Counting Statistics Memorandum

15 February 2010

To: Mainsail Taylor—Project STRUT
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Re: Random Processes—^{137}Cs β-Decay
Date: 15 February 2010

1 Background

The success of Project STRUT requires that data packets be sent at steady rates in order to facillitate smooth communication, but also that the time between individual packets be unpredictable, so that they cannot be reliably intercepted. You asked us to find a way to determine the rhythm of packet-sending. We propose that the radioactive decay of ^{137}Cs is a good candidate for a process that is "predictably random" on the time-scales relevant for Project STRUT.

2 Theory

Cesium-137, a radioactive isotope with a half-life near 30 yr, beta decays (i.e. emits an electron) into Barium-137 which emits a detectable 662 keV gamma ray. We can detect the gamma ray with a scintillation counter, which contains a detecting material that fluoresces each time it is struck by ionizing radiation. The flash of light is captured by a photomultiplier tube, which effectively converts light signals into electrical signals that can be recorded by a computer. We describe below that the average rate of beta decay in ^{137}Cs is quite steady, but the individual emissions are apparently independent of each other.

Our idea is that beta decay of a radioactive ^{137}Cs source could serve as regulator for outgoing packet flow for Project STRUT. Specifically, each detection of an emission could easily be used to send a data packet. Following are details of the statistical analyses that led us to this conclusion.

3 Procedure

For the collection of our data, we used a Spectech UCS30 computer spectrometer with a sodium iodide-based detector. It was calibrated with the desired ^{137}Cs sample in place using the "Auto Calibrate" feature present in the UCS30 computer software. (The appendix contains a schematic of the experimental setup.) We then set the spectrometer detection limits to record only the number of 662 keV gamma rays emitted, each of which signaled a ^{137}Ba nucleus—created through a beta decay of a ^{137}Cs nucleus—dropping to its ground state. It was this measurement which we expected to possess a Gaussian distribution.

Three distinct dwell times—the duration during which an individual count was determined—were employed to examine the impact of different amounts of data points. In order to maintain identical acquisition times, we collected one spectrum at a 200 ms dwell time (corresponding to 1024 total points), two at 100 ms (2048 points), and five at 40 ms (5120 points). In this way, a duration of 204.8 s was recorded for each dwell time, yielding three separate distributions of varying size. Statistical analysis of this data then ensued, with the aid of Microsoft Excel and simple probability theory.[1]

4 Discussion of Analysis

The core of our investigation centers around several key concepts of probability theory, specifically the characteristics of independent random variables. If our data display all the features of such variables, then ^{137}Cs decay can rightly be deemed a random process. Now the distribution of a truly random variable, in the limit of a large mean, can be approximated by the familiar normal—or Gaussian—distribution.[2] And the two most important parameters in this comparison are the mean and standard deviation for each of the three dwell times; from these, a standard Gaussian curve can be constructed. Roughly speaking, if the counts lie—within error—on the given Gaussian, then the objective is met; we have uncovered a random process which can be easily measured and applied to meet your desired functionality.

Yet for more rigorous quantitative certainty, the data itself should staisfy several mathematical criteria. First, the mean count rate should prove constant, and a running average should converge as the total number of data points increases; a graph can be used to verify this property. (^{137}Cs decays at varying rates based on the age of the sample, but its thirty-year half-life implies a virtually constant rate over the duration of a multi-hour experiment.) Likewise, the standard deviation should equal the squre root of the mean and accurately describe the distribution of data points. For example, approximately 50 % of the values must lie within 0.6745 standarad deviations of the mean, and 69 % within one. Fulfilling these criteria lends serious credence to the presence of random, independent events.

And finally, each data set as a whole can be compared with the other two, to ensure further uniformity in the process of ^{137}Cs decay. But since the dwell times—and therefore the averages—vary between data sets, exact match is not expected. Moreover, comparing the relative spread by the standard deviation alone would skew interpretation, as a distribution with a greater mean would by nature possess a deceptively greater spread. To alleviate this error, the coefficient of variation—standard deviation divided by the mean—can be used instead, thereby normalizing a data set for ease of comparison. And so we applied this theory to our data; our results show the radioactive decay of ^{137}Cs to be truly random about a given mean value.

5 Data Analysis

The data matched well the predicted values for random decay.[3] The mean counts measured were 377 ± 18.91 for 200 ms, 188.87 ± 12.93 for 100 ms and 75.44 ± 8.76 for 40 ms. The measured number of decays per unit time conformed ver closely to a normal distribution, with about 68 % of the data falling within $\pm\sigma$, 95 % of the data falling within $\pm 2\sigma$, and 99.7 % of the data falling within $\pm 3\sigma$. It fit the predicted Gaussian curve very well, within the error associated with independent random decays.

The sample mean $\bar{x}$ and standard deviations were calculated using Excel functions. Also, $\sigma = \sqrt{\mu}$ was calculated and closely approximated s. For some of the data σ was above s, and for some of the data σ was below s, but in general, s is expected to be slightly higher since it is not calculated using an infinitely large sample size and thus has greater uncertainty than the theoretically ideal σ.

Taking $P = 0.6745\sigma$, the P values were calculated and, as expected, about 50 % of the data fell within P of μ.

Table 1: Summary of Important Metrics

Dwell Time [ms]	Mean μ [counts]	Population Deviation s	Standard Deviation σ
40	75.44	8.76	8.69
100	188.87	12.93	13.74
200	377.69	18.91	18.81

Dwell Time [ms]	Probable Error P	Fraction within P of μ	Fraction within s of μ
40	5.91	0.5082	0.6914
100	8.72	0.4824	0.6948
200	12.76	0.5137	0.6924

6 Conclusions

In conclusion, the statistical data gathered from all three phases of the experiment displayed the properties of a random variable with a normal distribution. The data closely fit the Gaussian curve and expectations regarding the percentage of data predicted to fall within the uncertainties and within P. It is safe to say that the measured ^{137}Cs decay, in the experimental runtime, closely followed expectations for a random variable.

Notes

[1] Our procedure followed closely that put forth in P. LeClair, "PH 255: Modern Physics Laboratory" Spring 2010, 13-26. This document should be examined for any additional clarification.

[2] J. R. Taylor, *An Introduction to Error Analysis*, Ch. 11.

[3] For explanations of our terminology and abbreviations, the reader is referred to `http://math2.org/math/stat/distributions/z-dist.htm`.

7 Appendix A—Experiment Schematic

Figure 1: A schematic representation of experimental setup. The spectrometer was connected to a computer as well, which is not shown.