The Photoelectric Effect

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Abstract

The photoelectric effect is exploited for a simple determination of Planck's connstant which agrees closely with the usually accepted value.

Contents

1 Introduction

1.1 Purpose of the Experiment

Since Planck's introduction of light quanta into physics, his eponymous constant of proportionality of the energy of these quanta to their frequency has proven to be one of the most important physical constants for modern physics. We find it to be related to the fine-structure constant, to the Bohr magneton, even to the Avogadro constant. To precisely measure—or to at least estimate the magnitude of—this number is to gain an understanding of the nature of quantum effects around us.

1.2 Description and Theory of the Experiment

Planck supposed that we can assign to light photons an energy E proportional to their frequency f, i.e. $E = hf$ where the proportionality constant h is known as Planck's constant, on the order of 10^{-34} J·s. We should expect radio waves ($f = 30\,\mathrm{MHz} \to E = 1.2 \times 10^{-7}\,\mathrm{eV}$) to have a lower energy level than visible light ($f = 726\,\mathrm{THz} \to E = 3\,\mathrm{eV}$), which is less energetic still than x-rays ($f = 3 \times 10^{18}\,\mathrm{Hz} \to E = 12\,\mathrm{keV}$).

We can exploit the photoelectric effect to determine Planck's constant h by measuring the kinetic energy of the electrons ejected from a metal sample by incident photons. Einstein related the energy K of the ejected electrons to the photon frequency f by $hf = K + \phi$. Here, ϕ, the so-called *work function*, is an additive constant determined by the composition of the sample target. As an example, if a certain metal with a work function of $\phi = 2.5\,\mathrm{eV}$ is illuminated by monochromatic light of wavelength 350 nm, the maximum kinetic energy of the electrons ejected in the photoelectric effect is about half an electron volt. The work function is analogous to the ionization energy of an atom,[1] and represents the minimum energy required for an electron to be emitted. The work functions for common metals run from $\sim 2\,\mathrm{eV}$ for cesium to almost 6 eV for osmium.[2] Metals with low work functions, such as potassium ($\phi = 2.29\,\mathrm{eV}$), tend to be favored because the photoelectric effect can be observed using photons from the visible range of light.

Directly measuring the kinetic energy of electrons is difficult, so we determined K indirectly by using the emitted electrons to charge a small capacitor. When the potential difference V across the capacitor reaches a maximum (is fully charged) we can equate the energy of the electrons (charge e) with the kinetic energy they would gain by crossing the gap, i.e. $K = eV_{\mathrm{max}}$. Einstein's relation then becomes

$$hf = eV_{\mathrm{max}} + \phi.$$

So by measuring the stopping potential of the capacitor and using a known wavelength of light to eject electrons, we can measure determine h as the rate of change of the stopping potential with the inverse of frequency. In other words,

$$\frac{d}{d\lambda}\left[eV_{\mathrm{max}}\right] = ch,$$

where, λ is the wavelength of the incident light, and c is, of course, the speed of light.

2 Procedure

The first portion of the experiment included proper setup of the h/e apparatus and corresponding equipment. Figure 1, a schematic of the experimental mechanisms, can be referenced for clarification. Since the light sources in this lab required time to warm up, we first turned them on—both mercury and sodium vapor sources—and waited approximately 15 minutes before conducting measurements. While waiting, we blocked the rear openings of teh vapor souces and used a multimeter to measure the battery voltage of the h/e apparatus. It exceeded the recommended value of 17 V,[3] so we plugged the multimeter into the voltage output of the h/e apparatus. Thereafter, the light aperture assembly was secured on the Hg light source. The lens/grating assembly then connected to this, adjusted so that the emitted light was incident directly on the center of the lens. Finally, we attached the base of the h/e apparatus to the coupling bar of the Hg light source.

A hood was pulled over the experimental devices to eliminate external light interference, and adjustments ensued. The lens was moved on its support rods until it yielded the sharpest image of the light source aperture on the photodiode mask. In order to accomplish this, the cylindrical covering was opened to allow full viewing of the photodiode mask. After turning on the h/e apparatus, masurements followed.

We rotated the h/e apparatus so that the first-order spectral lines struck the exterior reflective mask. Deciding first to examine the blue wavelength's behavior, we adjusted the apparatus so that only this spectral line was incident on the photodiode mask, using the same procedure as in the previous paragraph. The cylindrical covering was returned, and a transmission filter was placed on the reflective mask so that the blue beam passed through the 100 % transmission window. We then pressed the red discharge button on the h/e apparatus, releasing it when the multimeter read 0 V. The potential reading rose, eventually arriving at a steady value; the result was deemed stable after one minute of no fluctuation. And after recording this voltage, the device was again discharged. But on this occasion we instead recorded the length of time required to reach the previously determined stopping potential; as before we ensured that the multimeter gave a constant value for a minimum of one minute. The transmission filter was subsequently moved so that the beam passed through its 80 % window, and the above procedure was repeated; this process continued for the 60 %, 40 %, and 20 % transmission windows. Thereafter, we rotated the h/e apparatus to examine the green spectral line. Its procedure proved identical to that above, except that a green filter was affixed to the external mask of the h/e apparatus underneath the transmission filter, in accordance with the lab manual's advice. In this way the stopping potential as a function of light intensity was determined for two separate wavelengths of light—completing the first part of the experiment.

The transmission and green light filters were then removed, and the h/e apparatus was aligned using the methodology noted earlier, this time so that the first-order ultraviolet line struck the window of the photodiode. We recorded the subsequent stopping potential on the multimeter, again waiting for a minute to ensure stability. And this process of alignment and measurement continued for each of the other four first-order spectral lines, with the addition of appropriate filters for the colors yellow green. Then the h/e apparatus was rotated, and each of the second-order lines received the same scrutiny—alignment and measurement—as their first-order counterparts. Finally, the sodium light source replaced the mercury one, and the above procedure was repeated for the only visible spectral line, a yellow doublet. And thus the in-lab work was completed.

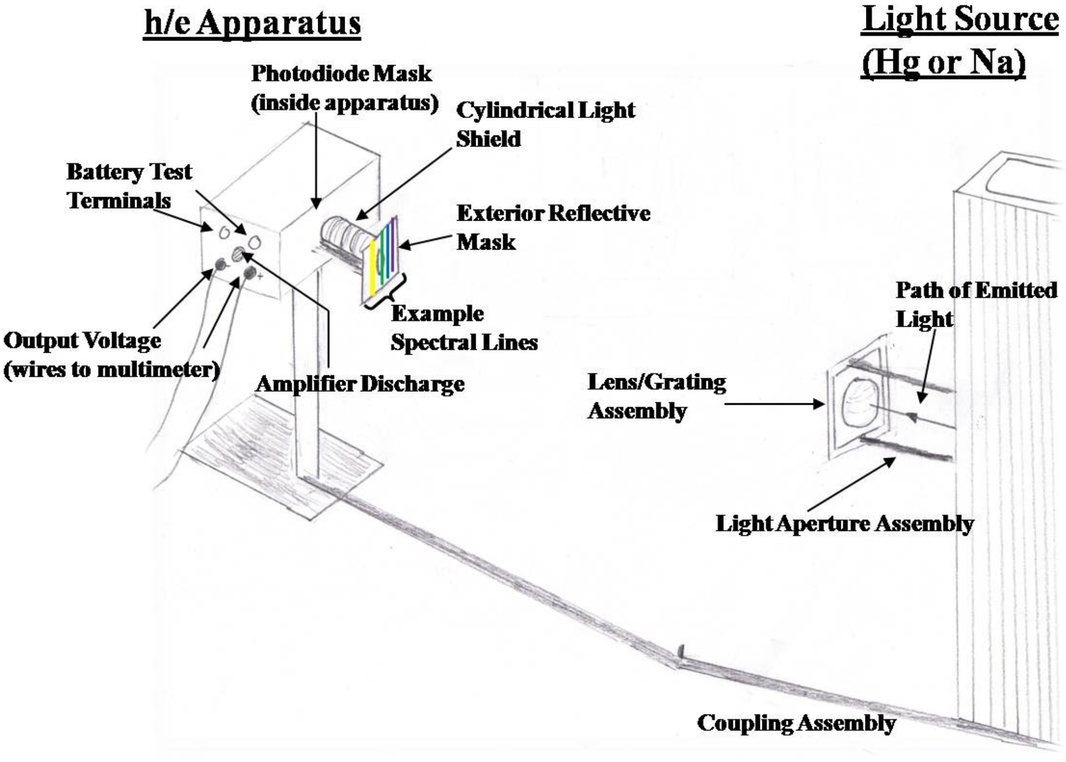


Figure 1: Schematic representation of experimental setup.

Table 1: Stopping Potentials at Various Relative Transmission Intensities

Relative Intensity $\frac{I}{I_0}$	$V_{\max}$ ($\lambda = 687.9\,\mathrm{nm}$) [V]	$V_{\max}$ ($\lambda = 549.0\,\mathrm{nm}$) [V]
1.0	1.556	0.951
0.80	1.549	0.949
0.60	1.549	0.943
0.40	1.544	0.927
0.20	1.526	0.894

3 Observations

3.1 Data

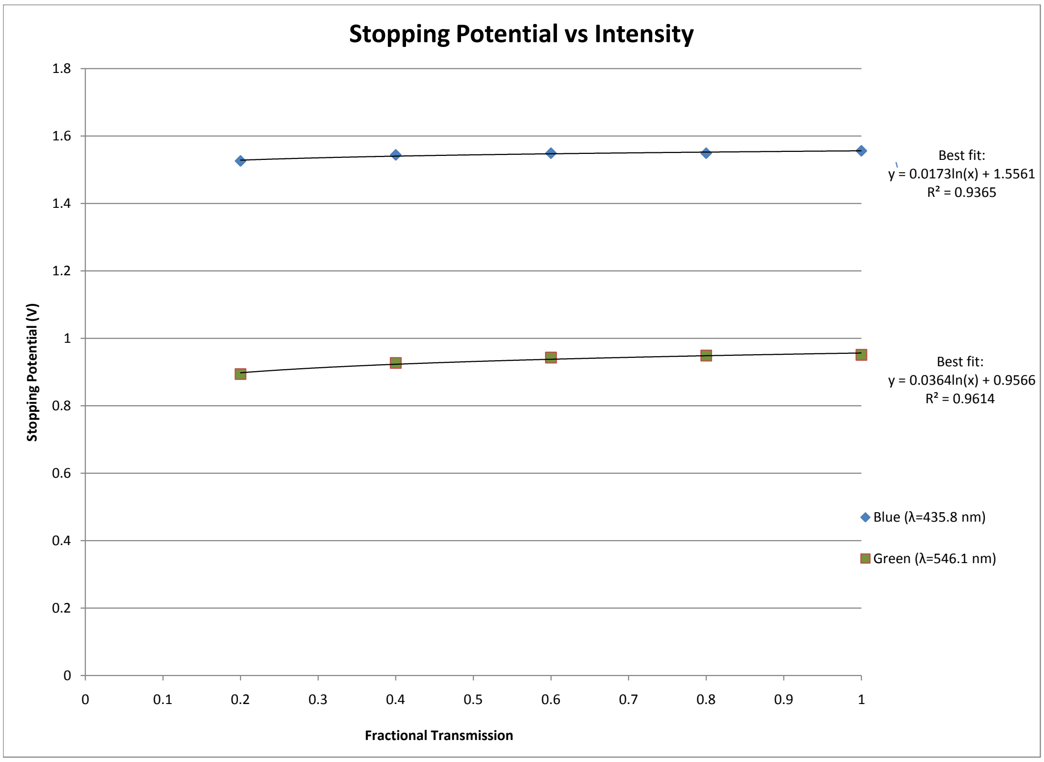
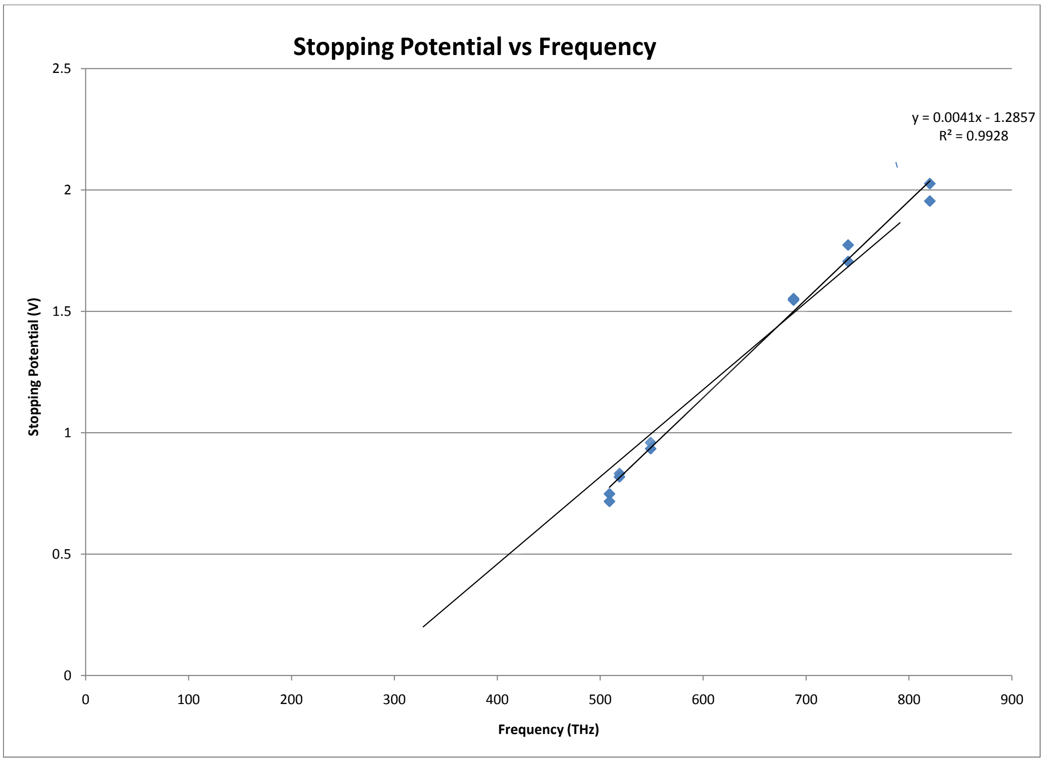


Table 2: Stopping Potentials for Several Incident Wavelengths

Wavelength λ [nm]	Stopping potential V_{max} [V] $(n = 1)$	V_{max} $(n = 2)$
820.3	2.026	1.954
740.9	1.773	1.705
687.9	1.552	1.546
549.0	0.959	0.934
518.7	0.832	0.818
509.0	0.748	0.717



3.2 Sample Calculations

Calculations were straightforward.[4] The best fit for the intensity curves was logarithmic, but at

$$I = 0.0173 \log\left(\frac{I}{I_0}\right) + 1.5561 \quad \text{(coefficient of determination } R^2 = 0.9365\text{)}$$

and at

$$I = 0.0364 \log\left(\frac{I}{I_0}\right) + 0.9566 \quad (R^2 = 0.9614)$$

both curves were nearly horizontal (where I is the intensity of transmitted light, and $\frac{I}{I_0}$ is obviously the fractional transmission). A best-fit line (again with a coefficient of determination near unity) for the wavelength experiment to solve Einstein's Nobel-prize winning equation for stopping potential V_{max} in terms of the fundamental constants h and e and the material's work function ϕ

$$V_{\text{max}} = \frac{h}{e}f - \frac{\phi}{e},$$

yielding $\phi = 1.29\,\text{eV}$ and $h = 6.56 \times 10^{-34}\,\text{J} \cdot \text{s}$ after taking the accepted value of $e = 1.602 \times 10^{-19}\,\text{C}$.

For uncertainty, the equations $\sigma_h = \sigma_{V_{\text{max}}} \sqrt{N/\Delta}$ and $\sigma_\phi = \sigma_{V_{\text{max}}} \sqrt{\sum f^2/\Delta}$, where $\Delta = N \sum f^2 - \left(\sum f\right)^2$, where $N = 12$ since there were twelve measurements, where f was the frequency, and where V_{max} was the measured stopping potential. To calculate $\sigma_{V_{\text{max}}}$ we followed the formula

$$\sigma_{V_{\text{max}}} = \sqrt{\left(\frac{1}{N-2}\right) \sum V_{\text{max}_i} + \phi - \frac{h}{e}f}.$$

3.3 Uncertainty Analysis

There were a number of causes for uncertainty, but most were relatively minor. Essentially the same uncertainties existed for each experiment. The material used to obtain the electrons was not an elemental substance, and there is some chance that the work function may not be compltetely homogeneous in the material. Additionally, there was some spreading of light from individual spectra, causeing very slight imperfections in measurement. Most of this, however, was accounted for by using light filters. Some degree of uncertainty exists in the exact wavelengths of the atomic spectra themselves. There was a small degree of systematic error, because the resistance in the zero gain amplifier, while high, is not infinite, so some charge was able to leak off. This was especially apparent in the intensity experiment, where the stopping potential dropped off slightly at low transmission percentages.

Most of these uncertainties were very small compared to the uncertainty in the stopping potential. The voltmeter has a precision of but one millivolt, and there were some slight fluctuations. There were slight movements of the apparatus as it was handled during the experiment, which seemed to cause a marginal change in voltage readings. Moreover, though enclosed in a dark setting, some ambient light from the room was allowed into the experimental environment, so there may be some minor effects from ambient light. However, the readings from the voltmeter seemed fairly consistent with a high degree of precision, so the uncertainty for each measurement was taken to be only $\pm 2\,\text{mV}$.

The real numerical uncertainty, however, came from the scattering of data about the regression line (and the fact that there were just twelve data points). The uncertainty $\sigma_{V_{\text{max}}}$ calculated from the greatly superseded the uncertainty measured from the data, and by far contributed the most to measured uncertainties in h and ϕ/e.

4 Results and Discussion

The results of this experiment thoroughly match quantum theory. And they do so twice: first by contradicting classical theory, and second by corroborating the quantum alternative. The first portion of the laboratory addressed the former, and its data diverge significantly from classical prediction. For whereas classical theory projects a strong linear correlation between stopping potential $V_{\max}$ and incident light intensity, our data instead show almost no relationship. That is, the stopping potential remains virtually constant independent of intensity. Admittedly, the experimental curve actually displays a slight logarithmic relationship, not the horizontal line of quantum theory, but his is no cause for concern. At lower intensities the measured stopping potential decreases because of capacitor discharge. The decreased level of photoelectric current cannot charge the circuit's capacitance as quickly, and thus the capacitor has more time to release its charge as current ghrough the amplifier: the lower the photoelectric current, the lower the equilibrium potential. Moreover, this small correlation pales in comparison to that of the wavelength, for the difference between average stopping potentials of the green and blue curves is much greater—by more than a factor of 10—than the differences resulting solely from intensity variation.

Nor does the variation in charge time with intensity contradict the quantum model. While $V_{\max}$ whould be independent of intensity, the photoelectric current should not; greater intensity means that more electrons are emitted, and thus current and intensity should possess a positive correlation. The voltage across a capacitance is proportional to the time integral of the current through it. And therefore greater current implies less charge time to reach a given potential. Although it would prove difficult to use the rough data from the lab to display this relationship quantitatively, our results nonetheless reveal this expected trend.

The data from the second part of this experiment also lend credence to the quantum model, but in this case with explicit numerical confirmation. Quantum theory predicts a linear relationship between $V_{\max}$ and frequency of incident light—offset by a constant unique to the material, known as the work function. Our data show such a relationship, and the experimental value of Planck's constant h matches the accepted value of 6.63×10^{-34} J·s within experimental error: ours is $(6.6 \pm 0.2) \times 10^{-34}$ J·s. Likewise, the measured work function falls within the predicted range of values for the h/e apparatus used in the lab: our determination is 1.3 ± 0.2 eV, while the manufacturer gives a range of 1.3 ± 0.08 eV.[5] The model provided by quantum theory is thus confirmed by our experimental data; it is light frequency—not intensity—that determines the kinetic energy of emitted electrons.

Therefore the results of this experiment fully endorse the quantum model of light, so the lab proves entirely successful in uniting theory and experiment. For this reason, no improvements seem necessary for future measurements. But if greater precision is required, the experiment should instead be conducted in a completely dark environment, for stray light did interfere during our procedure, even with the addition of a hood over the devices. And a more precise voltmeter could be employed to further limit uncertainty. Yet the experiment as it is proves absolutely sufficient for our purposes. In it the success of the quantum theory of light shines brightly through, illustrating a historic achievement of the quantum revolution.

Notes

[1] T. Engel and P. Reid, *Physical Chemistry* 2ed., 2010.

[2] From the Wikipedia: `http://en.wikipedia.org/wiki/Work\_function`

[3] P. LeClair, "PH 255: Modern Physics Laboratory," 101–114, Spring 2010.

[4] For discussion of the techniques used here, see J. R. Taylor, *An Introduction to Error Analysis*, 2ed.

[5] PASCO Support's Tech Note 303 Detail reports that the photocathode is made of Cs_3Sb. See `http://www.pasco.com/support/technical-support/technote/techIDlookup.cfm?TechNoteID=303`