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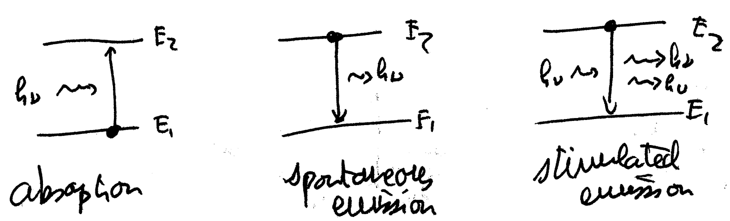
Optics Exam II

1. **(a)** What are the 3 main types of lasers? Briefly discuss and compare their principal characteristics? **(b)** What are the two main physical conditions on the laser system that must be satisfied for lasing to occur? If you do not know the exact terminology, try to explain it.

Solution: **(a)** The three main types of lasers are *gas*, *solid state*, and *semiconductor*. One may contrast them by noting that gas lasers typically have the largest size, the highest power, and lowest efficiency, while semiconductor lasers typically have the smallest size, lowest power, and largest efficiency.
(b) The two main physical conditions for lasing to occur are population inversion (gain) and the presence of a cavity for optical density enhancement.

2. **(a)** Describe the 3 optical transitions that can occur in a laser. You can use diagrams to illustrate. **(b)** Express these transitions using the Einstein coefficients. **(c)** Express the two conditions of lasing using the Einstein coefficients.

Solution: **(a)** Here is a quick drawing:



In the first, absorption, a single photon of energy $hf = E_2 - E_1$ induces an electron to transition upward from level E_1 to E_2. In the second, spontaneous emission, an electron in the excited state E_2 transitions to the lower state E_1 while emitting a photon of energy hf. In the third, stimulated emission, an incident photon of energy hf induces an electron in the excited state E_2 to transition downward to the lower state E_1, which results in the emission of a second photon of energy hf.

(b) Let N_1 and N_2 be the number of atoms in states 1 and 2, respectively, and $u(\nu)$ the density of radiation of the incident field at frequency ν. The rate of chance of N_1 can then be written

$$\frac{dN_1}{dt} = -B_{12}N_1u(\nu) \qquad \text{stimulated emission} \tag{1}$$

$$\frac{dN_1}{dt} = A_{21}N_2 \qquad \text{spontaneous emission} \tag{2}$$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt} = B_{21}N_2u(\nu) \qquad \text{absorption} \tag{3}$$

$$B_{12} = B_{12} \tag{4}$$

(c) Population inversion: stimulated emission rate greater than absorption rate

$$B_{21}N_2u(\nu) > B_{12}N_1u(\nu) \tag{5}$$

$$B_{21}N_2 > B_{12}N_1 \tag{6}$$

$$N_2 > N_1 \qquad \text{since } B_{21}=B_{12} \tag{7}$$

Cavity enhancement: simulated emission rate greater than spontaneous emission rate

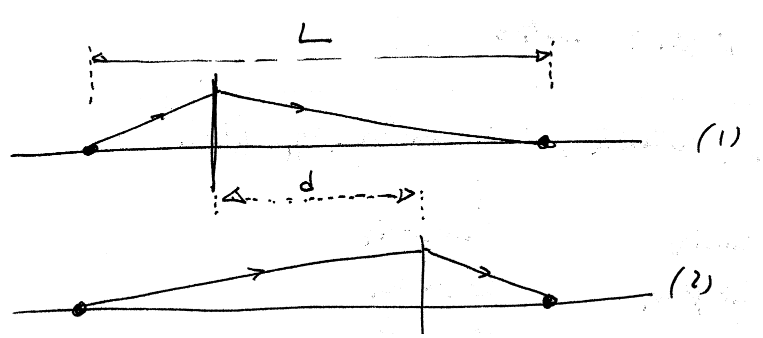
$$B_{21}N_2u(\nu) > A_{21}N_2 \tag{8}$$

$$\Longrightarrow \qquad u(\nu) > \frac{A_{21}}{B_{21}} \tag{9}$$

3. A convenient way to measure the focal length f of a positive thin lens is to make use of the following fact. If a pair of object and image points are separated by a distance $L > 4f$, there will be two locations of the lens for which the same pair of object/image are associated. Let this distance be d. Show that:

$$f = \frac{L^2 - d^2}{4L} \tag{10}$$

Solution: Consider the figure below illustrating the two locations:



The two situations are symmetric because of the reversibility of the optical ray. From the upper figure,

$$\frac{1}{f} = \frac{1}{\frac{1}{2}(L-d)} + \frac{1}{L-\frac{1}{2}(L-d)} \tag{11}$$

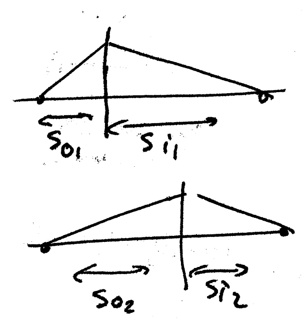
Simplifying,

$$\frac{1}{f} = \frac{2}{L-d} + \frac{2}{L+d} \tag{12}$$

$$\frac{2}{f} = \frac{L+d+L-d}{L^2-d^2} \tag{13}$$

$$\implies \qquad f = \frac{L^2-d^2}{4L} \tag{14}$$

There is a somewhat longer and more fussy approach, of course. Consider the two generic situations below.



If the two situations are to have the same focal length,

$$\frac{1}{f} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \tag{15}$$

Let the total distance from image to object be L, and recall that d is the separation of the two lens positions.

$$L = s_{o1} + s_{i1} = s_{o2} + s_{i2} \tag{16}$$

$$d = L - s_{o1} - s_{o2} \tag{17}$$

Substituting into our lens equation,

$$\frac{1}{f} = \frac{1}{s_{o1}} + \frac{1}{L-s_{o1}} = \frac{1}{s_{o2}} + \frac{1}{L-s_{o2}} = \frac{1}{L-d-s_{o1}} + \frac{1}{d+s_{o1}} \tag{18}$$

Now eliminate s_{o1}:

$$\frac{L}{s_{o1}\left(L - s_{o1}\right)} = \frac{L}{\left(L - d - s_{o1}\right)\left(d + s_{o1}\right)} \tag{19}$$

$$s_{o1}\left(L - s_{o1}\right) = \left(L - d - s_{o1}\right)\left(d + s_{o1}\right) \tag{20}$$

$$s_{o1}L - s_{o1}^2 = Ld + Ls_{o1} - d^2 - ds_{o1} - s_{o1}d - s_{o1}^2 \tag{21}$$

$$2ds_{o1} = Ld - d^2 \tag{22}$$

$$s_{o1} = \frac{L - d}{2} \tag{23}$$

Substituting into the first part of Eq. 18, we arrive at the result:

$$\frac{1}{f} = \frac{2}{L - d} + \frac{2}{L + d} \tag{24}$$

$$\frac{1}{f} = \frac{L^2 - d^2}{4L} \tag{25}$$

4. The headlights on automobiles are approximately 1.5 m apart. If the diameter of the pupil of the eye is 3 mm and an average wavelength of visible light is $\lambda = 500$ nm, what is the maximum distance at which the headlights will be resolved (i.e., appear as two separate lights) assuming that diffraction effects at the circular aperture of the eye are the limiting factors? Note that for a circular aperture the resolving power is $\alpha = 1.22\lambda/D$ where D is the diameter of the hole.

Solution: The minimum angle the eye can resolve owing to its circular aperture is α, which means that the angle the two headlights make with the center of the eye must also be α at minimum. If the headlights are a distance R away, and the separation of the headlights is h, the angle the headlights form is approximately (assuming α is small)

$$\sin\alpha \approx \alpha \approx \frac{h}{R} \tag{26}$$

One can either assume the triangle is roughly a right triangle of side h opposite angle α and hypotenuse R, or approximate h as the arc length of a circle of radius R over an angle α. Equating this to the minimum angle resolvable by the eye, the minimum distance at which one can resolve two separate headlights is

$$\alpha = 1.22\frac{\lambda}{D} \approx \frac{h}{R} \tag{27}$$

$$R \approx \frac{Dh}{1.22\lambda} \approx 7.4\,\text{km} \tag{28}$$

5. You are looking through a piece of square woven cloth at a point source ($\lambda = 600$ nm) 20 m away. If you see a square arrangement of bright spots located about the point source, each separated by an apparent

nearest-neighbor distance of 12 cm, how close together are the strands of cloth?

Solution: We need only consider the spacing of the fibers along one axis, since if the cloth has a square weave the spacing will be the same along either axis. Or, if you like, since the two fiber axes are orthogonal, diffraction from one set of fibers can't affect what happens due to fibers running in other direction. The interference condition for a fiber spacing d is

$$n\lambda = d\sin\theta \tag{29}$$

where θ is the angle at which the spots are observed, and n is the diffraction order. The spacing between adjacent spots is thus

$$\sin\theta_{n+1} - \sin\theta_n = \frac{\lambda}{d} \tag{30}$$

Given a spacing of 12 cm at a distance 20 m, the angles involved are very small, and we may approximate $\sin\theta \approx \theta$ for at least the first several order spots:

$$\sin\theta_{n+1} - \sin\theta_n \approx \theta_{n+1} - \theta_n = \frac{\lambda}{d} \tag{31}$$

Thus, the spots are evenly spaced at angles of $\Delta\theta \approx \lambda/d$. In order to find the spacing of the fibers, we can consider only the first order spot. If the horizontal distance to the first order spots is R and their separation along the lateral (orthogonal) direction from the direct beam is h, then

$$\sin\theta_1 \approx \tan\theta_1 \approx \frac{h}{R} \tag{32}$$

$$\sin\theta_0 = 0 \tag{33}$$

$$\Delta\theta \approx \frac{h}{R} \approx \frac{\lambda}{d} \tag{34}$$

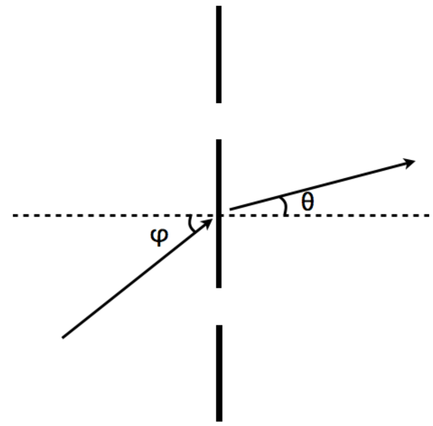
Thus, the spacing d is

$$d \approx \frac{\lambda R}{h} = \frac{\left(6\times10^{-7}\,\text{m}\right)(20\,\text{m})}{0.12\,\text{m}} = 10^{-4}\,\text{m} = 100\,\mu\text{m} \tag{35}$$

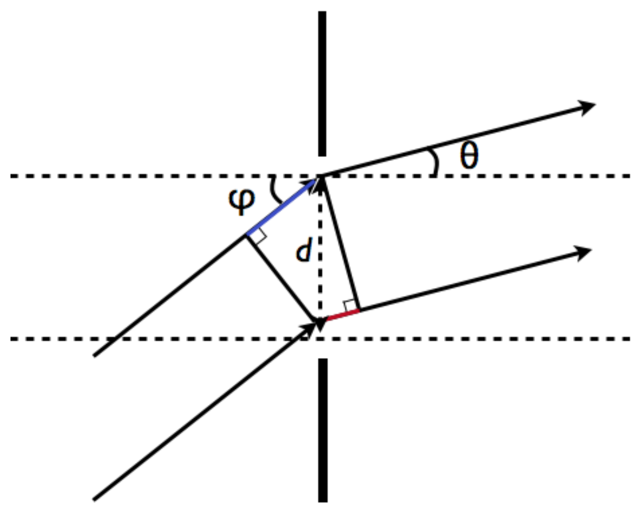
6. Light of wavelength λ is obliquely incident on a pair of narrow slits separated by a distance d (see figure below). The angle of incidence of the light on the slits is φ. Show that the diffracted light emerging at an angle θ interferes constructively if

$$|d\sin\theta - d\sin\varphi| = 0, \lambda, 2\lambda, \ldots \tag{36}$$

Solution: We may assume that the incident light is arriving from far enough away that the incident rays/waves are parallel. The key is to recognize that there are two path differences in this problem: the



upper ray travels an extra distance in getting to the slits, the lower travels an extra distance after going through the slits. This is shown in the figure below: the upper ray travels the extra distance indicated by the blue line, while the lower slits travel the extra distances indicated by the red line. The total path difference for the two slits is the difference between these two distances. (In the figure we have eliminated the portion of the screen between the two slits for clarity.)



The blue segment has length $d \sin \varphi$, and the red $d \sin \theta$. The total path difference for the two incident rays is thus

$$d \sin \varphi - d \sin \theta \tag{37}$$

This total path difference must be an integral multiple of the incident wavelength λ for constructive interference, and thus

$$|d \sin \theta - d \sin \varphi| = 0, \lambda, 2\lambda, \ldots \tag{38}$$

Here the absolute value just signifies that it doesn't matter whether the angles are positive or negative, the problem is symmetric about the horizontal axis.