

Electron Charge to Mass Ratio e/m

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Abstract

We have repeated with some modifications an 1897 experiment by J. J. Thomson investigating the cyclotronic motion of an electron beam. From the empirical data obtained, we arrive at a value for the ratio of charge to mass of an electron comparable to that commonly accepted.

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1 Introduction

1.1 Purpose and Description of Experiment

In an evacuated glass chamber (back-filled with low-pressure helium gas), electrons thermionically emitted from a heated metal filament are accelerated through a potential difference. The cyclotronic motion of the resulting electron beam due to an applied magnetic field (from a Helmholtz coil apparatus) is investigated to determine the ratio of the electrons' charge to their mass. Assuming the filament temperature to be around 4000 K, we can expect the electrons to be emitted with an average energy similar to the energy level of atoms in a gas at the same temperature,¹ i.e. $\frac{3}{2}k_B T \approx 0.5$ eV. This energy is small compared to the 300 to 500 eV imparted by the accelerating potential, so we approximate the electrons as being emitted with no kinetic energy.

Using a common calculation for the mean free path l in a gas (molecular diameter d) at low pressure P , $l = k_B T (\sqrt{2}\pi d^2 P)^{-1}$, given that the pressure in the glass chamber is ~ 1.3 Pa we find that the mean free path of the electrons is on the order of 10 cm, comparable to the circumference of the path described,² so we may treat the electrons as if in vacuum.

1.2 Theory

A Helmholtz coil apparatus was used to produce an essentially uniform magnetic field perpendicular to the path of the electrons. Griffiths³ gives the magnetic field B near the center of the coils as

$$B = \frac{8\mu_0 I}{5\sqrt{5}a}$$

where μ_0 is the magnetic constant, I is the current through the coil wires, and a is the coil radius. This makes sense, as a smaller radius and larger current would serve to increase the field, as expected. At the low pressures of gas used in the experiment the magnetic permeability does not deviate appreciably from that of vacuum. The advantage of this particular geometry is that the second derivative of the magnetic field vanishes at the midpoint between the two coils, producing a field strength reliably uniform near the midpoint.

Now the magnetic force on the electron is proportional to its velocity, which, after acceleration is on the order of 10^7 m/s. Using the relation that

$$r = \frac{vm}{eB}$$

(here, v , m , and e are the electron's velocity, mass, and charge respectively, and r is, of course, the path radius), we note that a magnetic field on the order of 1 mT will produce a path of about 5 cm, corresponding roughly to the size of the glass chamber. Luckily, that is precisely the regime of the Helmholtz coils employed.

2 Procedure

The experiment began by connecting the power supplies to their appropriate components. A 6 V AC supply was connected to the heater filament to warm the cathode for

thermionic electron emission. Another supply established a potential between the anode and cathode of the electron gun, providing the accelerating voltage V_{acc} used to impart emitted electrons with kinetic energy. A third source was connected to the Helmholtz coils, yielding a current and producing the required magnetic field.

As the cathode heated, the voltage of the Helmholtz coils was adjusted to establish a current in them of about 1.95 A. When the electron beam had grown strong enough for experimental work, the accelerating voltage V_{acc} was tweaked until a circular electron path with an approximate radius of 5 cm arose; this occurred at a voltage of about 300 V. As recommended,⁴ we determined to work with as high an accelerating voltage as possible, so V_{acc} was set to 325 V, and the measurements began. One group member sat down in front of the electron tube, and—aligning his dominant eye such that the beam exactly covered its reflection—he averaged the two radius measurements and recorded the value. We then increased the accelerating potential to 350 V, and another experimenter repeated the above procedure, measuring the new radius. This process continued for each successive value of V_{acc} . Thereafter, the current through the Helmholtz coil was lowered to 1.86 A, and utilizing the same accelerating voltages as before, we logged the electron path radius at each potential.

The system was then reconfigured by connecting a DC supply to the deflection plates. Maintaining an accelerating voltage of about 250 V, we varied the potential difference of the deflection plates from 0 to 50 V. As this value increased, the electron beam curved progressively upward, which was as expected, for the negatively charged electrons should experience a net force toward the positively charged upper plate. After turning off the equipment, the calculations described below yielded values for the desired ratio e/m .

3 Observations

3.1 Data

Table 1: Measurements for $I = 1.95$ A

V_{acc}	r [m]	e/m [$\times 10^{11}$ C/kg]	σ [$\times 10^{11}$ C/kg]
325	0.045	1.390	0.131
325	0.041	1.674	0.171
350	0.042	1.718	0.172
358	0.046	1.465	0.135
375	0.044	1.678	0.161
400	0.046	1.637	0.151
425	0.046	1.739	0.160
437	0.048	1.576	0.140
450	0.047	1.764	0.160
475	0.054	1.411	0.113
500	0.055	1.432	0.113

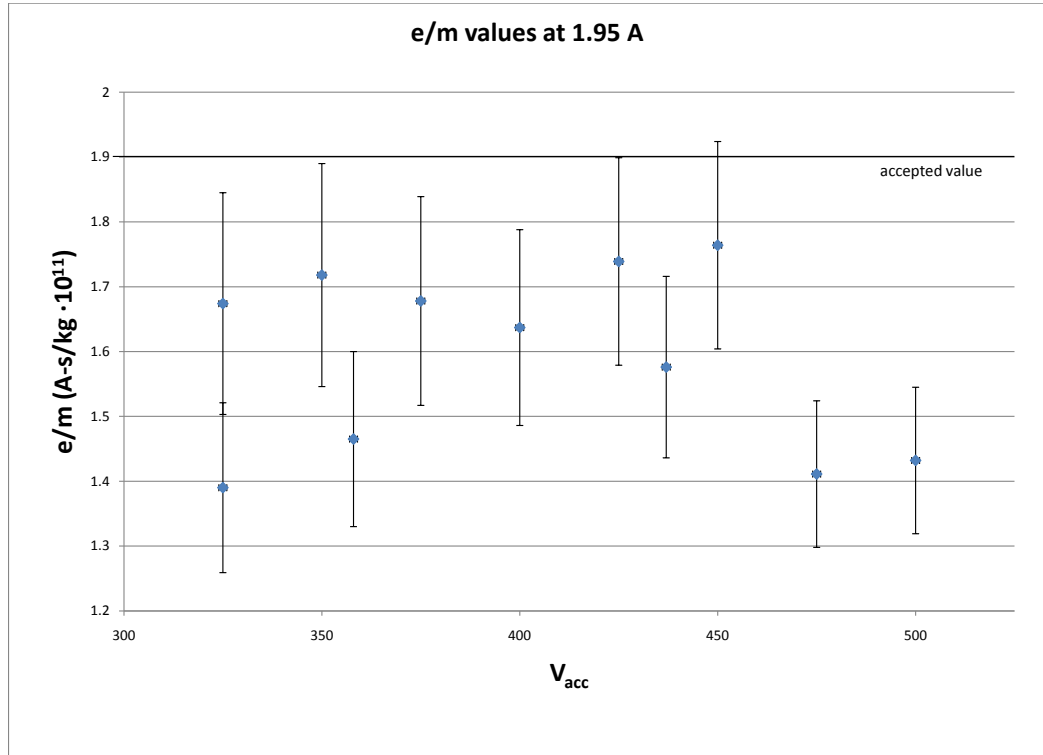
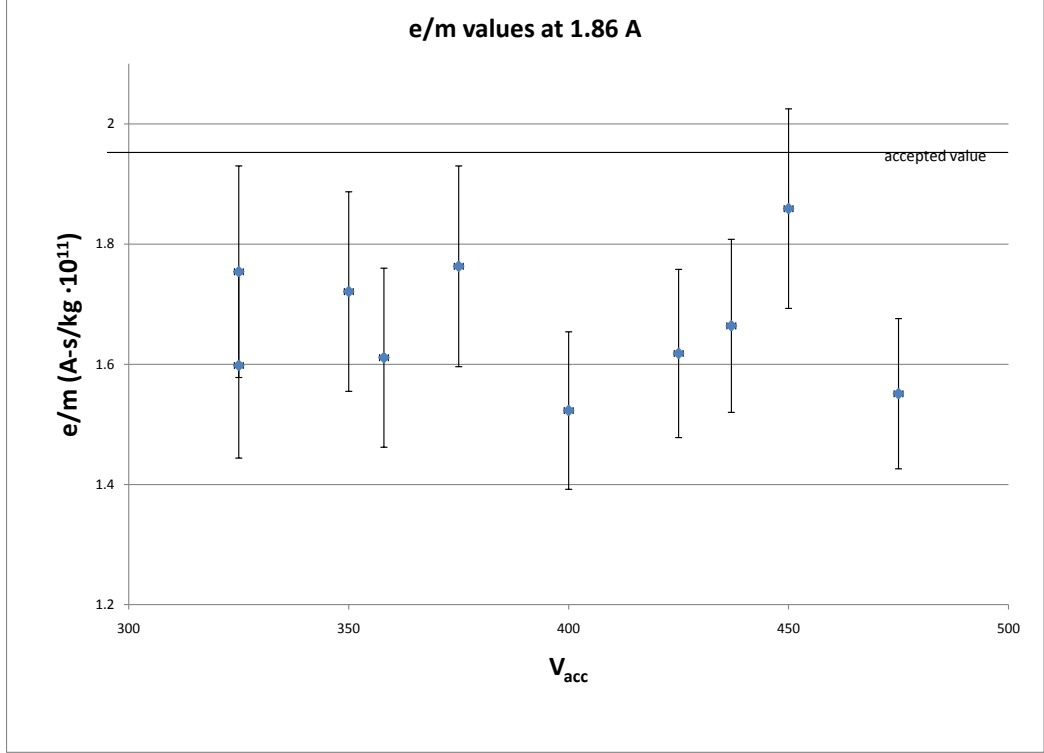


Table 2: Measurements for $I = 1.86$ A

V_{acc}	r [m]	e/m [$\times 10^{11}$ C/kg]	σ [$\times 10^{11}$ C/kg]
325	0.042	1.754	0.176
325	0.044	1.598	0.154
350	0.044	1.721	0.166
358	0.046	1.611	0.149
375	0.045	1.763	0.167
400	0.050	1.523	0.131
425	0.050	1.618	0.140
437	0.050	1.664	0.144
450	0.048	1.859	0.166
475	0.054	1.551	0.125



3.2 Uncertainty Calculations

The equation

$$e/m = 2V_{acc} \left(\frac{4}{5}\right)^3 \frac{a^2}{(N\mu_0 I r)^2}$$

was used as the calculation, taking $N = 130$, $a = 0.15$ m, $I = 1.95$ A and (as explained below) V_{acc} and r being the experimentally measured values. Uncertainty was calculated using standard techniques⁵ taking

$$\frac{\delta q}{|q|} = \sqrt{\left(2\frac{\delta I}{I}\right)^2 + \left(2\frac{\delta r}{r}\right)^2 + \left(\frac{\delta V_{acc}}{V_{acc}}\right)^2},$$

where q is the charge-to-mass quotient.

The final values were calculated using a weighted average, where each value was weighted in proportion to δ^{-2} , where δ is the uncertainty in each measurement, and the final uncertainty δ_{wav} was calculated as

$$\delta_{wav} = \frac{1}{\sqrt{\sum (\delta_i^{-2})}}.$$

The final tabulation yielded a weighted average value of $1.55 \pm 0.03 \times 10^{11}$ C/kg.

3.3 Uncertainty Analysis

There were a number of causes of uncertainty in this experiment. Some were purely based on the equipment. The V_{acc} was only measured to within one volt, so an uncertainty of ± 0.5 V was assigned for this measurement. The current varied over the course of the experiment, starting at 1.97 A and dropping to 1.94 A, so a measurement of 1.95 ± 0.03 A was given for the first trial. For the second trial, the current is measured as 1.86 ± 0.03 A. The second trial took far less time (thus, less time for variation in current supplied to the Helmholtz coils) but the current was only measured at the beginning, so the same uncertainty has been left for any rounding in the apparatus and any potential fluctuation that was not observed. Numerous sources of uncertainty were associated with measuring the radius. For instance, the equations are based on the assumption that the electrons are placed in a vacuum, which is not strictly true, as the chamber had been backfilled with helium. This would decrease the actual velocity of the electrons below the theoretical level, since they may collide with other particles.

Much uncertainty was associated with physically measuring the radius. The ruler that had been placed on the apparatus was not always in line with the diameter of the electron circle; often it was aligned with somewhere in the upper half of the circle, requiring the experimenters to “eyeball” the actual diameter. Measurement also became less precise as the radius of the circle increased and the electron path approached the edge of the containing bulb. The bulb being spherical, the rapidly changing thickness of glass through which the light passed muddled the path’s sharpness. Consequently, measurements at higher voltages (larger radii) are potentially less reliable. Also, another source of uncertainty was the fact that three different observers were used in this experiment; they may have employed slightly different estimation techniques, allowing the possibility of slightly inconsistent measuring. The uncertainty given for the estimation of the radius was ± 0.2 cm.

Finally, the magnetic field of the earth introduces some uncertainty in the experiment. However, Earth’s magnetic field has a strength of roughly $50 \mu\text{T}$ at this location on the earth,⁶ while the field produced by the experiment has a strength on the order of 2 mT. Moreover, the Helmholtz field was aligned very nearly east-to-west, so that the largest component of the Earth’s field was parallel to the electron velocities, further diminishing its effect on measurements. Clearly, therefore, the influence of the earth’s field is quite small compared to other uncertainties and can be justifiably ignored in all calculations. Moreover, relativistic effects may play a role at higher accelerating voltages (i.e. higher electron speeds) (>10 kV⁷), but are ignored for the voltage levels used in this experiment (<500 V).

4 Concluding Discussion

Admittedly, the calculated values of the electron’s charge-to-mass ratio e/m diverge appreciably from the accepted value. At both Helmholtz currents, the uncertainties in e/m fail to capture the established value of 1.76×10^{11} C/kg. Such deviation implies the presence of serious systematic error. Therefore the experimental measurements—namely, the accelerating voltage, Helmholtz current, and electron path radius—all would require closer examination in future experiments. The voltmeter and ammeter used in this laboratory

should be calibrated with standards and checked for accuracy, as consistently low voltage readings or high current measurements could certainly cause the observed error.

But the source of this discrepancy likely does not stem from these electrical measurements, which should prove fairly accurate; rather, it is the electron path radius that appears the most probable cause of error. Discerning the radius length for each trial introduces the potential for considerable human error. Tasks such as properly lining up one's eye or reading the radius value can prove difficult and imprecise, especially with little practice. Moreover, the electron charge-to-mass ratio is proportional to the inverse square of the path radius, so a systematic error, even if relatively small, would multiply in the calculation of e/m . The best way to alleviate this error would be to obtain a larger electron tube, for larger radii are easier to measure accurately. Or perhaps all that needs to be done is simply to magnify the image of the electron path. Either method should improve measurements, lowering the possibility for systematic error and hopefully yielding more accurate results for the electron charge-to-mass ratio.

Yet despite any systematic error present in this particular experiment, this electron tube apparatus proves to be very useful, not only for calculating the electron charge-to-mass ratio, but also for determining the polarity of a charge. Applied with a potential difference, the deflection plates create an electric field, which deflects charged particles in its path. And so the second portion of this experiment proved that the electron is negatively charged; the beam deflected away from the negatively charged plate toward the positively charged one. This methodology is not limited to electrons, for the process of deflection could be used to determine the nature of the charge on many different particles—with the setup adjusted accordingly, of course.

Likewise, the principles exploited in measuring the e/m ratio can be easily extended to more general particles, laying the foundation for a mass spectrometer. Charged particles traveling perpendicular to a magnetic field undergo acceleration orthogonal to both the field and their motion. So ions accelerated through a potential difference can be forced into a circular path in the same manner as the electron beam in this experiment. Then, either by exciting a noble gas (the case in this experiment) or using a detector, the radius of the ensuing path can be determined; this, when coupled with a knowledge of the accelerating potential and magnetic field, yields the desired charge-to-mass ratio. Additionally, if the charge of the particles is known, determining the mass from this ratio is trivial. And therefore the theory applied in this experiment proves to be a valuable tool in examining the nature of charged particles.

Notes

¹O. W. Richardson, *Phil. Mag.* XVI 890, 1908.

²http://en.wikipedia.org/wiki/Mean_free_path Accessed 24 Jan 2010.

³D. J. Griffiths, *Introduction to Electrodynamics* 3ed., 249.

⁴P. LeClair, “PH 255: Modern Physics Laboratory” Spring 2010, 27-36.

⁵J. R. Taylor, *An Introduction to Error Analysis*.

⁶NOAA National Geophysical Data Center.

<http://www.ngdc.noaa.gov/geomagmodels/IGRFWMM.jsp>

⁷A. Anderson, *Phys. Educ.* 11 455, 1976