Gamma Ray Attenuation

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2 Introduction

The term "gamma ray" (or γ-ray) is an approximate classification for photons of energies $E \sim 100\,\text{keV} - 10\,\text{MeV}$ ($f \sim 10^{19} - 10^{24}\,\text{Hz}$; see Fig. 7, Sec. A). When gamma rays pass through matter, their intensity may be attenuated by interactions with the matter just as with any other photons. In this experiment, we investigate the attenuation of monochromatic gamma rays through different materials. Specifically, we determine the mass absorption coefficients for lead, copper, and aluminum for a fixed gamma ray energy (662 keV) to ascertain their utility in shielding ionizing radiation.

Gamma rays have three primary mechanisms of interaction: photoelectric effect, Compton scattering, and pair production. If gamma rays are emitted in a narrow beam toward a block of intervening matter, these three interactions cause a loss of intensity along the beam direction either by deflection along a different direction or absorption. These processes are shown schematically in Fig. 1 below.

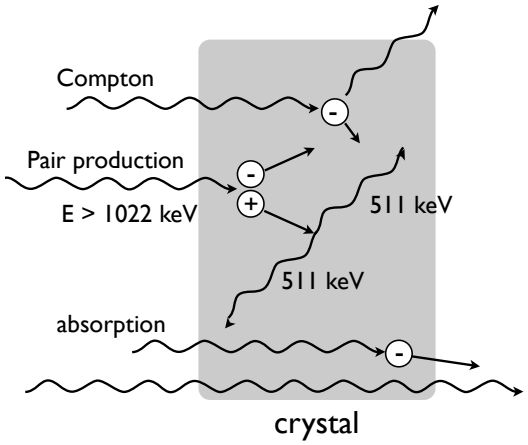

Figure 1: *Gamma rays can interact with crystals in a number of ways: Compton scattering, pair production, absorption, and photoeffect (not shown). The combination of these effects is a dispersion in energy, as well as a deflection of many gamma rays from the original beam direction.*

Thus, the interaction of high-energy X-rays and γ rays involves three different mechanisms, which we describe briefly below:

1. **Photoelectric effect:** the photons in the incident radiation are of high enough energy to release electrons from atoms and molecules in the absorbing or detecting material. Photons in the beam are thus annihilated, leading to a corresponding reduction in intensity. This photoeffect will lead to a peak in the γ energy spectrum, known as the photopeak or full absorption peak. This corresponds to gamma rays depositing all of their energy in the detector material. The energy of the gamma rays can be determined from the photopeak energy and compared to an energy level diagram for the parent nucleus and its decay products.
2. **Compton effect:** when a beam of high-frequency electromagnetic radiation passes through a material containing free electrons, an interaction takes place between the incident photons and the

free electrons. In this interaction, inelastic photon-electron scattering, energy and momentum are transferred from the photons in the incident beam to the electrons. X-ray and γ-ray energies are rather large compared to the binding energies of the outermost electrons of any atom, such that the outermost electrons can be treated as essentially free. As a result of energy transfer to the electrons in the absorbing material, both intensity and energy of the beam is reduced.

3. **Pair production:** When a positron and electron collide, they annihilate each other and produce photons. Energy conservation dictates that the total energy of the emitted photons equal that of the annihilated particles. Given that the rest energy of an electron or positron is 511 keV, the energy of the emitted photons must be at least 1.022 MeV. When high-energy photons (>1.022 MeV) interact with matter, the reverse process can also occur: the photon be annihilated into an electron-positron pair. This process conserves charge as well as linear and angular momentum, and is usually referred to as *pair production*. As with the photoelectric effect, the annihilation of photons results in a reduction of gamma intensity.

Two other mechanisms are worth noting, though they do not change the essential physics of the problem.[i] A fourth mechanism, Rayleigh scattering, corresponds to the elastic (photon-energy-conserving) scattering of light, which simply changes the direction of incident photons. Though no energy is lost, in contrast to inelastic (Compton) scattering, this process will reduce the measured intensity of radiation simply because some photons will be deflected out of the detection region. Essentially, at a given energy this may be regarded as a geometrical factor.

Finally, in principle must also consider direct absorption of incident photons by the nucleus itself, photonuclear absorption. This process usually results in the ejection of one or more neutrons and/or protons. This interaction can contribute $\sim 5-10\%$ to the total photon interaction, though within a fairly narrow energy region usually occurring somewhere between 5 MeV and 40 MeV. At the energies of the present experiment (<2 MeV), this effect is negligible.

2.1 Gamma Absorption

In the simplest model, we do not distinguish between the different types of interactions discussed above, but simply presume that at a given incident γ energy, the number of gamma rays ΔN removed from the incident beam of original intensity N_0 is proportional to the number of electrons and nuclei along the gamma ray's path through the material. That is, we assume a constant attenuation coefficient for the beam, which may be viewed as an energy-dependent weighted sum of separate attenuations from all three mechanisms above.

The number of electrons and nuclei encountered is itself proportional to the density of the material ρ, and the path length traveled in the material x. The coefficient of proportionality is known as the *mass*

[i] Of course, there are still other mechanisms, such as nuclear-resonance scattering and Delbrück scattering, but they are negligible from our point of view.

attenuation coefficient μ, and has units of inverse distance. The loss of gamma ray intensity can then be expressed as

$$\Delta N = -N_o (\rho x) \left(\frac{\mu}{\rho}\right) \tag{1}$$

Typically, one deals with the density-normalized mass attenuation coefficient μ/ρ, which has units of cm^2/g, rather than the raw mass attenuation coefficient. Similarly, the quantity ρx is known as the "mass thickness," with units of g/cm^2. In any case, the solution to this first-order differential equation is well-known:

$$N(x) = N_o e^{-(\rho x)(\mu/\rho)} \tag{2}$$

$N(x)$ is the number of gamma rays remaining after passing through a material of thickness x. Two reduced quantities determine the gamma ray attenuation: the density-normalized attenuation coefficient μ/ρ, and the mass per unit thickness (often called "mass thickness") ρx, which has units of g/cm^2. An exponential law is not unexpected in this case, one obtains the same solution for any system in which the primary quantity grows or diminishes at a fixed rate.

2.2 Attenuation Coefficient

The simplistic model outlined above would certainly seem reasonable for a single energy-independent mechanism for beam attenuation. Considering all three interactions, however, we must at least consider the density-normalized attenuation coefficient to be a function of energy. Typically, the quoted coefficients are a sum of attenuation coefficients for the principle photon interactions:

$$\mu = \mu_{pe} + \mu_{coh} + \mu_{incoh} + \mu_{pp} \tag{3}$$

Here μ_{pe} is the coefficient for the photoelectric effect, μ_{coh} for coherent (Rayleigh) scattering, μ_{incoh} for incoherent (Compton) scattering, and μ_{pp} for pair production. Overall, when dealing with *monochromatic* radiation, one may simply consider an effectively constant μ at that particular energy, and not worry about the detailed contributions to the effective μ.

However, each process has a characteristic energy dependence, making μ a strongly varying function of energy. The photoelectric attenuation decreases rapidly with increasing energy, leading to an overall decreasing background for $\mu(E)$. The Compton effect has a roughly constant attenuation until ~ 0.1–1 MeV, after which it decreases rapidly with increasing energy. Pair production is possible only above 1.022 MeV, and increases rapidly for higher energies. Taken together, $\mu(E)$ is a strongly decreasing function of energy until a few MeV, after which it increases gently, shown in Fig. 2. At lower energies, one must also factor in characteristic x-ray absorptions.[ii]

[ii] The X-ray absorption energies correspond to incident photons matching the energy level spacing of the atoms in the detector material. This leads to strong absorption at these particular energies, and hence, strongly increased attenuation.

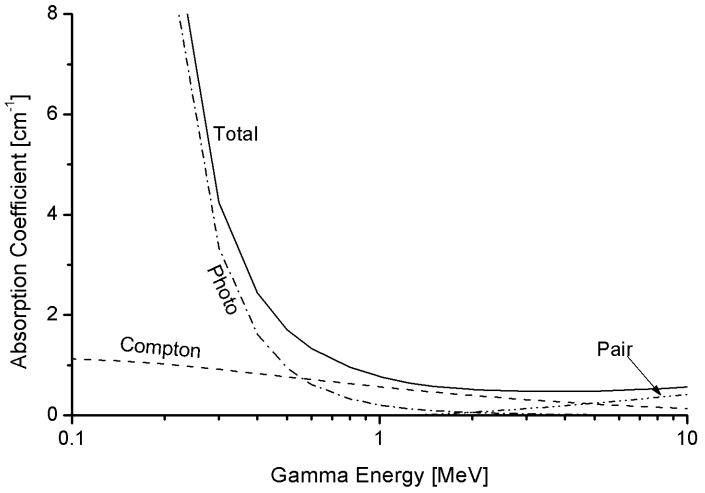

Figure 2: *The total absorption coefficient of lead (Z = 82) for gamma rays. From* `http://en.wikipedia.org/wiki/Gamma_ray`

Figure 3 shows the actual density-normalized mass attenuation coefficient for Pb as a function of photon energy. From this one can see the superimposed effects of Compton scattering, photoelectric effects, and pair production as well as characteristic X-ray absorption energies. At the energies used in this experiment (^{57}Co and ^{137}Cs gammas), the value of μ should vary by approximately a factor of $20-30$.

3 Methods

A Spectech UCS-30 spectrometer with a NaI(Tl) detector and multi-channel analyzer (MCA) was used to measure gamma ray energies and intensities.

3.1 Energy Calibration

Quantitative gamma ray spectroscopy first requires that a proper energy calibration be performed. In order to perform a calibration, it is first necessary to acquire a known spectrum from a specific isotope. In this case, a ^{137}Cs source was used to calibrate the spectrometer over the range of $0-1$ MeV, sufficient for the present experiment. The 32 keV X-ray and 662 keV ^{137}Cs photopeak were used in conjunction with the "Auto Calibrate" routine provided in the Spectech software. This routine performs an acquisition sequence that attempts to determine the optimum detector voltage and gain settings using known positions of the X-ray and photopeaks. Once the peaks are located and positioned, the energy calibration coefficients are calculated to provide a two-point calibration with the 32 and 662 keV peaks.

In this experiment, we are only looking at the relative intensity of 662 keV gamma rays. Since the 662 keV photopeak is the most prominent feature in the ^{137}Cs spectra and easily identified, the absolute energy calibration is not of critical importance in this case.

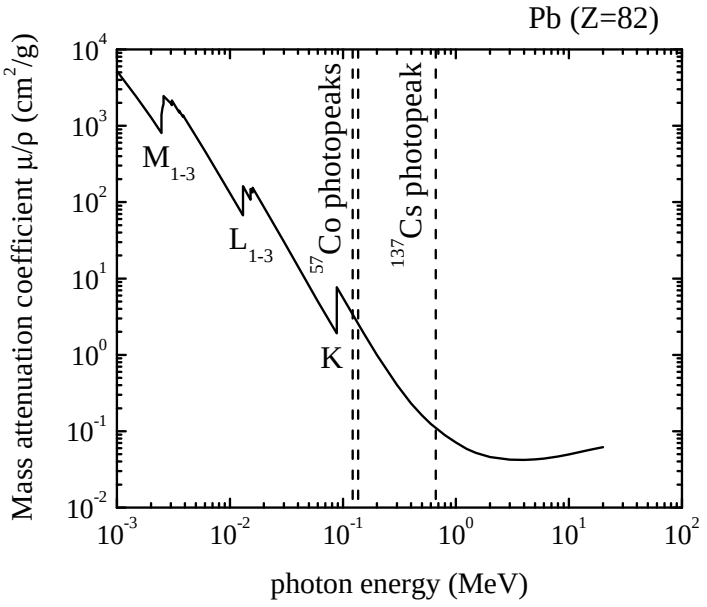

Figure 3: *Density-normalized mass attenuation coefficient μ/ρ for Pb ($Z = 82$) as a function of photon energy. Note the logarithmic scales. The energies of primary ^{57}Co and ^{137}Cs gamma ray photons are noted, as are the K, L, and M absorption energies for Pb.[?]*

3.2 Procedure

Once the energy calibration was performed, we recorded the intensity of the ^{137}Cs photopeak as a function of the thickness of intervening Al, Cu, or Pb following the procedure below.

1. The NaI detector was placed on top of the slotted stage, with a surrounding Pb shield.
2. The ^{137}Cs source was placed in the fifth slot from the top of the slotted stage using a plastic holder, a distance of approximately X cm from the NaI(Tl) detector.
3. A spectrum of the ^{137}Cs source alone was acquired for 300 s, shown in Fig. 4, and the intensity of the 662 keV photopeak was recorded.
4. With the source in the same position, a sheet of Pb was inserted between the source and detector. The thickness of this sheet was measured with the calipers provided to an accuracy of ± 0.1 mm.
5. Another spectrum was acquired for 300 s using the steps above, and the 662 keV peak intensity was recorded.
6. This procedure was repeated for increasing numbers of Pb sheets (whose thicknesses were individually measured) until the peak intensity was reduced by roughly a factor of 10 compared to that without Pb sheets.
7. A second spectrum without Pb sheets was recorded to verify that the unattenuated intensity had not changed.
8. The steps above for sheets of Cu and Al.

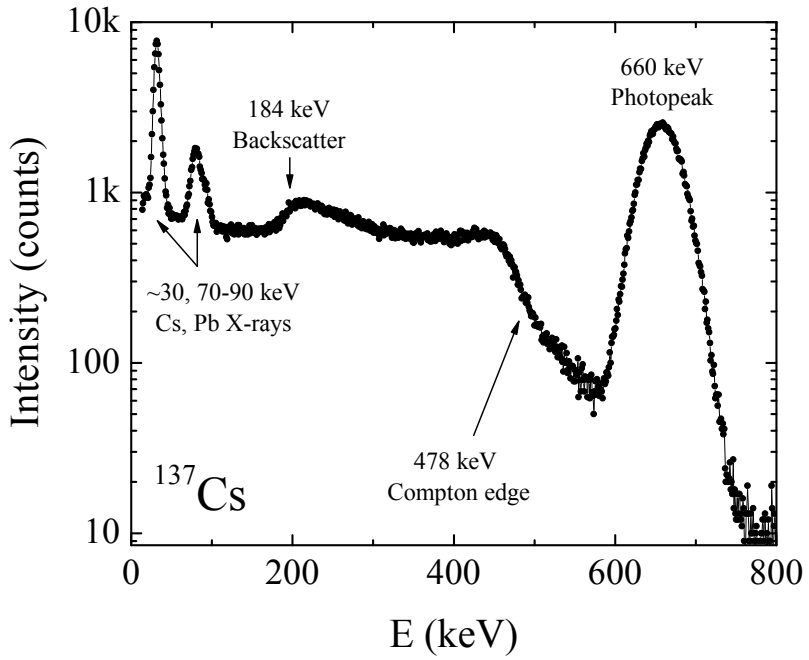

Figure 4: *Gamma spectrum of ^{137}Cs, acquired for 300 s at a distance of X from the ^{137}Cs source. Aside from the main photopeak at ≈ 660 keV, clear Compton edge and backscatter features are visible, as well as X-ray absorption edges. Note the logarithmic scale on the vertical axis.[?]*

4 Results

As discussed briefly above, a narrow beam of monoenergetic photons with an incident intensity I_o[iii], penetrating a layer of material with thickness x and density ρ emerges with intensity I given by an exponential attenuation law

$$\frac{I}{I_o} = \exp\left[-\left(\frac{\mu}{\rho}\right)(\rho x)\right] \tag{4}$$

We can put this in a form more amenable to analysis by taking the natural logarithm of both sides:

$$\ln I = \ln I_o - \frac{\mu}{\rho}(\rho x) \tag{5}$$

Given a material of known density and thickness, it is clear that if we plot $\ln I$ (y-axis) versus ρx (x-axis), we will obtain a straight line of slope $-\mu/\rho$. Thus, by measuring the gamma ray intensity as a function of intervening material, we can extract the density-normalized mass attenuation coefficient.

For each attenuating material, a table was made recording the thickness of the material x and the number of counts for the 662 keV photopeak, including $x = 0$ corresponding to the ^{137}Cs source without any intervening material. From this table, we constructed a lot of $\ln I$ versus the product of material thickness and density ρx, also known as the "mass thickness." A weighted linear regression analysis was used to determine the slope of the plot, which according to the model above yields $-\mu/\rho$. In order to compare

[iii] Intensity is proportional to the number of photons detected, so this is not at odds with our previous discussion.

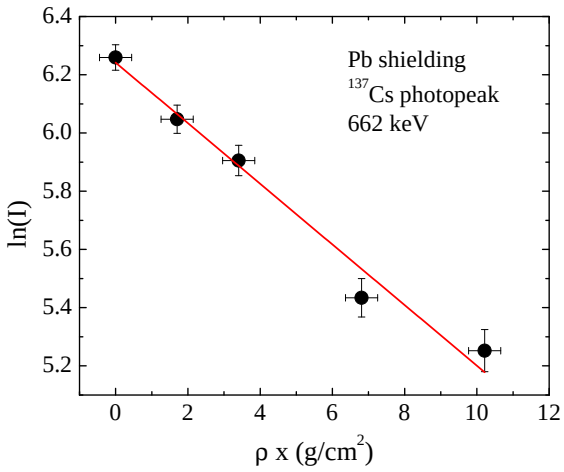

Figure 5: *Natural logarithm of ^{137}Cs gamma ray (661 keV) intensity as a function of lead mass thickness ρx. The accepted value of μ/ρ for Pb at 661.6 keV is 0.105 cm^2/g.*

with the NIST database, we have used units g and cm. The uncertainty in the material thickness was determined from the accuracy of the calipers used, while the uncertainty in the photopeak intensity was determined from the square root of the peak intensity, presuming a Gaussian distribution. The uncertainty in the logarithm of the peak intensity can then be found by standard uncertainty propagation to be $\delta(\ln I) = \sqrt{I}/I$. The uncertainty in density was neglected, as it was several orders of magnitude smaller than any other uncertainties present in the experiment.

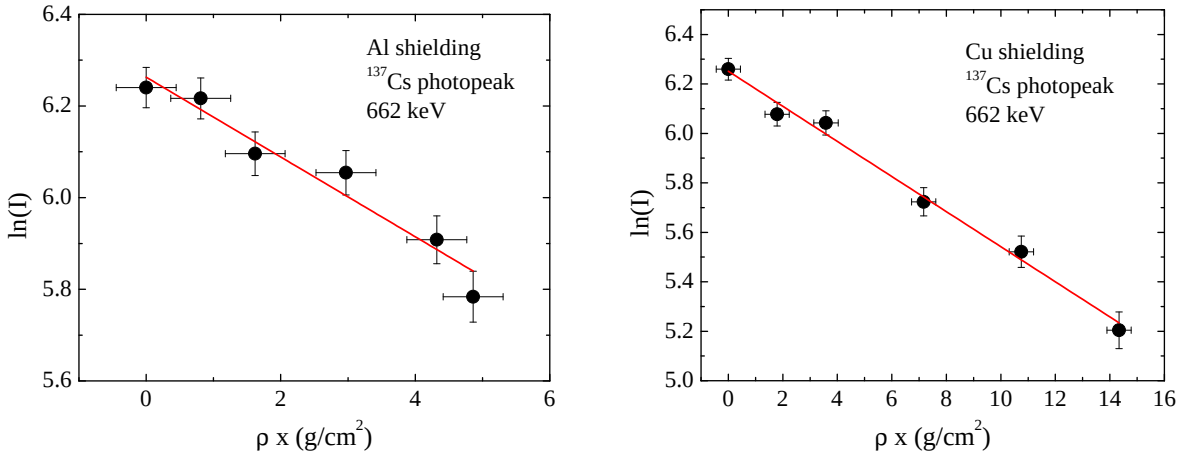

Figure 6: *Natural logarithm of ^{137}Cs gamma ray (661 keV) intensity as a function of Al and Cu mass thicknesses ρx. The accepted value of μ/ρ for Cu at 661.6 keV is 0.105 cm^2/g.*

The values of μ/ρ were compared with accepted values from the NIST database, using linear interpolation between table entries to find the effective mass attenuation coefficients at 662 keV. Our results and the accepted values are summarized in the table below.

In addition to the value of μ/ρ determined from the weighted linear regression analysis of ln I versus ρx,

Table 1: *Mass attenuation coefficients at* $662\,keV$

	μ/ρ (g/cm^2)		
Element	**Experiment**	**Accepted**	R^2
Al	0.087 ± 0.010	0.0731	0.938
Cu	0.071 ± 0.004	0.0750	0.986
Pb	0.104 ± 0.008	0.1136	0.976

we have included the standard error in slope determination (as an uncertainty for μ/ρ) and the R^2 values for the regression.

5 Discussion

For all three materials studied, the gamma ray intensity as a function of intervening mass thickness were well-described an exponential decay, consistent with the model presented in the introduction. The measured mass attenuation coefficients μ/ρ for Pb and Cu agree with the standard NIST values within the experimental uncertainty, accounting for both statistical uncertainty in the gamma ray intensity and thickness of the intervening plates. However, the coefficient for Al is approximately 1.4 times the standard error from the accepted value ($1.4\sigma_{\overline{x}}$). The probability of a discrepancy this large or larger is only 16% based on the normal distribution, i.e., the discrepancy is significant at the 16% level. In our view, given that the coefficients for Cu and Pb are within approximately $\sigma_{\overline{x}}$ of the accepted values, the measurement for Al may suffer from one or more systematic uncertainties.

The more significant deviation of the Al coefficient from the accepted value has, in our opinion, one primary origin. The range of Al mass thicknesses ρx covered in the experiment is a factor of two or more less than that for Cu or Pb, and as a result the gamma ray attenuation is less than a factor three for even the largest thickness studied. This increases the relative error in determining μ/ρ, but also makes the regression analysis more questionable – a similar discrepancy would be observed in the Pb data, were we restricted to $\rho x < 6\,g/cm^2$ as we are with the Al data. Put simply, the decrease in gamma ray intensity should ideally be observed over at least a decade for an accurate determination of ρ/μ, but this would result in unwieldy thicknesses of Al in the present experiment. A related problem is the limited number of points: we have data for only 5 separate thicknesses of Al, which necessarily limits our accuracy in determining ρ/μ.

As a practical "figure of merit" for characterizing the shielding properties of Al, Cu, and Pb against 662 keV gamma rays, it is a simple matter to calculate the thickness required for a factor of 10 reduction in intensity from Eq. 4. If the required thickness is x_{10}, simple algebra yields

$$x_{10} = \frac{-\ln 0.1}{(\mu/\rho)\,\rho} \approx \frac{2.303}{(\mu/\rho)\,\rho} \tag{6}$$

where x_{10} is in cm if μ/ρ is in cm^2/g and ρ is in g/cm^3. A secondary consideration is the mass per unit area for the given thickness x_{10}, which is given by ρx_{10}. Table 2 summarizes our analysis.

Table 2: *Thickness for factor 10 reduction in 662 keV gamma intensity*

Material	x_{10} (cm)	ρx_{10} (g/cm^2)
Al	9.55	25.8
Cu	3.58	32.1
Pb	2.01	22.8

6 Conclusions

The attenuation of 662 keV gamma rays by Al, Cu, and Pb were studied as a function of mass thickness. We found that for all three materials, the gamma ray attenuation is well-described by an exponential decay, with Pb yielding the largest mass attenuation coefficient. For Pb and Cu, the mass attenuation coefficients are in agreement with accepted values (within the standard error of the experiment), while for Al the mass attenuation coefficient is somewhat larger than expected (1.4 times the standard error). The discrepancy is tentatively attributed to the limited range of Al mass thicknesses available for study in the present experimental geometry.

While Pb has superior shielding properties for a given thickness, the smallest minimum thickness for a factor of ten reduction in intensity, and the lowest weight at minimum thickness, the restrictions on the use of hazardous substances makes its use problematic. The use of a Cu shield incurs a 50% increase in weight, but the extra cost may be offset by considerable savings in processing, safety, recycling, disposal, etc.

7 References

G. Audi and A.H. Wapstra, "Masses, Q-values and nucleon separation energies: The 1995 update to the atomic mass evaluation," Nuclear Physics, vol. A595, pp. 409-480, 1995. `http://nucleardata.nuclear.lu.se/nucleardata/toi/`.

J. Hubbel and S. Seltzer, "Tables of x-ray mass attenuation coefficients and mass energy-absorption coefficients." NISTIR 5632. Available online at `http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html`.

R.S. Peterson,"Experimental γ ray spectroscopy and investigations of environmental radioactivity," 1996. Hard copy available in the laboratory; soft copy available on spectrometer computer.

J.I. Pfeffer and S. Nir, *Modern Physics*. Imperial College Press (London), 1st ed., 2000.

Spectrum Techniques, "UCS-30 Universal Computer Spectrometer," software and hardware manual. Hard copy available in the laboratory; soft copy available on spectrometer computer.

J.R. Taylor, *An Introduction to Error Analysis*. University Science Books (Sausalito, Ca), 2nd ed., 1997.

A Electromagnetic Spectrum

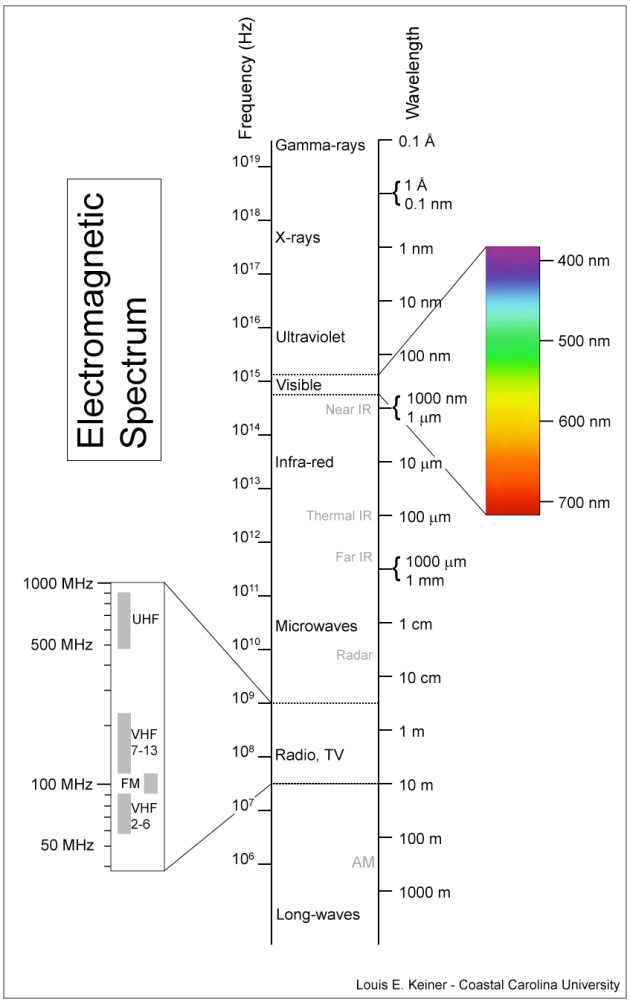

Figure 7: *The electromagnetic spectrum. From* `http://en.wikipedia.org/wiki/Electromagnetic_spectrum`.

B Density of shielding materials

Table 3: *Densities of absorber materials*

Element	Density (g/cm^2)
Al	2.70
Cu	8.96
Pb	11.35

C Additional data for ^{137}Cs

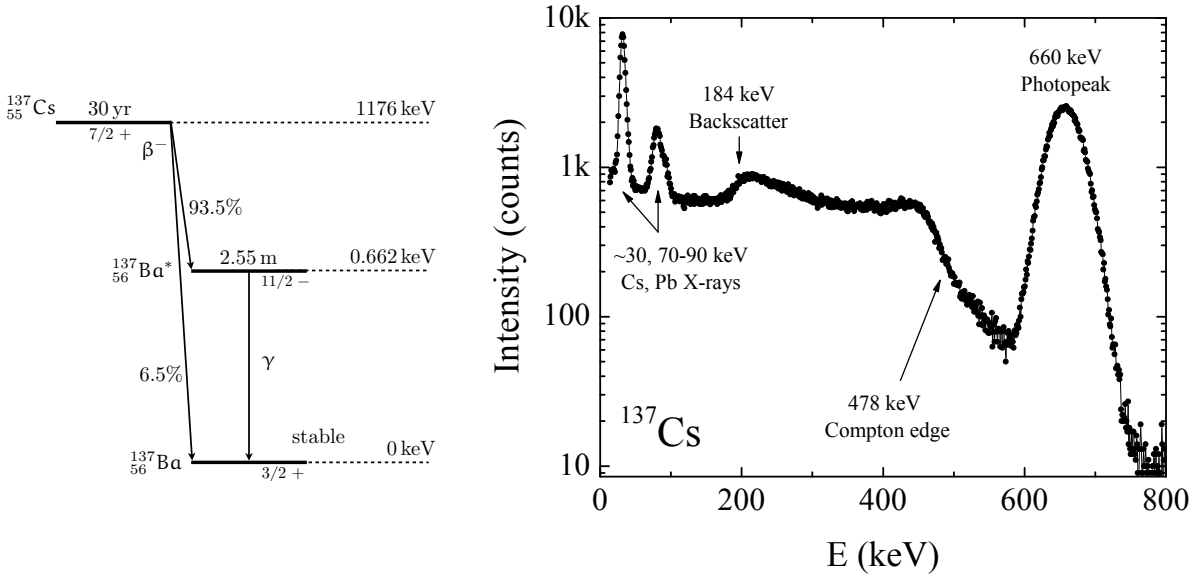

Figure 8: Left: *Energy level diagram showing the decay of ^{137}Cs to ^{137}Ba. The ^{137}Cs decays via two paths, 93.5% follow a two-step process resulting in the emission of a 662 keV gamma ray. The decay of the Ba nucleus from its excited state is an internal transition, and contributes to the decay percentages but not the gamma decay factor.* **Right:** *Gamma spectrum of ^{137}Cs. Aside from the main photopeak at ≈ 660 keV, clear Compton edge and backscatter features are visible, as well as X-ray absorption edges. Note the logarithmic scale on the vertical axis.*

A ^{137}Cs nucleus can decay via two routes to the ^{137}Ba ground state, shown schematically in Fig. 8. First, a single beta emission of 1.176 MeV can occur with no subsequent gamma emission, bringing the ^{137}Cs directly to the ground state. However, this single-step process occurs for only about 6.5% of all nuclei. The other 93.5% decay via two-step process. First, a 514 keV beta is emitted, resulting in metastable and short-lived ^{137}Ba*. The metastable ^{137}Ba* subsequently (2.55 min half life) decays to the ^{137}Ba ground state by emission of a 662 keV gamma.