Experiment in Muon Decay

J. Lukens, B. Reid, A. Tuggle
PH 255-001, Group 4

3 May 2010

Abstract

We describe a procedure for detecting and timing muon decays in a scintillation detector, calculating a value for the muon's mean lifetime. In addition, the collected data is used for several other calculations, including for the ratio of positive to negatively charged muons on Earth's surface and the Fermi coupling constant.

Contents

1   Introduction

Muons, elementary particles with no known internal structure, are negatively charged leptons (though positively charged antimuons are also called "positive muons," as herein), sometimes referred to as "heavy electrons" due to the similarities in their properties. Formed high in Earth's atmosphere by energetic cosmic rays, muons are interesting to physicists for several reasons, most obviously because of their status as a fundamental building block of the universe. In addition, muons are one of the most-stable and longest-lived, of the elementary particles—only free neutrons live longer. They can also be found in abundance here on Earth's surface, without having to be produced in a laboratory setting.

1.1   Purpose of the Experiment

The purpose of this particular experiment is to study the decay of muons. We have investigated the distribution of decay times among muons to confirm that the decays are independent, i.e. that the decay rate is proportional to the number of muons present. These studies allow us to report a mean muon lifetime—this is the principle result of this experiment. From the mean muon lifetime, we determine a value both for the ratio of anti-muons to muons and for the (pre-Standard Model) Fermi coupling constant.

1.2   Description and Theory of the Experiment

Muons are formed in the upper atmosphere by cosmic rays incident on molecules in the air. The high energy rays produce a shower of particles, among which are charged pions, which decay into muons of the same charge accompanied by neutrinos. It is these muons which comprise nearly all of the specimens for this experiment. Muons have a mass of about $106\,\mathrm{MeV}/c^2$, so the rays producing them must carry at least $106\,\mathrm{MeV}$ of energy, for muons to be produced. The muons rain down through the atmosphere, but are themselves unstable and eventually decay, though, due to relativistic time dilation, appear to live longer to observers in the Earth's reference frame than in their own.[1] They would otherwise be significantly less likely to reach ground level.

As the muons travel toward the earth, a simple scintillation detector is all that is needed for detection. We defer to T. Coan *et. al.* [2] for details concerning the detection and discrimination of muon decays, but will briefly summarize the important points. The scintillator consists of polyvinyltolulene doped with the organic semiconductor anthracene, which fluoresces when

impinged on by radiation. Using photomultiplier tubes to measure this fluorescence gives indication of a muon entering the device.

But muons decay into an electron or positron (again with neutrinos, which are far beyond our detection limits) which also cause scintillation fluorescence. Monitoring the signals from the photomultiplier tubes and timing durations between them allows us to measure the lifetime of muons in the scintillator. But muons are not the only thing the scintillator will detect— almost any energetic radiation can cause anthracene to fluoresce, and though we can use a voltage discriminator to screen out very low energy events, there will definitely remain a background of unwanted radiation.

In addition, remember that there are two types of muons, positive and negative, with different properties, including, most relevantly, different mean lifetimes. The observed mean lifetime, then, is a weighted average of the lifetimes of the two different muon types. Given the lifetimes of each of these particles, the observed mean lifetime then tells us about their relative abundance. Muons (negative), because they can interact with protons through the electroweak force, have shorter lives on average than antimuons.

2 Procedure

The procedure for data collection began with familiarization of the experimental equipment and software, for no explicit lab guidelines were given. Therefore our first day of work consisted primarily of experimenting with various sets of data, during which we sought to gain proficiency with the muon apparatus and to determine experimental objectives. In particular, we utilized the instrument titled "Muon Physics" produced by TeachSpin.[3] Consisting of a plastic scintillator, photomultiplier tube (PMT), and signal amplifier, this apparatus measures the time between successive light pulses, which amounts to the raw data taken as the muon lifetime. A muon entering the scintillator imparts some of its energy to the fluorine molecules present, exciting electrons to higher energy states which then emit light as they return to their original levels. Likewise, when a muon decays, a newly formed electron—a product of such decay—emits light as well. And therefore the time between pulses is a measure of the muon's lifetime. Yet not all successive pulses represent decay, for particles other than muons can prove responsible, and some muons can easily pass through the scintillator without decaying at all. The equipment, however, at least partially compensates for these isssues: if a triggering pulse is not followed by a successive signal within the allotted time frame of $20\,\mu\mathrm{s}$[4]—most likely signifying that the muon has failed to decay within the scintillator—the timer resets. Moreover, the accompanying

software employs an algorithm to remove superfluous information from the raw data times, freeing us from tedious manual inspection of the original file.

The basic setup follows in Figure 1. After ensuring proper connections on the HV power supply and PMT output, we opened and ran the computer program `muon` to begin collecting data. Special attention was given to the "Monitor" panel on the graphical interface; Figure 2 provides an image of the complete program display. This section furnished a running total of the number of muons observed to decay and the total muons detected—which actually corresponded to the number of light triggers, not necessarily due to muons alone. As expected, our initial analysis revealed the exponential behavior of such decay times. Actual muon data previously collected and saved on the computer were then loaded into Microsoft Excel and graphed. But the raw data failed to yield a mean muon lifetime comparable to the accepted range near $2\,\mu\text{s}$; this issue was corrected, however, through use of the executable file `sift`. This program claimed to eliminate any data records incompatible with actual muon decay. With this correction, the exponential distribution describing the data yielded a best-fit curve with a mean of approximately $2.19\,\mu\text{s}$, well within plausible limits. And therefore we had discovered the basic procedure necessary to obtain meaningful data: use the program `muon` to record the prospective muon decays; then run `sift` on the resultant raw data file to yield the actual decay distribution.

At this point, with a greater handle on the capabilities of our equipment, we took time to formulate the experiment's objectives. Drawing largely from "Muon Physics" we selected four goals, all pertaining to the specifics of muon behavior:

1. Determine the mean lifetime τ from the distribution of decay times.

2. Calculate the Fermi coupling constant G_{F} using an accepted value for the muon mass.

3. Compute the ratio of positive to negative muons at ground level.

4. From the total muon counts, estimate muon flux at the experimental location.

Unfortunately, one of the most fascinating applications of this muon apparatus—confirmation of relativistic time dilation—could not be implemented, for our experimental equipment was confined to a single elevation over the duration of data collection. Nonetheless, the objectives we selected represented a broad spectrum of important muon data, so our chosen goals were deemed worthwhile.
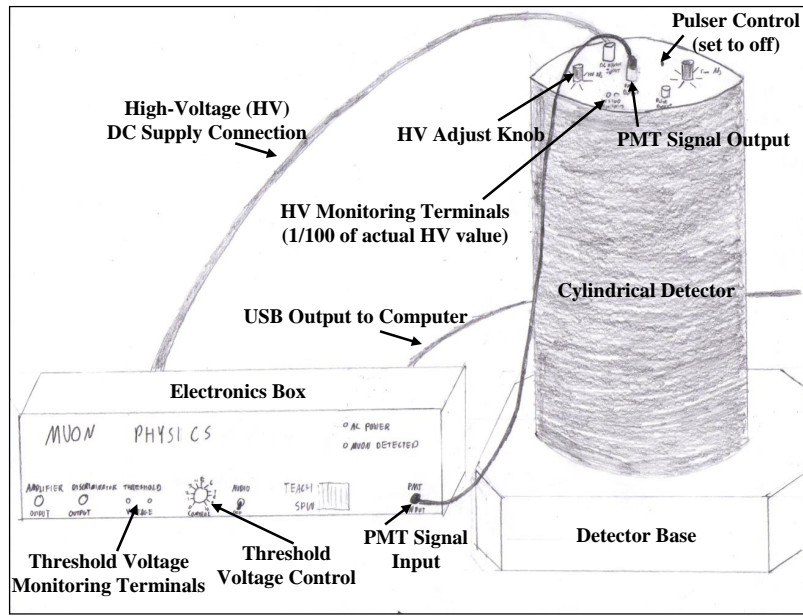
Figure 1: Schematic of experimental setup. For convenience, only those components relevant to the current experiment receive explicit labels.
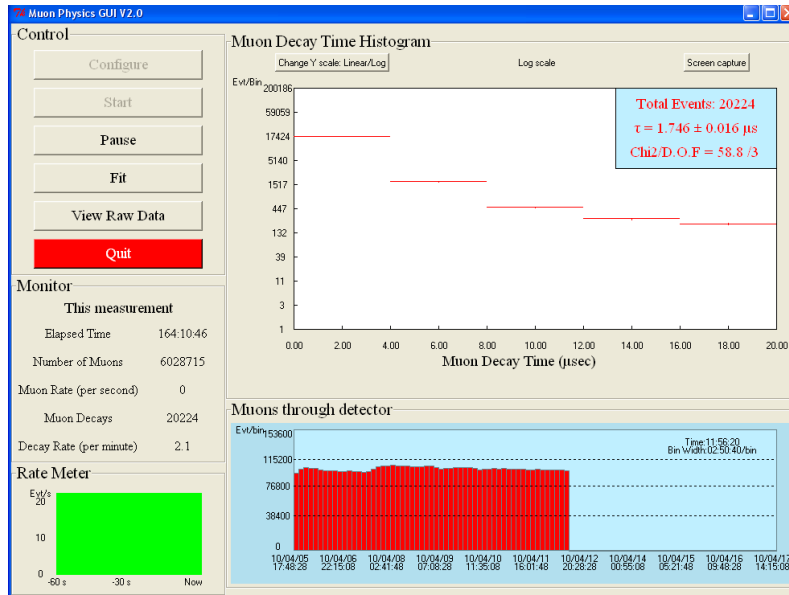
Figure 2: Graphical interface of the program `muon`. The section titled "Monitor" located on the left side of the screen proved the most useful in calibrating the apparatus. Since more precise fitting was required, the muon lifetime τ given here was discarded in favor of more careful analysis. (Image taken after terminating data collection.)

Yet before actual data collection could ensue, one more variable needed to be examined: the voltage of the high-voltage (HV) supply to the PMT and the threshold potential of the discriminator itself. Since the functions of both were coupled for the experiment—that is, the HV supply controlled the size of the amplified pulse, and the discriminator determined the threshold voltage triggering the counting of an event—we set the HV supply to 1170 V, and we varied only the discriminator. Although the attached software in theory could eliminate all extraneous detections, we did not set the discriminator threshold to zero, but instead adjusted it to purge some of the obvious non-muon events. In particular, the threshold was tweaked to 87.4 mV, for at this point the muon counter gave an approximate flux of 1 muon $\mathrm{min}^{-1}\mathrm{cm}^{-2}$ (the approximate value at sea level). And with these levels established, we commenced final data collection. Starting a fresh round of decay time calculations, we let the program `muon` run for the duration of the following week in order to obtain a comprehensive data file; from this would stem our final analysis.

When we returned to the laboratory seven days later, the collection program was halted and the data file saved. The application `sift` then yielded a revised record of muon decays which eliminated superfluous data. However, glitches nonetheless remained, as some of the lower times possessed extremely high counts, so these were manually purged from the distribution; likely the detector was overly sensitive, sometimes counting the same event twice and thus generating inflated high counts in the low time range. Moreover, Excel proved insufficient in generating a best-fit curve; not only was it unable to produce a trendline combining an exponential and a constant—the distribution which best accounted for any background interference in our data—but it also could not weight the fit according to the square root of the number of counts. For this reason Origin software found use instead, which successfully fit the distribution accounting for both a constant shift and relative error. And from this trendline we extracted the mean muon lifetime τ and its corresponding uncertainty; straightforward calculations—as given in the following section of this report—then yielded our experimental values for the Fermi coupling constant and the ratio of positive to negative muons at ground level, and the procedure proved completed.

3 Data, Calculations, and Uncertainty

3.1 Mean Muon Lifetime τ

Lifetime was taken from a best-fit plot using an equation of the form

$$N(t) = Ae^{-\frac{t}{\tau}} + B.$$

Data for time in the range of 0–20 μs were plotted as a histogram, using a bin size of 80 ns. Several adjustments had to be made before the data were deemed acceptable. The first 80 ns or so, unlike the rest of the graph did not display an exponential decay relationship for the data. Rather, they were increasing and deemed to be unreliable data. Moreover, the data for high time measurements were deemed less reliable; fewer decays were observed so the $\delta N/N$ ratio was higher (taking $\delta N \equiv \sqrt{N}$). Therefore, these data were less significantly significant and were weighted less accordingly. The final value was tabulated to be $\tau = 2.14 \pm 0.03$ μs, in accordance with the expected value of somewhere between 2.043 μs (for negative muons) and 2.197 μs (for positive muons).

Some uncertainty existed in our measurements from the possibility of spurious events (muon "decays" detected may not be true muon decays). Based on the histogram data, there were about 5 spurious decays per bin; with a bin size of 80 ns, the total number of spurious events turned out to be roughly 5 decays per bin times 20 000 ns divided by 80 ns per bin—approximately 1250 spurious events. This is a noticeable percentage of the roughly 20 000 events recorded.

It is natural to ask the question where such spurious events come from. Any particle with enough energy to be picked up by the detector could conceivably be detected as a muon decay. If an electron or photon had enough energy, it might be picked up. How many spurious detections would result from these particles?

First we look at how much time was actually spent "looking" for muon decays. This would be the number of muons observed multiplied by the observation time for each muon: $6\,028\,717 * 20$ μs $= 120$ s. Thus only 120 s out of the whole week was actually spent where the machine would interpret a "hit" as a decaying muon rather than a newly detected muon. Thus, the probability that any given time was ready for a muon detection is 2.03×10^{-4}. Thus, these alleged electrons causing spurious events should actually be setting off the device far more often than the observed 1250 spurious events; in fact, the number should be 6.16×10^{-6} of the supposed detections.

However, there is a more likely source of the extra muons. The machine records a hit whether a new muon enters or a decay takes place. Taking

the total detected muons divided by the total (week-long) detection time, we have 10.13 muons detected per second. This is equivalent to an average of 2.03×10^{-4} muons detected in the standard "wait time" of $20\,\mu\text{s}$. At such low numbers, this should be close to the probability that at least one muon is observed during this time frame, and thus that the muon detector could detect an energy burst from a second entering muon and interpret it as a decay of the original muon during the observation time. Multiplying probability by the total number of trial periods (6 028 715 muons detected), roughly 1200 spurious events would be expected, matching very well with the spurious events count previously estimated. We consider extra muon appearance to be the most likely source for the vast majority of the spurious muon "decays."

Additional systematic uncertainty exists from the fact that certain energy parameters were set for muon detection. They were designed to cover the general range expected for muons, but high and low energy values could be ignored; thus, some muons could have escaped detection.

3.2 Charge Ratio

To determine the ratio ρ of positively charged muons to negatively charged muons, we take the formula

$$\rho = -\frac{\tau^+}{\tau^-}\left(\frac{\tau^- - \tau_{\text{obs}}}{\tau^+ - \tau_{\text{obs}}}\right),$$

where $\rho = N^+/N^-$ and τ values represent the muon lifetime of positive muons, negative muons, and the experimentally observed lifetimes, respectively, viz. $\tau^+ = 2.197\,\mu\text{s}$; $\tau^- = 2.034\,\mu\text{s}$; $\tau_{\text{obs}} = 2.14 \pm 0.03\,\mu\text{s}$.

Uncertainty was considerable; it was calculated as

$$\delta\rho = \frac{\partial\rho}{\partial\tau_{\text{obs}}}\delta\tau_{\text{obs}},$$

since τ_{obs} had considerably more uncertainty than either of the other (accepted) τ values.

Unfortunately, these do not yield precise results, coming out to $\rho = 2 \pm 2$ with appropriate significant figures. Ignoring significant figures, however, we have 1.83 ± 1.53, a better result that leaves a range of 0.3 to 3.36. Though far less precise than desirable, it is notable that the expected value of about 1 fits comfortably in this range, being the geometric mean of the low and high values (each is off by a factor of about 3.3 from the expected value).
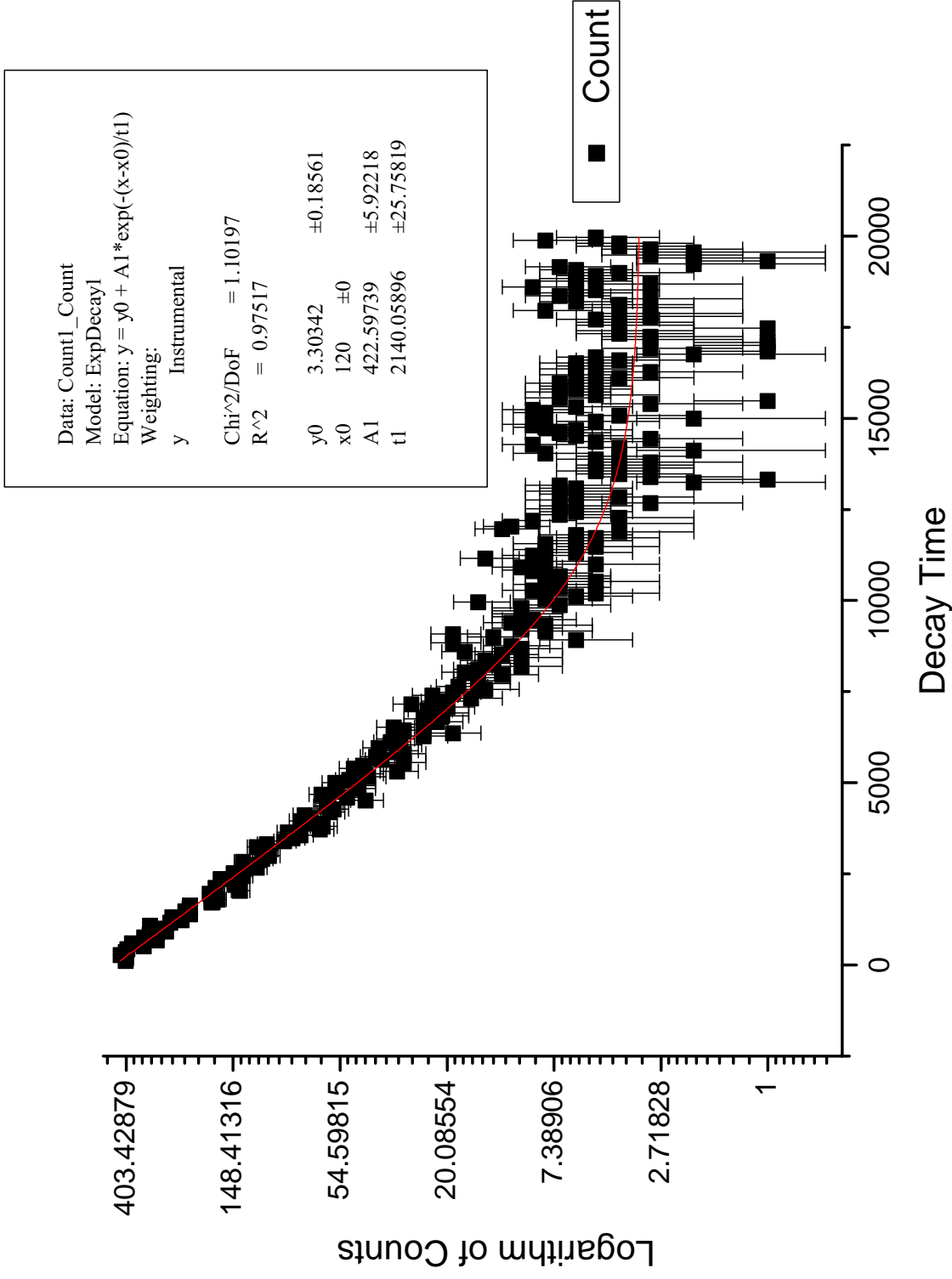
Figure 3: Semi-log plot of the number of decays against decay time. Note the increasing statistical uncertainty due to the scarcity of decays taking longer than $\sim 8000\,\mathrm{ns}$.

3.3 Muon Flux

We attempted to calculate the muon flux Φ at the location of our experiment using the equation

$$\Phi = \frac{N}{st},$$

where N is the number of muons counted, t is the time period over which they were counted, and s is the surface area of the scintillator, taken to be $589\,\text{cm}^2$. The total number of muons was measured as $6\,028\,715$ over a time span of $164\,\text{hr}\,10\,\text{min}\,46\,\text{s}$. The surface area was calculated to be $589\,\text{cm}^2$.

Uncertainty was taken based on uncertainty in the number of muons observed N. The standard uncertainty $\sqrt{N}$ and the estimated number of spurious events (about 2500 over the time observed) were summed in quadrature to find δN. The calculation was straightforward, taking

$$\delta\Phi \equiv \frac{\partial \Phi}{\partial N}\delta N = \frac{1}{st}\sqrt{N + 1250^2}.$$

The result was $1.0391 \pm 0.0005\,\text{muons}\,\text{min}^{-1}\text{cm}^{-2}$, which is close to the accepted value of about $1\,\text{muon}\,\text{min}^{-1}\text{cm}^{-2}$ at sea level.

A number of additional systematic uncertainties are also possible, however. The experiment was conducted in a building, and the material may interfere to some degree with the muons. Moreover, the experiment was conducted above sea level. The two aforementioned causes of uncertainty are working in the opposite direction; the building would be likely to decrease the number of muons that reached the apparatus, while the fact that the experiment is carried out above sea level means the muons have had less time to decay. Another cause of uncertainty is that one direction may be preferred over another for muon flux. For the accepted value measured at sea level, it is likely that the majority are assumed to be travelling downward, but much of our calculated surface area came from the sides of the apparatus. Few muons would be expected to enter from the bottom of the apparatus. Thus, the shape of the container may to some degree affect the measurements of flux.

3.4 Fermi Coupling Constant G_{F}

To find the Fermi coupling constant, we rework the equation

$$\tau = 192\frac{\pi^3 \hbar^7}{G_{\text{F}}^2 m^5 c^4}$$

to

$$\frac{G_{\mathrm{F}}}{(\hbar c)^3} = \sqrt{\frac{192\pi^3\hbar}{m^5c^{10}}},$$

where m is the muon mass, τ is again the lifetime, and everything else are accepted constants. Uncertainty is taken as a partial derivative of this function. The final result is that $G_{\mathrm{F}} = 1.172 \pm 0.004\,\mathrm{GeV}^{-2}$, close to the accepted value of $1.166\,\mathrm{GeV}^{-2}$.

4   Results and Discussion

Although several objectives comprised this experiment, the core of our analysis centered on our determination of the mean muon lifetime τ, for it was this value that proved the lone variable in our calculations of the Fermi coupling constant and the ratio of positive to negative muons. Statistically, the result of $2.14 \pm 0.03\,\mu\mathrm{s}$ is excellent; simply by assuming a uniform distribution to represent any background interference, our fit gives a coefficient of determination R^2 value of nearly 1—specifically 0.975. Without a doubt, our data demonstrate the exponential behavior of the muon decay distribution, validating the predicted theoretical behavior derived under the assumption of a uniform decay rate. Yet despite the excellent self-consistency of the collected statistics, the accepted value for τ of $2.197\,\mu\mathrm{s}$ fails to fall within the boundaries of our uncertainty. Nonetheless, this incongruity is an inherent consequence of the scintillator employed, for negative and positive muons behave differently in the presence of matter. Specifically, negative muons can react with protons in the plastic scintillator and vanish prior to the spontaneous decay characterized by the free space mean lifetime. Positive muons share no such potential for matter interaction. Therefore the decay times observed for our apparatus—which makes no distinction between oppositely charged muons—should prove less than the accepted value, for any negative muons incident upon the scintillator may conspire with matter and accelerate their demise. And so the smaller mean lifetime obtained in this experiment is just as expected as a consequence of such muon-matter interaction.

Although the general trend makes sense from purely qualitative considerations, the actual quantitative implications of our result surface in the calculation of the ratio of positive to negative muons, represented by ρ. Unfortunately, propagation of the error in τ yields an extremely large fractional uncertainty in ρ; with proper significant figures, it is 100% of the nominal value of 2. However, this uncertainty nonetheless encompasses the expected ratio, which lies between 1.2 and 1.4 according to the most recent information from the Particle Data Group.[5] For this reason, our calculated ρ matches

prediction, albeit with unimpressive precision. Thus more data must be collected in order to diminish the uncertainty in τ; the high sensitivity of ρ on τ prevents particularly meaningful calculations with the present level of precision.

Our calculated value of the normalized Fermi coupling constant $\frac{G_{\mathrm{F}}}{(\hbar c)^3}$, although close to the accepted value, fails to match within the stated uncertainty. But such deviation is intrinsic to the nature of the experiment. For as noted above, the mean lifetime gathered from this data represents the average over both positive and negative muons, with the latter decaying more quickly in the presence of scintillator atoms; therefore our τ should not be expected to match the mean lifetime in a vacuum. Yet it is precisely this free-space value which is specified in the equation for G_{F}. Thus our smaller mean lifetime gives an appreciably larger G_{F}—since G_{F} is proportional to the inverse square root of τ—and this proves no fault of the experiment itself. In fact, to match perfectly would imply that negative muons do not interact with matter in the apparatus, a false conclusion. Therefore this discrepancy is not only acceptable but required to conform to known muon behavior.

And our final objective of calculating muon flux—the only computation independent of the mean muon lifetime—also matches closely to expected values; accepted data for muon flux by terrestrial location is surprisingly sparse, however, and so we compared our calculation to the flux at sea level. Yet the merit of this result is admittedly open to dispute. Since our threshold on the discriminator voltage was chosen in order to eliminate non-muon events, the detector was set to record approximately the "right" number of muons, so the fact that the resultant flux matches accepted ranges is not remarkable. But since the total detections gathered cannot be easily sifted through as can the decay times themselves, flux calculations with the present measuring techniques will remain only approximate, and our value is definitely plausible for rough estimation.

The results of this experiment confirm the expected behavior for muon decay; the exponential distribution of decay times is unmistakable, and the resultant mean muon lifetime fits well with the accepted value—corrected, of course, for the accelerated decay of negative muons. While our flux calculation and determination of the Fermi coupling constant do not prove particularly valuable, due to the nature of the τ calculated, the apparatus's ability to determine the averaged muon lifetime is undeniably useful. With this parameter, the charge ratio of positive to negative muons can be uncovered, which—as we found—is an active and untamed field of modern research. And so without a doubt, this muon experiment has proven an instructive experience, providing a glimpse into the fascinating world of the muon—a fundamental building block of the universe.

Notes

[1]Frisch, David H. and James H. Smith, *Am. J. Phys.* **31** 342-355, 1963.

[2]T. Coan, T. Liu, and J. Ye, arXiv:physics/0502103v1 [physics.ins-det]

[3]T. E. Coan and J. Ye, "Muon Physics," `http://www.mathphys.com/muon_manual.pdf`

[4]D. Van Baak, "Conceptual Tour of Muon Physics," `http://www.teachspin.com/newsletters/Muon%20ConceptualIntroduction.pdf`

[5]C. Amsler, "Cosmic Rays," revised August 2009 by T. K. Gaisser and T. Stanev, `http://pdg.lbl.gov/2009/reviews/rpp2009-rev-cosmic-rays.pdf`