o[e^{\frac{eV}{kT}}] \tag{6}$$

By taking the natural log of the current generated across the junction, we see that

$$\ln(I_{net}) = \ln(I_o) + \frac{eV}{kT} \tag{7}$$

And by plotting it against the potential difference, we obtain a linear graph with slope $e/_{kT}$. By multiplying by temperature of the semiconductor (assumed to be the ambient temperature of the lab), we are able to obtain the ratio e/k_B.

3.1.3   The Ideality Factor

Due to a large number of unforeseeable variables in the use of a simple p-n junction, it is common practice to alter the exponential in (6) by introducing the so-called ideality factor n, which is determined experimentally and can change the value of the exponential by as much as a factor of 2.5 [1]. Therefore, a simple p-n junction is not suitable for this experiment. However, if a transistor is used instead, enough of the problem is eliminated by the bleed-off of the base current that the experiment becomes relatively accurate.

3.2   Procedure

We connected a 2N1724 High Power NPN Silicon Transistor to the Keithley 220 current source, running the current through a Hewlett Packard 3457A Multimeter to measure the voltage. Using the PH 255 Transport program, we measured the potential difference and the current across the transistor using logarithmic sweeps across six orders of magnitude. We made five measurements, each time varying the sweep parameters slightly.

3.3   Results

Figures 1-5 are the graphical results of our five measurements, with the natural log of the current plotted against the potential difference. The data points have been fitted with a linear trendline, which displays the slope of each. According to our model, the value of the slope should be equal to e/k_BT.

As predicted, the graphs are beautifully linear with very high R^2 values. Our mean value for the slope was 33.7 V^{-1}, with a standard deviation of 0.3 V^{-1}. We estimated the ambient temperature in the room to be $295 \pm 5°$ K. By multiplying these two values, by our model we arrive at the ratio e/k.

$$(33.7 \pm 0.3 \, V^{-1}) * (295 \pm 5° \, K) = 9940 \pm 260 \, K/V$$

The accepted value for e with negligible uncertainty is

$$e = 1.60 * 10^{-19} \text{ C}$$

Using this value, we determine k_B to be

$$k_B = \frac{1.60 * 10^{-19} C}{9940 \pm 260° K/V} = (1.61 \pm 0.04) * 10^{-23} J/K$$

3.4 Discussion

The accepted value for Boltzmann's constant with negligible uncertainty is

$$k_B = 1.38 * 10^{-23}$$

This puts our mean measurement off by approximately 17%. Our mean R^2 value obtained from the statistical analyses of our trend lines indicates that 99.9% of the variance in the data can be accounted for by statistical fluctuation. Because of our still relatively high error, we suspect a systematic problem is at fault. Attached are two graphs (Figures 6 and 7) testing the calibration of the equipment used in the experiment, at resistances of 1 kΩ and 100 kΩ. The errors were approximately .007% and 0.2%, respectively. This leads us to conclude that the equipment was not at fault, and discrepancy between our result and the accepted value for k_B is a result of a fundamental error in the experimental model.

There are, of course, other possible sources of error. By far the most tenuous assumption was the rough estimation of the temperature of the semiconductor. We assumed that the temperature of the lab was the temperature of the transistor, but it is very possible (actually, very likely) that this was not the case. Our estimation of the temperature was subjective and not measured directly. Additionally, aside from the fact that the air may not have been in thermal equilibrium with the silicon transistor, Joule heating resulting from the current flow may have slightly raised the temperature of the transistor. However, the 2N1724 transistor was chosen specifically for its capacity to withstand relatively large currents without much power dissipation (its technical specifications can be found in the PH 255 Lab Manual [1] or online [2]), so the prospect of Joule heating creating much error in our result is highly unlikely.

3.5 Conclusion

By measuring the current flow and potential difference across a transistor and sweeping across several orders of magnitude, we accumulated the raw data describing their relationship as a

function of voltage. By taking the natural log of the current and plotting it against voltage, we obtained five highly linear graphs with, from our model, slopes of e/k$_B$T. By estimating the temperature of the transistor and using the accepted value for *e,* we determined that
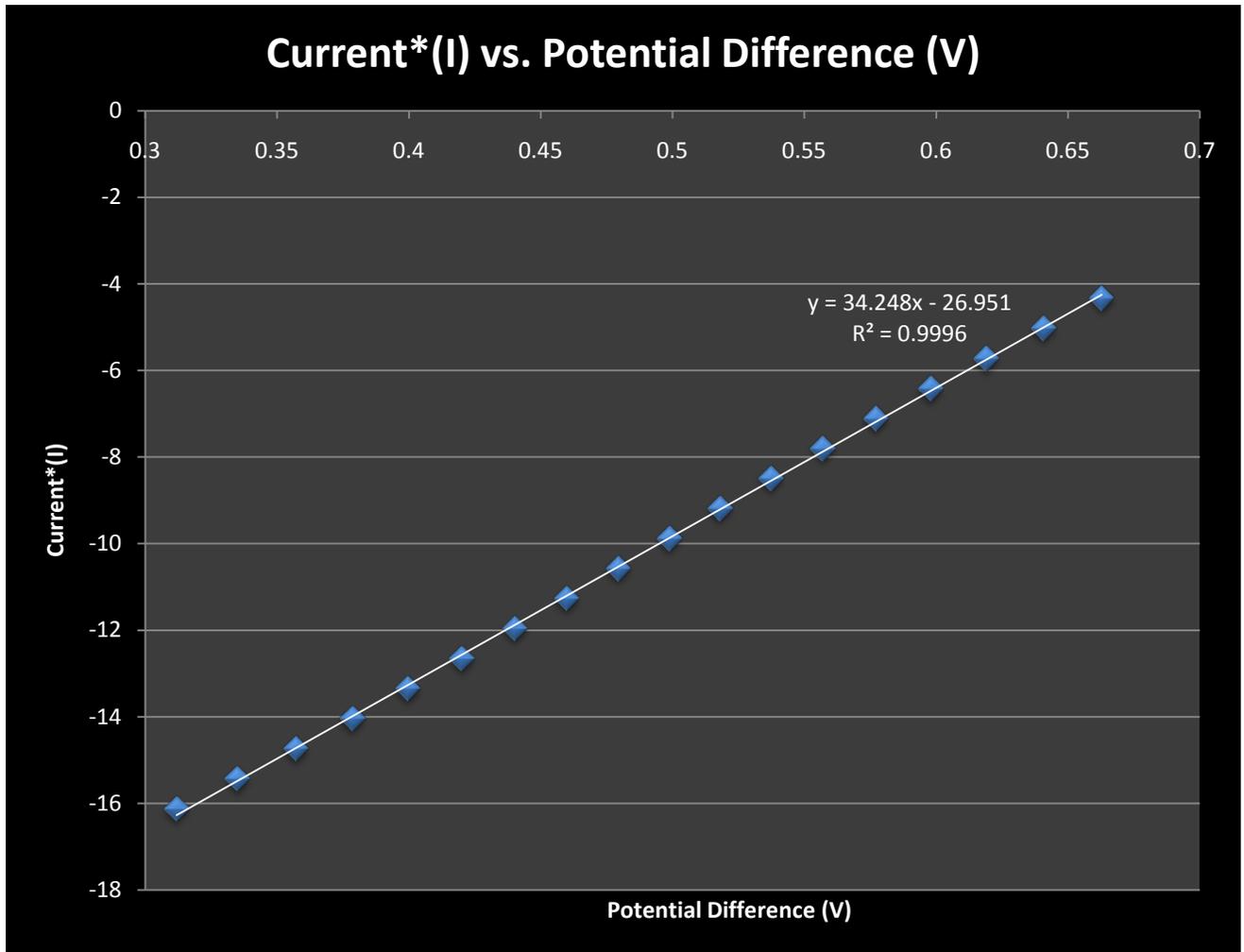
$$k_B = (1.61 \pm 0.04) * 10^{-23} J/K$$

Though our data fit beautifully together, we calculated a mean final result that differed from the accepted value by approximately 17%. Because statistical error cannot account for this discrepancy, we conclude that the model for this experiment is too simplistic to satisfactorily measure the ratio of e/k$_B$.

References

[1] P. LeClair, PH 255 Laboratory Manual, Spring 2010

[2] Specifications of the 2N1724 Silicon Transistor, http://www.digchip.com/datasheets/parts/datasheet/300/2N1724.php

Appendix A: Graphical Results

Current*(I) vs. Potential Difference (V)

$$y = 34.248x - 26.951$$
$$R^2 = 0.9996$$

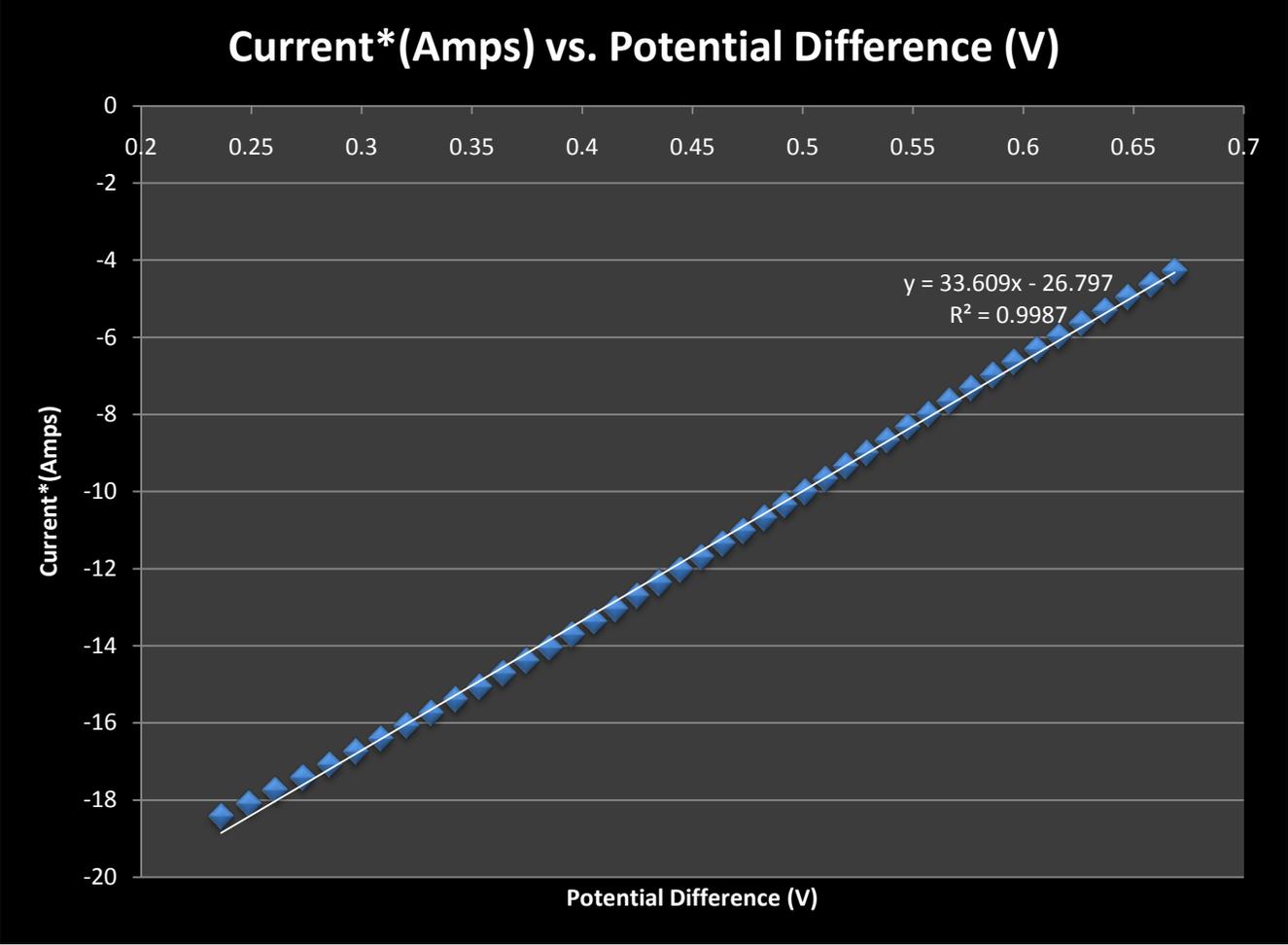*x-axis: Potential Difference (V)*

y-axis: Current(I)*

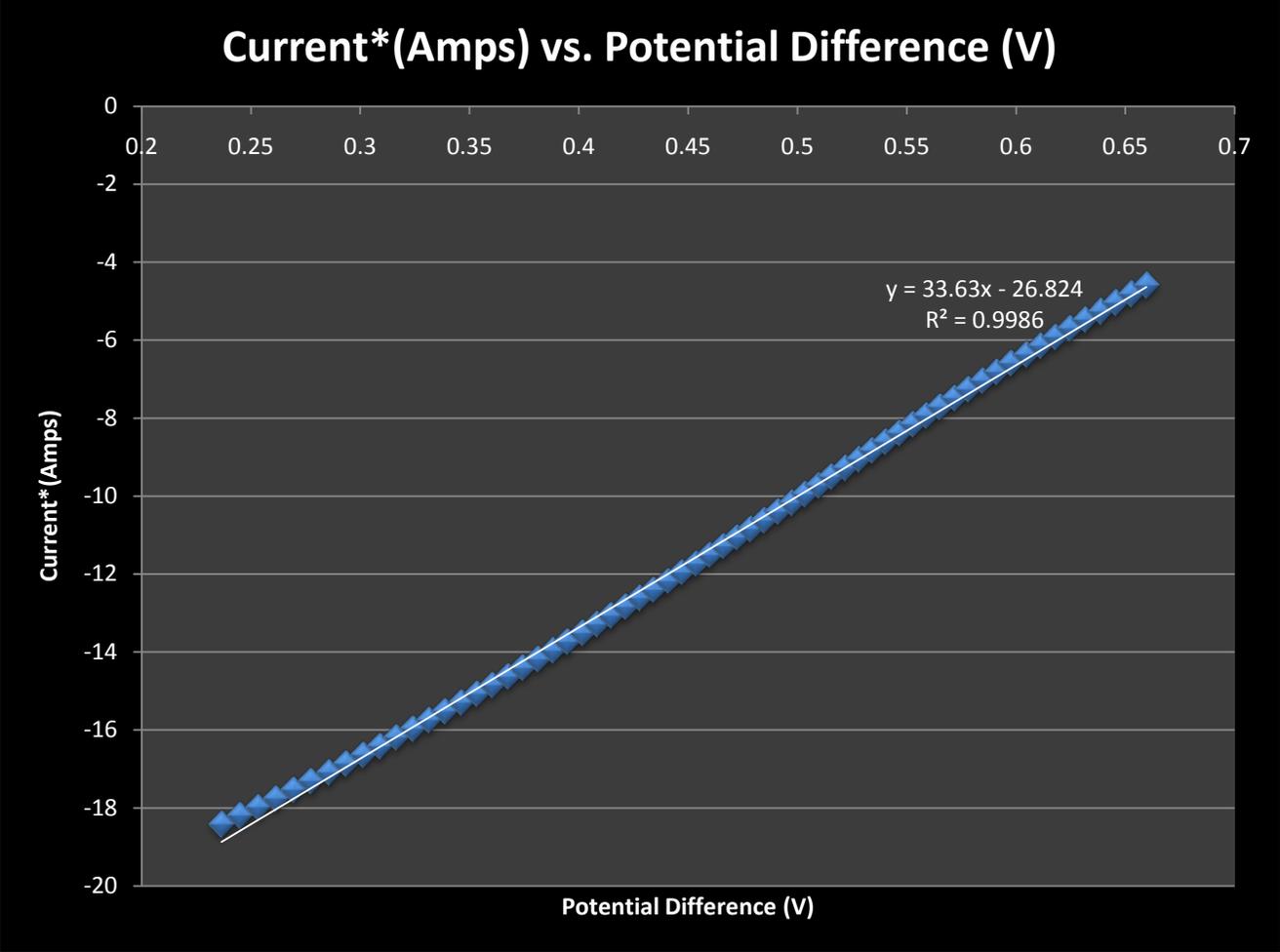**Figure 1** Natural Log of Current vs. Potential Difference, Logarithmic Sweep, Interval: 2
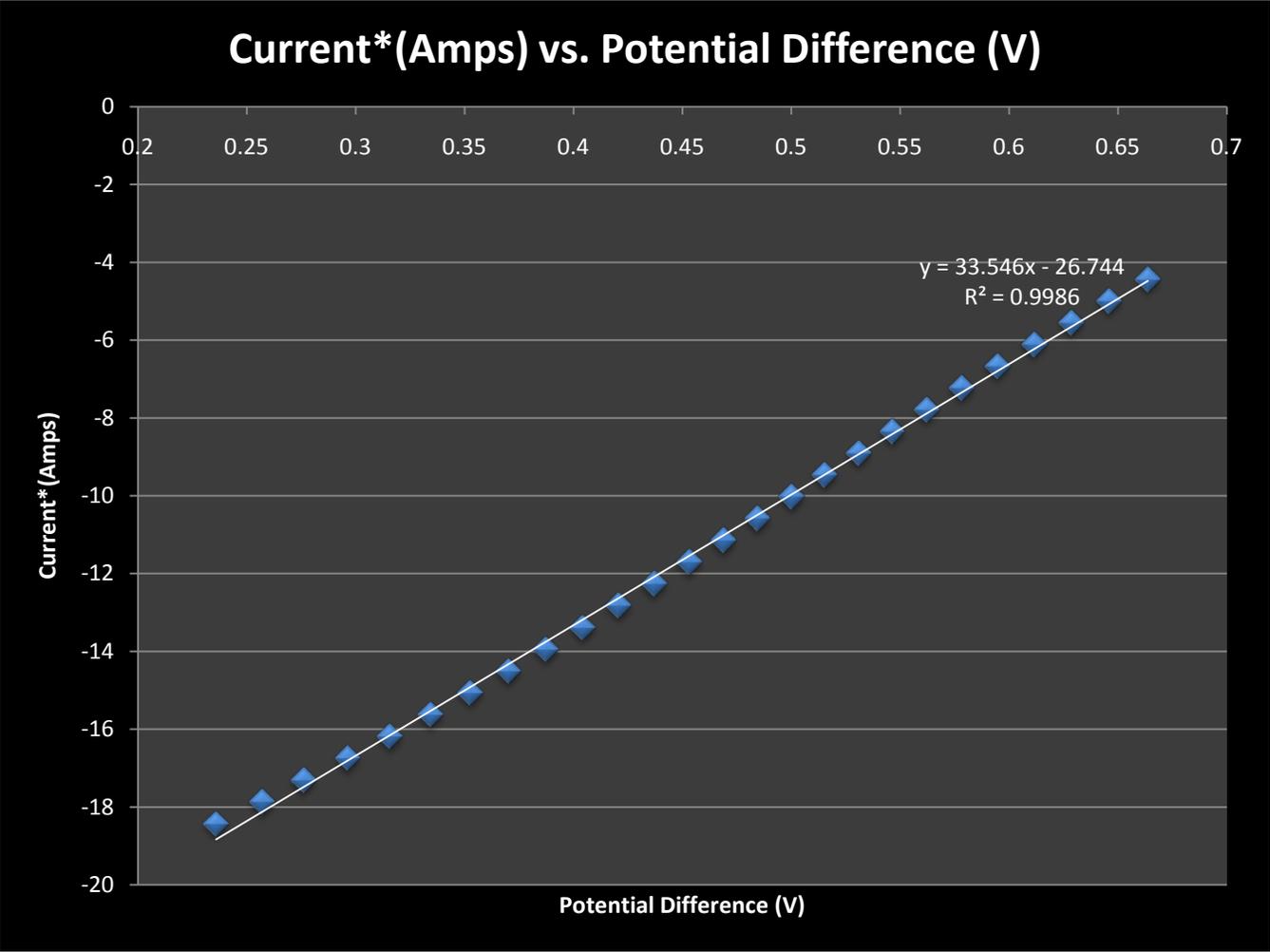
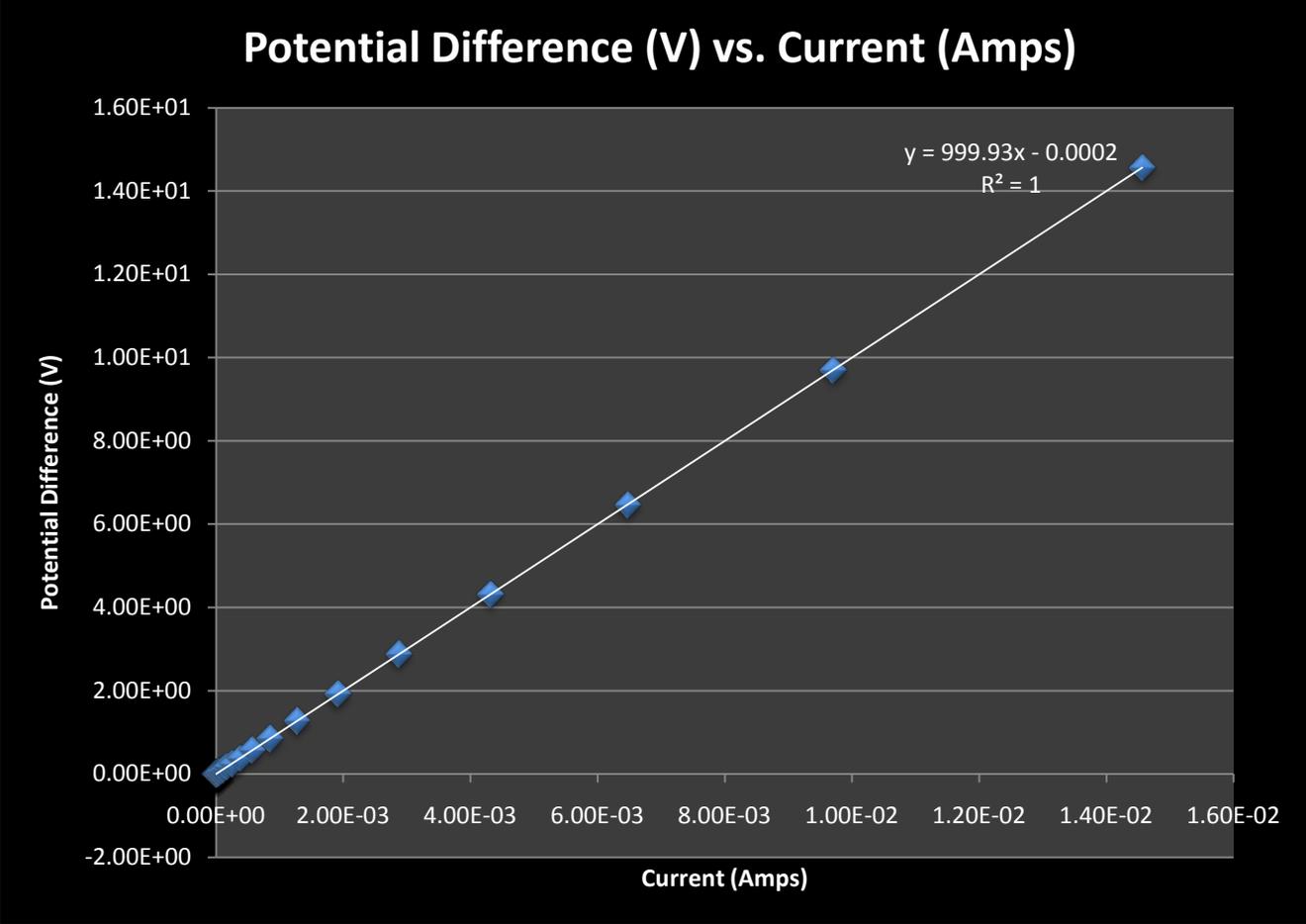**Figure 2** Natural Log of Current vs. Potential Difference, Logarithmic Sweep, Interval: 1.75
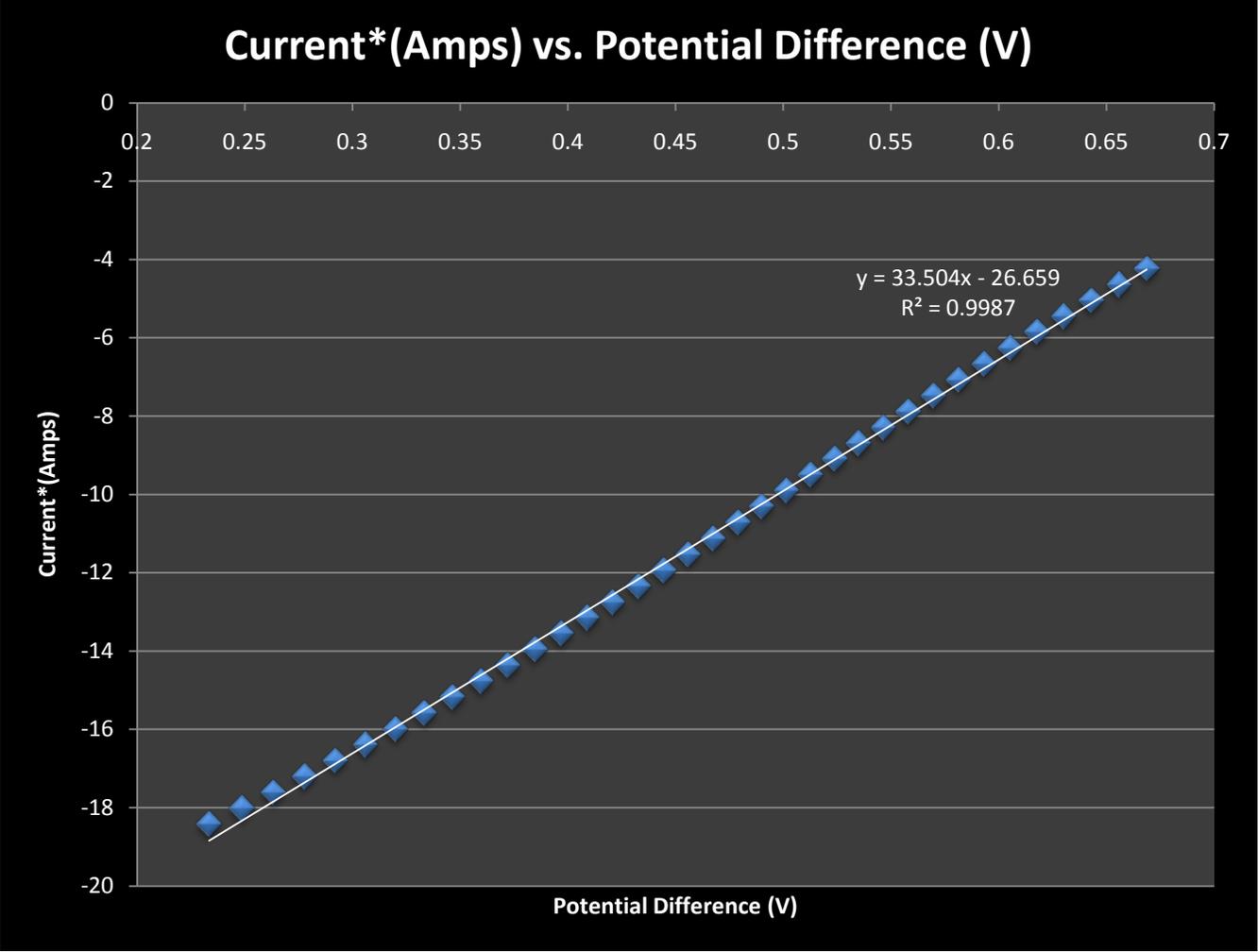
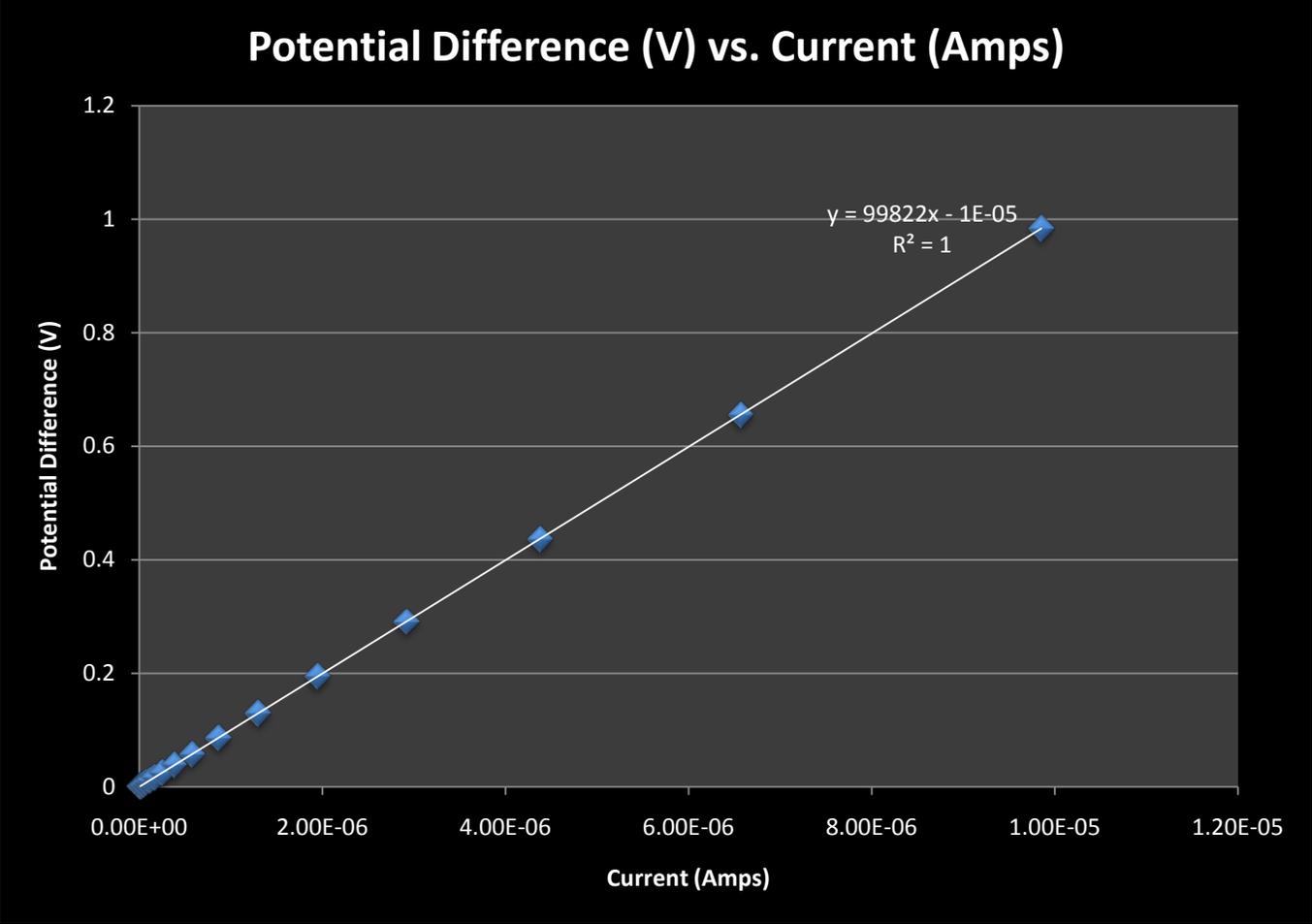**Figure 3** Natural Log of Current vs. Potential Difference, Logarithmic Sweep, Interval: 1.5

Figure 4 Natural Log of Current vs. Potential Difference, Logarithmic Sweep, Interval: 1.4

Figure 5 Natural Log of Current vs. Potential Difference, Logarithmic Sweep, Interval: 1.25

Figure 6 Potential Difference vs. Current, Calibration test, Resistance: 1 kΩ

Figure 7 Potential Difference vs. Current, Calibration test, Resistance: 100 kΩ